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PRACA DOKTORSKA

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GEOMETRY CHANGES IN THE ELASTIC-PLASTIC ANALYSIS OF STEEL SPATIAL SKELETAL STRUCTURES

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- To my parents
  - To my brothers
    - To my wife
      - To my children

I offer this modest effort

HOSAIN

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Glossary of Symbols

ξ	Load multiplier ($\xi=1$ for reference domain)
ξ_E	Elastic-limit multiplier
ξ_{SD}	Shakedown multiplier
ξ_L	Limit-load multiplier
Ω_R	Reference domain
$\partial\Omega_E$	Elastic-limit surface
$\partial\Omega_{SD}$	Shakedown surface
$\partial\Omega_L$	Limit load surface
β_i	Time scalar parameters (loading process)
$\underset{\sim}{\lambda}$	Plastic multipliers (i.e., measure of plastic flow)
$\underset{\sim}{\sigma}$	Stresses
$\underset{\sim}{\varepsilon}$	Strains
$\mathbf{p}$	Generalized load vector
$\mathbf{q}$	Generalized strains vector
$\mathbf{q}^e$	Elastic part of the total strain
$\mathbf{q}^p$	Plastic part of the total strain
$\mathbf{s}$	Generalized stress vector
$\mathbf{s}^e$	Elastic part of total stress
$\mathbf{s}^r$	Residual part of stress
$\mathbf{u}$	Generalized displacements
$\mathbf{u}^e$	Elastic part of the total displacement
$\mathbf{u}^r$	Residual part of displacement
$\mathbf{d}$	Elastic stress vector
$\mathbf{f}$	Yield functions
$\mathbf{k}$	Vector of yield moduli (plastic capacities)
$\mathbf{C}$	Compatibility matrix
$\mathbf{C}^T$	Equilibrium matrix (i.e., transpose of $\mathbf{C}$)
$\mathbf{E}$	Elasticity matrix
$\mathbf{H}$	Hardening matrix
$\mathbf{K}$	Stiffness matrix of assembled structure
$\mathbf{N}^c$	Matrix of exterior unit normals (cross-section)
$\mathbf{N}^{(e)}$	Matrix of exterior unit normals (element)
$\mathbf{N}$	Matrix of exterior unit normals (structure)
$\dot{(\ )}$	Time derivative (rate)
$(\ )^T$	Matrix transpose
$\underset{\sim}{(\ )}$	Vector (under Greek symbol)

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GEOMETRY CHANGES IN THE ELASTIC-PLASTIC ANALYSIS OF STEEL SPATIAL SKELETAL STRUCTURES*

Summary

Principal objective of this study is to furnish a tool for a unified formulation and numerical analysis of all the history-independent classes of response of elastic-perfectly plastic space skeletal structures. We aim at determination of load domains corresponding to *elastic*, *shakedown* and *collapse* behaviour for structures in initial and deformed configurations.

Numerical program SDLAS for post-yield and inadaptation (non-shakedown) analysis of elastic-plastic space skeletal structures has been prepared. It accounts for biaxial bending, torsion and axial forces contributing to yielding of beam elements. The program permits to determine a safe domain for either proportional or variable repeated loads.

The proposed method accounts for nonlinear geometrical effects. In the post-yield analysis the conditions for plastic flow are considered at subsequent deformed configurations. For the inadaptation analysis the similar procedure in applied. Automatic generation of all possible plastic deformation modes allows to select the most dangerous mechanisms and to determine a critical plastic deformation path.

Finally, the post-yield or inadaptation curve is obtained showing the dependence of load (load domain) multiplier on the plastic deformation.

These problems are formulated as a sequence of linear programing tasks and are solved by a step by step procedure.

The proposed method allows to determine the limit multiplier and the shakedown (adaptation) multiplier accounting for geometric effects.

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CHAPTER 1

INTRODUCTION

1.1 Inelastic behaviour of structures and technological motivation of the thesis

Accounting for inelastic properties of materials permits the structural analysis to simulate better the structure behaviour. It leads to a more realistic assessment of the safety of the structure designed and, therefore, permits to reduce the safety margins needed with respect to design limit states. The above is obvious and generally acknowledged, especially when besides service loads the structure may undergo excessive (catastrophic) overloading.

To make our further analysis clear, let us recall some main phases of the structure response under increasing load intensity. Then are visualized symbolically in the Fig. 1.1. The following is restricted to problems where the time may be considered as a parameter determining only the order of events, without any particular physical meaning. The types of the structure response may be specified as follows:

1. Elastic response - terminating at an *Elastic Limit* load;
2. Elastic-plastic behaviour (constrained plastic flow) with:
   either (2a) elastic response under repeated loading:
   * for one-parameter loading - occurs always (in the absence of plastic deformation at unloading),
   * for multi-parameter loading - in a restricted domain limited by the *Shakedown Load*,

   or (2b) repeated plastic deformation (inadaptation), i.e.:
   * alternating plasticity, leading to *Low-Cycle Fatigue*,
   * ratcheting, leading to *Incremental Collapse*
3. Unconstrained plastic flow (immediate plastic collapse) appearing for perfectly plastic material under *Limit Load* (*Collapse Load*)
4. Post-Yield behaviour of perfectly plastic structures in the presence of geometrical hardening, terminating at *Ultimate Load* (unstable behaviour or rupture).

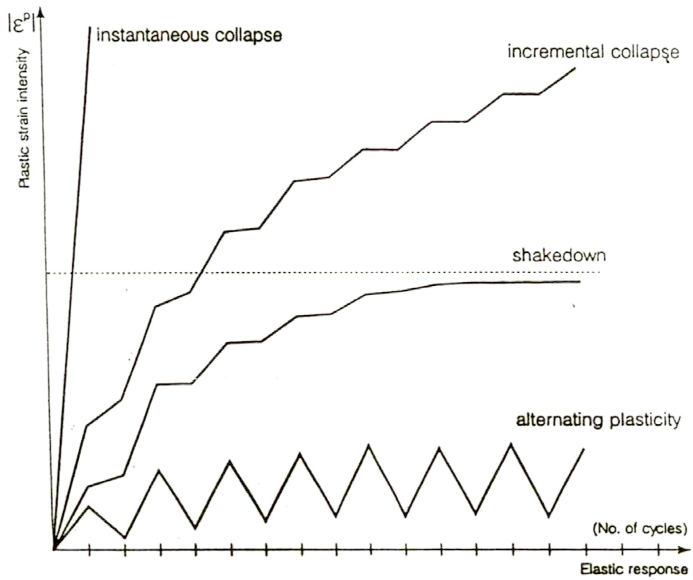


Fig. 1.1 Types of behaviour of elastic-plastic structures

Analysis of the last type of the structure behaviour requires taking into account geometrical non linearities, first of all considering equilibrium of the structure in the deformed configuration. The geometrically non linear analysis may be needed, of course, even for purely elastic slender structures.

Accounting for physical and/or geometrical non linearities results in serious complication of the analysis that made it, in the past, of little utility for real complex structures. However, computation facilities (hardware, FE techniques, incremental procedures) available nowadays permit an analysis of large-scale engineering systems involving even very sophisticated constitutive laws, large deformations and complex boundary conditions.

However, all the procedures aiming to solve non linear problems have an important weak point: their history dependence. In many practical cases and probably in majority cases of civil engineering structures the loading history is unknown and only the domain for an arbitrary variation of loads may be specified. This fact makes very difficult and expensive, or frequently impossible, a direct analysis needed for a realistic design.

However, in addition to the classical domain of linear elastic response, two classes of inelastic behaviour of mechanical systems are independent of the loading history. These are unconstrained plastic flow and adaptation of the structures. It means that problems of the limit analysis and shakedown theory may be solved without knowledge of the loading path. Hence, results of these theories may be relatively easily implemented in the engineering practice.

The simplest and effective approach is, in these conditions, limit analysis. However, it gives incomplete information concerning deformation. Therefore, using it in the design practice raises serious objections of engineers because of a supposed danger of excessive deflections and/or of the material degradation due to the low-cycle fatigue. That is why such an easy tool may be accepted only if the safety margin against an immediate collapse under monotonic load is looked for. To complete the limit analysis results the incremental elastic-plastic analysis is needed. Unfortunately, it is history-dependent; it is neither always feasible nor reliable, since all the possible loading paths should be considered. In these conditions the shakedown theory appears advantageous. It permits to determine the safe domain that the structure will adapt to. An arbitrary load variation within this domain induces a purely elastic response, due to residual stresses generated in the initial elastic-plastic process. It means that the shakedown-based design insures elastic behaviour of the structure under loads exceeding its elastic-limit values.

If the load exceeds the safe (shakedown) domain, inadaptation appears. It means that the structure will collapse or became unserviceable because of the accumulation of plastic deformation leading to local rupture (*low cycle fatigue*) or to infinitely increasing displacements of the structure (*incremental collapse*). This process may be delayed or stopped by a sufficient strain-hardening of the material. On the other hand, geometry changes due to deformations may induce either a geometrical hardening or softening of the structure. That will result in expanding or shrinking of the safe (*shakedown*) or prior-to-collapse domains of loads variation. Therefore, the structure may fail by inadaptation or immediate collapse before the respective limit loads were attained. On the other hand, the (hardening) structure may sometimes withstand loads largely exceeding the limit values determined using geometrically linear approach. The above is shown in Fig. 1.2 symbolically, for the case of one-parameter loading of a perfectly plastic structure.

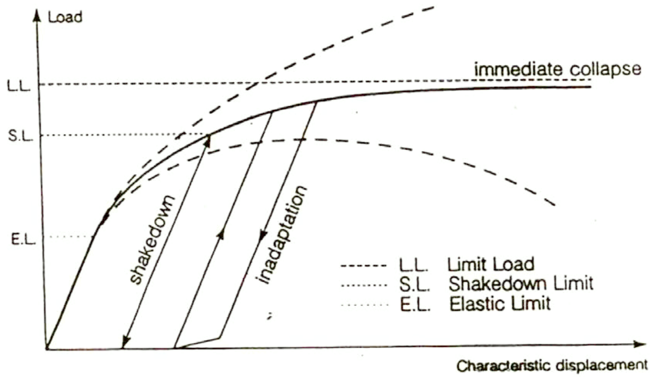


Fig. 1.2 Proportional loading-unloading of an elastic-perfectly plastic structure

Therefore, an information on the post-yield and inadaptation behaviour is needed, if our design is to be safe and realistic. The engineers' interest in this problem is exemplified in the so called P-Δ effect in portal frames (flexure increase due to horizontal displacements of their upper nodes), extensively studied in elastic and elastic-plastic domains. Unfortunately, each type of the behaviour is history-dependent when the geometrical non linearities are not negligible. That is why effective results concerning arbitrarily varying loads are until now rare and more extensive study of this problem seems necessary.

1.2 Limit analysis and post-yield behaviour

Limit analysis theory assumes initial undeformed geometry of the structure. The limit state, i.e., the limit load and the corresponding stress and velocity fields, appears then independent of the loading history and from elastic properties of the material (see, e.g., Sawczuk, [107]); therefore, its model may be assumed rigid-perfectly plastic. Such an interpretation contributes to mathematical clearness of the theory (Hill, [47]) but raises objections of engineers. Another interpretation of the collapse state considers it as a limit of elastic-plastic process (Prager, Hodge, [97]). However, this state of "unconstrained plastic flow" may be attained at considerable (sometimes infinite) displacements. That raises doubts on the soundness of the assumption of geometrical linearity of the problem.

That is why numerous attempts at extending the limit analysis to the domain of finite displacements were undertaken from its very beginnings. The so called Post-Yield Behaviour (**PYB**) approach was applied first probably by Onat [88]. In this approach the classical limit analysis is applied to a deformed configuration of the structure. This configuration corresponds to continuation of the incipient collapse mode at finite deflections. At each stage of the deformation process collapse load is determined corresponding to an instantaneous plastic flow of the structure. Thus, the geometrically non linear problem is replaced by a sequence of linear problems. In this way the load corresponding to a steady (inaccelerated) plastic flow of the structure may be plotted against its characteristic displacement Fig. 1.3.

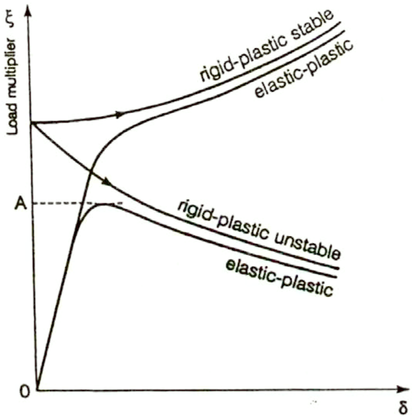


Fig. 1.3 PYB: Load intensity versus deflection δ.

Increasing curve indicates the so called geometrical hardening (by analogy to strain-hardening) and is well seen in the behaviour of plastic plates (see, e.g., [103]). The rigid-plastic increasing curve has small initial slope and relatively well approximates the real behaviour. Destabilizing effects may be more important and the decreasing curve (geometrical softening) has frequently a very steep slope (see, e.g., [54]). The peak load (ultimate load A in Fig. 1.3) may be considerably lower than the initial collapse load and than the current rigid-plastic collapse load corresponding to A.

This approach was extensively developed for arches [82], [89], plates [107], [54] and frames [30], [51], [52] and some extensions of the limit analysis theorems were proposed.

Based on the Hill's stability criterion [48] the condition for stability of initial plastic collapse process was derived [29] that permits analytical determination of the initial sensitivity of the limit load to displacements for simple structures. The latter problem was recently reconsidered by Gao [34] who seems to be unaware of Duszek's contributions [29], [31]. For more complex structures the stability should be studied by a sequential step-by-step analysis at specified deflections.

Taking into account changes in the collapse mode at increasing displacements was possible analytically only for simple structures [42], [55] but development of numerical techniques and, first of all, application of the mathematical programming permits a continuous modification of the modes (see, e.g., [19]). Moreover, because of the duality of the linear programming problem and its direct relation to the limit analysis theorems (see, e.g., Maier [75]) we obtain the exact mechanism at each deformation stage. Therefore, following [29] the theorems may be considered to hold also at finite displacements.

1.3 Shakedown analysis

Shakedown (*adaptation*) of the structure means that the plastic strain energy dissipated during the whole process is bounded, i.e., after a certain elastic-plastic process plastic increments must vanish. This physical effect prevents occurrence of two different phenomena shown in Fig. 1.1: unlimited progressive growth of the structure displacements (*incremental collapse*) and local cyclic plastic straining (*alternating plasticity*).

To illustrate the phenomenon of adaptation and its limit by incremental collapse, let us recall experimental results for a portal frame made of a mild steel [3] subjected to repeated load cycles consisting of four stages as in Fig. 1.4.

The vertical deflection δ of the beam is drawn against the number of cycles for various load intensities P. One can see that at loads less than the critical value the deflection δ stabilizes after each cycle of increased intensity. At P=430 N the deflection stabilizes but very small change of the load intensity results in a dramatic change of the structural response: the deflection does not stabilize any more.

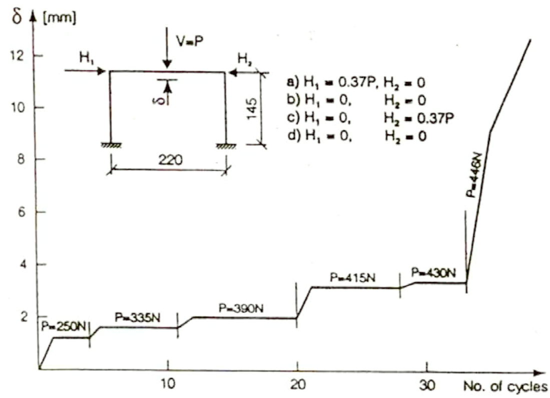


Fig. 1.4 Test of a portal frame:
Adaptation/inadaptation to load cycles of increasing intensity

The phenomenon of appearance of residual stresses in beams undergoing elastic-plastic bending was known to engineers from the very beginning of studies in plastic behaviour. These stresses insure elastic behaviour of the cross-section at reloading up to the maximal moment attained previously, if the range of moment variation does not exceed the double of the elastic limit moment. In 1932 Bleich [5] has taken into account this phenomena together with residual moments generated by plastic deformations of hyperstatic beams and formulated a criterion for adaptation of these structures. In 1938 Melan [79] formulated this criterion in general terms for an arbitrary elastic-plastic body. His theorem became a basis for the shakedown theory and is the fundamental tool for the analysis of the structure under variable loads.

The *Melan's theorem* (*theorem on adaptation*) states:
The structure will adapt to any loading path in a prescribed domain of load variation, if there exists such a time-independent residual stress field $\rho_{ij}(x)$ that the yield condition $\varphi(\sigma_{ij})=k$ is never violated under these stresses acting together with the stresses $\sigma^E_{ij}(x,t)$ determined in the structure considered perfectly elastic. That reads:

$$\varphi\left[s\left(\sigma^E_{ij}(x,t)+\rho_{ij}(x)\right)\right] \leq k \tag{1.1}$$

The kinematic approach was first proposed by Neal [83] in 1950 for frames and in 1956 Koiter [59] formulated a general kinematic theorem (*theorem on inadaptation*). It states:

The structure will not adapt to a certain loading program if there exists such a cycle of plastic strain rates $\dot{\varepsilon}_{ij}(x,t)$ resulting in compatible strain increments that plastic dissipation over this cycle is inferior to the work of stresses $\sigma^E_{ij}(x,t)$ (calculated for the elastic structure) on these strains.

It reads:

$$\int_{t_1}^{t_2}\int_{\nu} \sigma^E_{ij}(x,t)\ \dot{\varepsilon}_{ij}(x,t)\ d\nu dt > \int_{t_1}^{t_2}\int_{\nu} D(\dot{\varepsilon}_{ij})\ d\nu dt \qquad (1.2)$$

where $D(\dot{\varepsilon}_{ij})$ means energy dissipation rate of the plastic strain rates working on their associated (by the flow rule) stresses. The inverse form of this theorem states that the structure will adapt if for any loading path and for any deformation cycle the inverse inequality in (1.2) is satisfied.

It is easy to see that the above theorems are analogous to the well-known fundamental theorems of limit analysis (see, e.g., [60]). The limit state of immediate collapse may be considered as a particular case of the shakedown under the domain of load variation reduced to one point in the space of load parameters.

The inverse form of the Koiter's theorem (1.2) used for the particular case of alternating plasticity (plastic strain increments disappearing after each cycle) permitted König (see, [66]) to give a corresponding lower bound theorem:

$$\varphi \left[s\ (\sigma^E_{ij}(x,t) + \bar{\psi}_{ij}(x)) \right] \le k \qquad (1.3)$$

It differs from the Melan's theorem (1.1) by the term $\bar{\psi}_{ij}(x)$, which is an arbitrary (not necessarily residual, i.e., self-equilibrated) constant in the time stress field.

Whereas the Melan's theorem is easily and directly applicable (e.g., [66], [98]), the Koiter's theorem was for long very difficult in use. Only when it was shown by Gokhfeld [36] and Sawczuk [104] (see also [66]) that when incremental collapse is concerned the inequality (1.2) may be, under some restrictions, integrated over the time in a general way - direct applications became possible.

Reformulation of the fundamental theorems using generalized variables (in our case cross-sectional stress resultants **s**, and deformation of the beam axis **q**) is not so direct and obvious as in the case of the limit analysis (e.g., [50]). However, under some insignificant restrictions (see [101], [108]) it can be done for incremental collapse.

In the case of alternating plasticity the König criterion (1.3) cannot be directly rewritten in generalized variables. Besides the residual self-equilibrated generalized stresses $\mathbf{s}^R$, pseudo-residual stresses are present in plastically deformed cross-sections. Being self-equilibrated in the cross-section, they correspond to null stress-resultants and cannot be represented by generalized stresses. However, as it was shown by Pycko and Mróz [101] the alternating collapse does depend upon the constant part of the loads. Therefore, if the load variation domains are regular the alternating plasticity criterion can be reduced to the criterion of purely elastic behaviour under an appropriately reduced variable load. The latter occurs in the absence of residual (and pseudo-residual) stresses and may be, therefore, described in generalized variables. Details will be presented in Chapter 2.

Further important step in the development of the shakedown theory was linear programming formulation of the theorems in generalized variables for finite-element discretization (Maier [70]). The analogy between the fundamental theorems and linear programming duality, which is obvious in limit analysis (see, e.g., [19], [75]), may hold, under certain restrictions, in the above approach for shakedown. This formulation is used in our study and, therefore, it will be described in details in the next Chapter.

The shakedown problem is sometimes solved using an incremental analysis for selected loading paths (e.g., [11], [57]), especially since it can be shown [66] that if the structure adapts to cyclic loading processes covering all the vertices of a linear-shaped domain of loads, it will adapt to any load path in this domain. The same approach is used in the case of imperfect boundary conditions (like for slackened structures [35]) where no theorems exist until now.

The theorems of shakedown were generalized to more complex loading conditions as temperature changes (see, e.g., [37], [62], [66]), imposed displacements [100] and dynamic effect (e.g., [16], [20]). Contrary to the limit analysis, the shakedown theory may concern also strain-hardening materials. This case was already dealt by the Melan's theorem [79]. Many contributions concern generalization of the shakedown theorems and their applications to different types of hardening (e.g., [61], [78], [90], [96]), and the linear programming formulation [70], [71] is easily extended into the hardening range [72], [76].

Numerous bounding theorems and methods for displacements prior to the shakedown theory (e.g., [4], [12], [13], [27], [95], [117]), have been formulated, also with extensions to workhardening and/or dynamic cases (e.g., [14], [15], [21], [53]). However, these approaches either need an information on the loading history or are effective only for simple structures. Until now, up to the author's knowledge, no practical applications for complex structures are known.

Shakedown problems including creep phenomena were also studied (e.g., [92], [94].

Extensions of the shakedown theory to the case of finite displacements will be discussed in the next section.

Detailed discussion concerning the above topics may be found in monographs and synthetic state-of-the-art articles by Gohkfeld and Chernivasky [37], König [66], König and Maier [68], Maier [73], Polizztto [91].

Applications of the shakedown theory to frame analysis started already with Neal's studies [84] and are numerous up to now (e.g., [23], [43], [44], [51], [63], [80]). They entered into popular books on plastic design [45], [52], [85] and into commercial computer codes for general structural analysis of plane frames (e.g., CEPAO [86]).

However, all these studies and programs concerned plane frames with yielding influenced by bending moment only. Practical analysis including axial force became possible when linear programming formulation [70] was implemented [18], [109], [111] and entered into plasticity-oriented commercial codes (e.g., STRUPL [19]). No problems taking into account more than two generalized stresses were until now treated.

1.4 Inadaptation

When the structure does not adapt to the load program applied, plastic strains continue to reappear and develop, leading either to the *alternating plasticity* or to the *incremental collapse*. Although some results concern bounding of the post-shakedown deformation (the method of "fictitious holonomic solution" [22]), the analysis of these phenomena should be performed until now following the given loading programs. It concerns first of all the so-called limit cycle approach, namely looking for the cycle intensity insuring stationary stresses and constant deformation increment in the cycle. In this way incremental collapse behaviour became analogous to the alternating plasticity. The notion of limit cycle introduced by Armstrong [33] was developed by Mróz [81], Polizzotto [93], and Pycko and König [99]; the last paper also takes into account the geometrically nonlinear effects.

These nonlinear effects may appear important also in the prior-to-shakedown behaviour. Large displacements developed in this process, if taken into account, make the problem geometrically non linear. First proposition concerning extension of shakedown theorems to large deformations appeared already 20 years ago. Maier proved their validity for the "second order theory" in a discretized piecewise linear (PWL) description [72]. It consisted of updating the elastic stiffness matrix by terms (a priori known or estimated), responsible for linear displacement-dependent terms in the equilibrium equations. A criterion for the stability of the adaptation process, analogous to the Duszek's criterion [29] for the post-yield behaviour, is directly derived from the above formulation. The original version accounted for linear hardening. The approach was then extended [76] to an arbitrary hardening in the framework of nonlinear programming. The above approach corresponds to applying small perturbation to a pre-existing stress state and had until now no application to practical cases of complex structures.

A more fundamental and general approach concerns considering the problem at large elastic and plastic strains in Lagrangean description. In this domain ambitious contributions were recently presented concerning extension of the theorems, both assuming additivity of elastic and plastic strains by Weichert and Gross-Weege [40], [116], [118], [119] and by criticizing this decomposition by Stumpf [102], [115]. All these contributions concern large deformations under constant loads and small deformations generated by variable cyclic loads and until now they seem to be of interest rather for qualitative studies.

The most practical approach consists of application of the post-yield behaviour methods (see Section 1.2) to the case of variable loads. It means that the geometrical configuration changes are due to plastic deformations in fully plastic cross-sections only. The first studies of the shakedown of frames including deformations are due to Davies [23], [24]. König [64], [65] determined stability of frames by comparing the shakedown load intensity for the undeformed structure and for its configuration deformed following the corresponding incremental collapse mode. Namely, two modes valid for the same shakedown load are compared; the real mode to occur will be that corresponding to the most unstable behaviour.

This approach was developed by Siemaszko and König in [111], and extended into a sequential step-by-step linearized procedure with a program SSDH [69], [109] permitting to determine the shakedown response of strongly deformed structures and taking into account material hardening. The approach appeared to work even in the case of material softening due to damage [110] and was applied to optimization of plane frames [112], [113].

1.5 Aim, Scope and Main Assumptions.

Principal objective of this study is to furnish a tool for a unified formulation and numerical analysis of all the history-independent classes of response of elastic-perfectly plastic space skeletal structures. We aim at determination of load domains corresponding to *elastic*, *shakedown* and *collapse* behaviour for structures in initial and deformed configurations.

If the loading process may be represented by a finite number of independently varying in time scalar parameters β_i, i=1,...,m (what is true for nearly all conservative loads), the loading history presents a path in the m-dimensional space of these parameters. In this case domains of plastic Ω_L, shakedown Ω_{SD} and purely elastic Ω_E response are enclosed inside the convex surfaces: $\partial\Omega_L$ (surface of limit loads), $\partial\Omega_{SD}$ (shakedown surface) and $\partial\Omega_E$ (elastic-limit surface), respectively. This is shown in Fig. 1.5. Limit-loads ($\partial\Omega_L$) and elastic-limit ($\partial\Omega_E$) surfaces are unique, whereas the shakedown surface ($\partial\Omega_{SD}$) is determined following an arbitrary choice of the domain shape. It means that a reference surface ($\partial\Omega_R$) should be chosen and the shakedown surface ($\partial\Omega_{SD}=\xi_{SD}\partial\Omega_R$) is its homothetic increase by a multiplier ξ_{SD}.

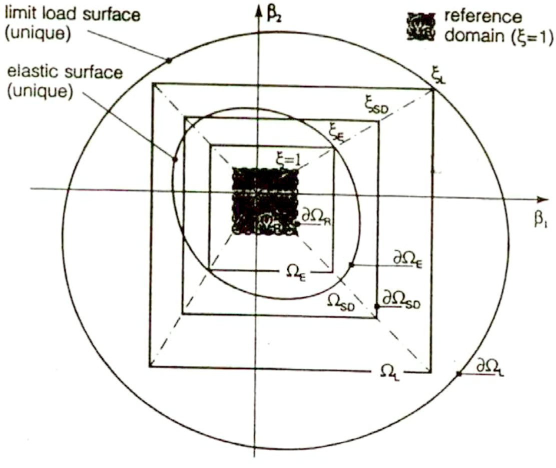


Fig. 1.5 Load domains in the space of load parameters

Determining the shakedown load means, thus, determining an intensity factor ξ_{SD} for a given domain configuration. Therefore, to permit determination of the corresponding multiplier for elastic limit ξ_E and for the limit (collapse) load ξ_L the maximum domains homothetic to the shakedown (and reference) domain are inscribed into the elastic-limit surface and surface of limit load, respectively.

The shakedown theory deals with arbitrary polyhedral load domains. However, because of the reasons discussed in the next Chapter, we shall restrict our considerations to "rectangular" (rectangular-hyperpolyhedral) domains.

Linear programming formulation of the shakedown problem in generalized variables by Maier [70] is used in its classical form (see, [19]) described in Chapter 2. Finite element discretization in lumped-compliance bar elements [10], [19], and solutions using the standard simplex procedure (e.g., [10]), are also classical ones. All actually successful programs (e.g., STIRUPL [19], SSDH [109]) are based on this kind of approach.

Description and the procedures adopted above correspond to geometrically linear formulation, the shakedown or limit-load multipliers concern the undeformed structure. To take into account effects of geometry changes occurring in the deformation process, it is assumed that the structure displacements are due mainly to the inadaptation or post-yield process. It means that displacements are deduced from the actual incremental-collapse mechanism. The mechanism is determined by duality together with calculation of the multiplier. The mechanism gives the displacement mode up to an undefined multiplier; therefore, the value of this multiplier is chosen to give a predetermined value for the maximum displacement. Such a displacement field is introduced to modify the configuration at the next step of the analysis. In this way the geometrically non linear problem is replaced by a sequence of step-by-step linearized problems. This approach, classical in limit analysis (post-yield behaviour), was proposed for shakedown in [64] and showed its applicability to plane frames [109]. The deformation mode modifying the current shakedown configuration concerns incremental collapse, therefore, if the alternating plasticity appears, the procedure stops, since the configuration in no more modified.

Using the plastic mode for updating the configuration means that the deformed shape is composed of undeformed bars undergoing rotations/extensions at plastic hinges. It is clear that such assumption may be considered justified only for structures not-too-deformable elastically, for which the overall elastic instability is not the principal danger. It should be noted, however, that even if elastic deformations are large, replacing the deformed shape by its secant piecewise linear approximation is frequently justified.

General theorems concerning large deformations mentioned in the preceding section do not furnish, until now, a tool for solving in one step the geometrically linear shakedown or limit-analysis problems. Using our approach seems to be the most reasonable since it permits to maintain at each step the essential feature of the collapse and shakedown behaviour, i.e., their history-independence. Moreover, the formulation used is already well discussed and verified, and classical procedures of linear programming may be applied.

The LP solution of the shakedown problem consists of multiple application of elastic analysis permitting local distortions. In the case of complex structures (like space frames) the efficiency of sub-routines for elastic analysis is essential. Therefore, using a professional commercial software is preferable. We used the SAP program, namely its updated Polish version POL(SAP) (see, e.g., [58].

Straight spatial beam element is used, of lumped-type compliance, with plastic distortion-like plastic strains concentrated at the end section (like in Borkowski approach [10]). Loads are applied at nodes. The influence of transverse forces is neglected both in elastic deformation and in yielding of cross-sections. Therefore, four active stress-resultants (generalizes variables) are present: axial force, torque, and two bending moments. Criteria for an entirely plastic ("Yield Surface") and entirely elastic ("Elastic Surface") cross-sections depend on these variables. Forms of the criteria are assumed to be known (from case studies or commercial catalogue data) and we proceed with an appropriate piecewise linearization of them. Studies are restricted to doubly symmetrical cross-sections, to avoid being concerned with problems of the cross-sectional elastic-plastic behaviour (shear centre in plasticity). Bernoulli assumption about flexure and St. Venant's about torsion are accepted, like in all classical approaches. Therefore, application of our approach to cold-rolled elements of thin-walled cross-section is rather disputable.

Taking into account strain-hardening should not pose serious problems at least from the conceptual point of view (see, [72], [76]), the more if a step-by-step updating of the yield criteria is applied [109]. However, hardening laws are still in the centre of discussion. Therefore, because of practical orientation of our work, this effect is excluded from considerations. For mild-steels the plasticity platform is considered sufficiently extended to justify this exclusion.

Let us resume principal assumptions described above:

1. Material is considered elastic-perfectly plastic and symmetric (the same response at compression and tension).
2. Classical SD theory is used in a linear programming formulation based on undeformed geometry on each step of the analysis.
2a. The above assumes loading process described by a finite number of scalar time-dependent parameters, with a given (rectangular) shape of the load domain.
3. Classical theory of beams (Navier-Bernoulli and unconstrained torsion) is applied in elastic analysis.
3a. Active generalized variables are: axial force, torque and two bending moments. The influence of transversal shear forces is neglected both in elastic analysis and at yielding.
4. Double symmetric cross-sections are considered and limit surfaces for full plasticity and first plasticity of cross-sections are assumed piecewise linear.
5. Finite element discretization is performed using straight lumped-compliance beam elements.
6. Geometry changes due to deformation are accounted for by a sequential linearized analysis for consecutive deformed configurations.
7. Configuration changes are due to plastic deformations.
7a. Plastic deformations are derived from the actual collapse mode with a selected step of the maximum displacement.

Some secondary assumptions, needing a detailed reference to formulae or procedures will be discussed later.

In *Chapter 2* fundamentals and details for the determination of load intensity multipliers as a linear programming problem are described. As special cases the descriptions concerns limit analysis, shakedown and purely elastic analysis. Subsequent steps for deformed configurations are determined.

In *Chapter 3* generalized variables for spatial beam elements are specified and the corresponding criteria for yielding and first plasticity are discussed. Different piecewise linear approximations of the corresponding surfaces in the 4-dimensional space of generalized stresses are compared.

In *Chapter 4* a finite element program SDLAS for solving the problem formulated in Chapter 2 is described.

In *Chapter 5* results for case studies are given. Because of the number of parameters influencing elastic-plastic response of spatial frames (elastic rigidities, elastic and plastic strength characteristics of the cross-sections, shape of load domains, etc.), possibly simple cases should be studied. With the exception of some more complex examples for testing efficiency of the program, all other studies concern one-span one-storey frames. Load multiplier versus control displacement curves were plotted. The first series concerned different PWL approximations of the yield surface and elastic surface, the next an influence of the frame height. The following series concerned, first of all, 2-dimensional sway modes being of importance in earthquake engineering. Finally, eccentrically braced plane frames were studied. They have a particular importance in aseismic design as they exhibit a significant phase of plastic response ("overall ductility") for energy absorption at exceptional situations. For such structures the shakedown analysis under service loads is essential.

In *Chapter 6* a simplified two-step version of the procedure is used to determine the sensitivity of the limit and shakedown loads to displacements, for a given control displacement value. The (sensitivity diagrams) permit the modification of standard safety factors needed, depending upon the collapse mode.

Finally, *Chapter 7* contains an overview and discussion of the results, conclusions, and some suggestions concerning possible course of future research.

CHAPTER 2

FUNDAMENTALS AND BASIC RELATIONS

2.1 Material model

For the sake of self-sufficiency of the presentation, basic notions and relations of the classical plastic flow theory (e.g., [106]) are recalled here. Elementary notions are visualized in the uniaxial-plasticity diagram: nominal stress σ versus linear strain ε in Fig. 2.1.

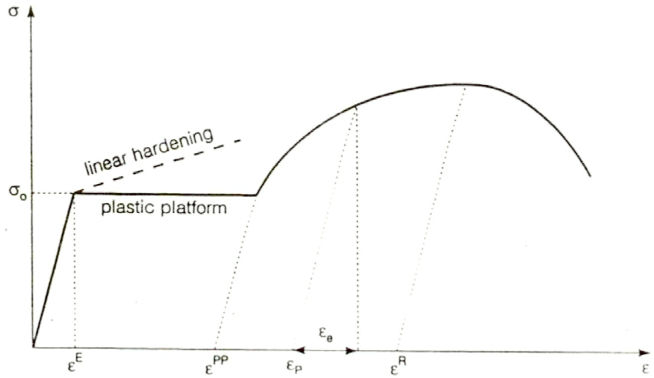


Fig. 2.1 Uniaxial plastic behaviour

Assumptions of additive strain decomposition ($\varepsilon = \varepsilon_E + \varepsilon_P$) and of pure elastic unloading are included in the classical theory. For perfectly plastic isotropic metal-like materials the plasticity condition is a scalar homogeneous symmetric function of stress components:

$$\varphi = f(\sigma_{ij}) - k = 0 \tag{2.1}$$

In the case of skeletal structures, under Bernoulli assumption only three non-vanishing stress components remain. They are, in local Cartesian coordinates (with x axis tangent to the beam axis): σ_{xx}, σ_{xy}, σ_{xz} and the plasticity function is reduced to an "ad hoc" expression (e.g., [121]):

$$f = (\sigma^2 + c\ \tau^2)^{1/2} \qquad (2.2)$$

with normal $\sigma_{xx} = \sigma$, and shear stresses $\sigma_{xy}^2 + \sigma_{xz}^2 = \tau^2$ due to bending and axial force, and to torsion and transversal forces, respectively. The formula (2.2) covers, e.g., the Huber-Mises (c=3) and the Tersca (c=4) plasticity condition.

Since the expression (2.1) may be given by several analytical functions it is convenient to formulate it in vector notation as:

$$\mathbf{\Phi} = \varphi_k(\underline{\sigma}) \leq \mathbf{0} \qquad (2.3)$$

The above takes the following matrix form in a piecewise linear (PWL) approximation:

$$\mathbf{\Phi} = \bar{\mathbf{N}}^T \underline{\sigma} - \mathbf{k} \leq \mathbf{0} \qquad (2.4)$$

In the above formula $\bar{\mathbf{N}}$ denotes matrix of the yield surface gradients defined, according to this formula, as:

$$\bar{\mathbf{N}} = \Phi_{\underline{\sigma}(n\mathrm{x}m)} = \begin{pmatrix} \frac{\partial\varphi_1}{\partial\sigma_1} & \frac{\partial\varphi_2}{\partial\sigma_1} & \cdots & \frac{\partial\varphi_m}{\partial\sigma_1} \\ \frac{\partial\varphi_1}{\partial\sigma_2} & \frac{\partial\varphi_2}{\partial\sigma_2} & \cdots & \frac{\partial\sigma_m}{\partial\sigma_2} \\ \frac{\partial\varphi_1}{\partial\sigma_n} & \frac{\partial\varphi_2}{\partial\sigma_n} & \cdots & \frac{\partial\varphi_m}{\partial\sigma_n} \end{pmatrix} \qquad (2.5)$$

The column matrix **k** contains the so-called plastic moduli. These are the functions of the yield stress σ_o and they define the distances at which the planes φ_K are located relative to the origin $\sigma = 0$.

Using the above notation, PWL classical models of strain hardening (e.g., isotropic or kinematical) may be described [72] by:

$$\mathbf{\Phi} = \mathbf{N}^T\sigma - \mathbf{H}\underline{\lambda} - \mathbf{K} = \mathbf{0} \qquad (2.6)$$

with **H** depending upon the type of hardening rule and on material data; the vector of plastic multipliers $\underline{\lambda}$ determines the intensity of plastic deformation.

The plasticity function being identified with the potential for strain rates, the strain rates are expressed by the "associated flow law":

$$\dot{\varepsilon}_{ij} = (E_{ijkl})^{-1}\ \dot{\sigma}_{kl} + \dot{\lambda}_k \frac{\partial \varphi_k}{\partial \sigma_{ij}} \tag{2.7}$$

with non-vanishing multipliers $\dot{\lambda} > 0$ if (2.1) is satisfied, i.e., with:

$$\dot{\lambda}\varphi = 0, \qquad \dot{\lambda}\dot{\varphi} = 0, \tag{2.8}$$

the latter excluding plastic deformations at unloading. The dot $(\dot{\ })$ denotes the time-derivative, the time being considered as parameter determining the order of events in the deformation process ("kinematical time").

For the PWL plasticity conditions (2.6) the above relations assume the following vector form [10]:

$$\dot{\underline{\varepsilon}} = \dot{\underline{\varepsilon}}_e + \dot{\underline{\varepsilon}}_p \tag{2.9a}$$

$$\dot{\underline{\varepsilon}}_e = \mathbf{E}^{-1}\dot{\sigma}, \tag{2.9b}$$

$$\dot{\underline{\varepsilon}} = \mathbf{N}\ \dot{\underline{\lambda}}, \tag{2.9c}$$

$$\dot{\underline{\lambda}} \geq \mathbf{0}, \tag{2.9d}$$

$$\mathbf{\Phi} \leq \mathbf{0}, \tag{2.9e}$$

$$\dot{\underline{\lambda}}^T\mathbf{\Phi} = 0, \tag{2.9f}$$

$$\dot{\mathbf{\Phi}}_a \leq \mathbf{0}, \tag{2.9g}$$

$$\dot{\underline{\lambda}}^T\dot{\mathbf{\Phi}}_a = 0. \tag{2.9h}$$

with $\mathbf{\Phi}_a$ concerning currently active planes of the plasticity surface.

Formulation in generalized variables deduced from the above relations is given in Section 2.3.

2.2 Description of load variation

When formulating the problem of safety of elastic-plastic structures under variable repeated loads one should take into account not only the maximum load magnitudes or their extreme combinations but, also, theoretically the whole history of load variations. However, as it was noted in Section 1.1 the collapse load (limit load) is history independent, similarly to the elastic behaviour. On the other hand, the shakedown limit, being also history independent, depends upon the domain that loads are permitted to vary arbitrarily within. Description of loads was shortly discussed in Section 1.5. Here we repeat this description in more details.

In limit analysis and in shakedown theory it should be assumed that loads may be described by a finite number of scalar parameters $\beta_i(t)$, $(i=1,....,m)$ varying independently in time, and by a spatial distribution (configurations) of "unit" body forces $\mathbf{b}_i(x)$ and surface tractions $\mathbf{t}_i(x)$ corresponding to these parameters. The distribution of loads in an instant of time is therefore:

$$\mathbf{b}(x,t) = \sum_{i=1}^{m} \beta_i(t)\mathbf{b}_i(x), \qquad \mathbf{t}(x,t) = \sum_{i=1}^{m} \beta_i(t)\mathbf{t}_i(x) \tag{2.10}$$

Some parameters β_i may concern both these types of loads, other only one. Nearly all loads happening in engineering practice can be described in this way or, at least, reasonably approximated.

The loading history describes now a path in the m-dimensional space of load parameters, as shown in Fig. 2.2a. This path should vary inside a certain domain,

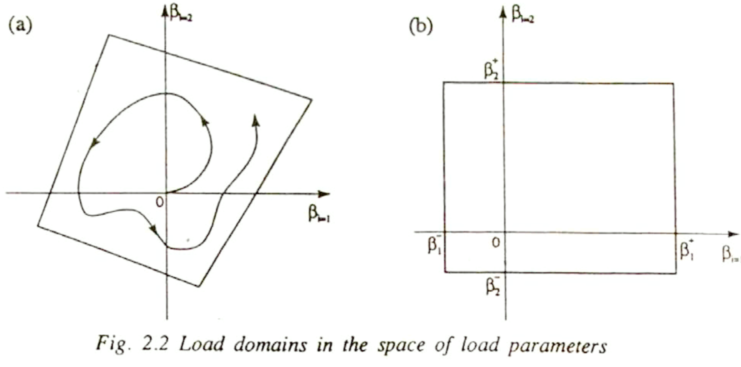


Fig. 2.2 Load domains in the space of load parameters

if the structure is looked to shakedown. The shape of this domain needs to be chosen, since the shakedown domain is not unique.

Corollaries to the fundamental shakedown theorems (see [66], [67]), permitting practical calculations, concern any polyhedral (PWL) load domains, see Fig. 2.2a. However, it seems that for practical analysis the domains may be restricted by each load parameter β_i being independently bounded by constant extreme values:

$$\beta_i^- \leq \beta_i(t) \leq \beta_i^+ \tag{2.11}$$

It means considering only the case of rectangular (rectangular-polyhedral) domains, Fig. 2.2b. It covers nearly all practical cases if load parameters were appropriately chosen, and it permits simpler LP solution (see [72], [109]) of the problem. Moreover, this permits a simple decomposition of loads into a "permanent" part determined by median coordinates:

$$\beta_i^o = \frac{1}{2}\left(\beta_i^+ + \beta_i^- \right) \tag{2.12}$$

and the part symmetrically oscillating around this new center of coordinate axes, Fig. 2.3, with an amplitude:

$$\beta_i^a = \pm\frac{1}{2}\left(\beta_i^+ - \beta_i^- \right) \tag{2.13}$$

$$\beta_i(t) = \beta_i^o + \beta_i^{'}(t); \qquad -\beta_i^a \leq \beta_i^{'}(t) \leq +\beta_i^a \tag{2.14}$$

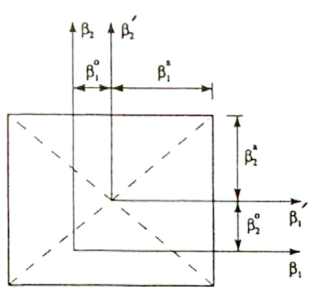


Fig. 2.3 Load decomposition

Only the oscillating part of the load influences the alternating-plasticity shakedown limit [101].

Solving the shakedown problem means determination of the common load factor ξ_{SD}, which transforms, by a proportional expansion, a given reference domain Ω_R (see Fig. 1.5) with prescribed dimensions:

$$\gamma_i^- \leq \beta_{iR}(t) \leq \gamma_i^+ \tag{2.15}$$

into the shakedown domain Ω_{SD}, with:

$$\xi_{SD}\, \gamma_i^- \leq \beta_i(t) \leq \xi_{SD}\, \gamma_i^+ \tag{2.16}$$

Following the decomposition (2.12-14) the reference domain (2.15) may be also described as:

$$\gamma_i^o - \gamma_i^a \leq \beta_{iR}(t) \leq \gamma_i^o + \gamma_i^a \tag{2.15a}$$

with:

$$\gamma_i^o = \frac{1}{2}\,(\gamma_i^+ + \gamma_i^-), \qquad \gamma_i^a = \frac{1}{2}\,(\gamma_i^+ - \gamma_i^-) \tag{2.15b}$$

When the alternating plasticity is concerned, the dimensions of the reference domain should be reduced by putting in (2.15a) $\gamma_i^o = 0$.

The history independence of the limit analysis problem and of elastic analysis induces uniqueness (see, e.g., [77]) of limit load and elastic limit surfaces ($\partial\Omega_L$ and $\partial\Omega_E$ in Fig. 1.5, respectively). Looking for load multipliers transforming the reference domain into the one insuring safety against immediate collapse:

$$\xi_L\, \gamma_i^- \leq \beta_i(t) \leq \xi_L\, \gamma_i^+ \tag{2.17}$$

or against occurrence of first plasticity:

$$\xi_E\, \gamma_i^- \leq \beta_i(t) \leq \xi_E\, \gamma_i^+ \tag{2.18}$$

means looking for the largest domains inscribed in the surfaces $\partial\Omega_L$ and $\partial\Omega_E$, respectively, homothetic (with respect to the coordinate origin) to the reference domain.

It should be noted that all the load limits (2.11) are considered to increase proportionally. Therefore, the case of fixed loads, not supposed to vary or to increase cannot be directly treated in this manner. That does not pose practical problems in determination of elastic ξ_E and collapse ξ_L multipliers, but the shakedown multiplier ξ_{SD} depends upon the shape of the domain, whereas this shape changes when one of the parameters maintains the constant value of its limit:

$$\xi_{SD}\,\gamma_i^- \le \beta_i(t) \le \xi_{SD}\,\gamma_i^+, \qquad \beta_c = \gamma_c^+ = \gamma_c^- \tag{2.19}$$

The easiest (computationally) way of treating this case is an iterative modification of the reference domain in the way that its initial shape corresponds to $\gamma_c' = \gamma_c / \xi_{SD}$ and, therefore, the surface expanded by ξ_{SD} has the needed fixed coordinate γ_c in the direction of the axis β_c.

2.3 Basic Relations

Basic relations for perfect plasticity given in Section 2.1, when formally rewritten using generalized variables (cross-sectional stress resultants **s** and corresponding axis deformations **q**) and assembled for all the elements of the structure, take the form given below. The manner of presentation and notation follow these form [111]. This description proposed by Maier [70] concerns discretization by finite elements and PWL form of yield criteria (criteria for fully plastic element cross-section). Loads are considered applied at nodes only.

Under assumptions of linear geometry, equilibrium and geometrical relations take the classical form, respectively:

$$\mathbf{p} = \mathbf{C}^T \mathbf{s} \tag{2.20}$$

$$\mathbf{q} = \mathbf{C}\,\mathbf{u}, \tag{2.21}$$

where **C** represents the compatibility matrix for the whole structure; **s** and **q** are supervectors that collect all the generalized stresses and strains, respectively, of all the elements; **p** and **u** denote the respective supervectors of loads and generalized displacements at nodes.

Elements are approximated by a lumped-compliance model [10] with plastic deformation concentrated at end sections of the element. The latter assumption justifies decomposition of generalized strain into elastic and plastic part, equivalent to (2.9a) but not so obvious as in standard variables:

$$\mathbf{q} = \mathbf{q}^{e} + \mathbf{q}^{p} \tag{2.22}$$

The elastic part $\mathbf{q}^{e}$ is related to the stress vector, through the structural elasticity matrix $\mathbf{E}$:

$$\mathbf{s} = \mathbf{E}\,(\mathbf{q} - \mathbf{q}^{p}) \tag{2.23}$$

The yield conditions are assumed for all elements to be piecewise-linear and can be expressed in the PWL form:

$$\mathbf{f} = \mathbf{N}^{T}\mathbf{s} - \mathbf{k} \leq \mathbf{0}\ , \tag{2.24}$$

where $\mathbf{N}$ is a rectangular hyperdiagonal matrix of the gradients of polyhedron faces, for all elements. The vector $\mathbf{k}$ represents the corresponding plastic moduli, with geometrical interpretation being the distances of the respective faces to the origin.

The associated flow rule (2.7) may be directly extended to generalized variables (e.g., [106]). So, the plastic part of generalized strain rates and the active process condition are as follows:

$$\dot{\mathbf{q}}^{p} = \mathbf{N}\,\dot{\underline{\lambda}}, \qquad \dot{\underline{\lambda}} \geq \mathbf{0} \tag{2.25}$$

Components of the vector $\dot{\underline{\lambda}}$ are intensities of plastic flow in the corresponding flow modes. Plastic flow may occur only if at least one row of Eq. (2.24) becomes equality. That induces:

$$\dot{\underline{\lambda}}^{T}\mathbf{f} = 0 \tag{2.26}$$

The vectors of stresses and displacements can be decomposed into elastic and residual parts:

$$\mathbf{u} = \mathbf{u}^e + \mathbf{u}^r \tag{2.27}$$

$$s = s^e + s^r \tag{2.28}$$

The elastic terms are determined using the elastic stiffness matrix:

$$\mathbf{K} = \mathbf{C}^T\mathbf{E}\mathbf{C} \tag{2.29}$$

$$s^e = \mathbf{E}\mathbf{C}\mathbf{K}^{-1}\mathbf{p}, \tag{2.30}$$

$$\mathbf{u}^e = \mathbf{K}^{-1}\mathbf{p} \tag{2.31}$$

The residual terms depend linearly upon the plastic deformation:

$$\mathbf{u}^r = \mathbf{K}^{-1}\ \mathbf{C}^T\ \mathbf{E}\ \mathbf{q}^p \tag{2.32}$$

$$\mathbf{s}^r = [\ \mathbf{E}\ \mathbf{C}\ \mathbf{K}^{-1}\ \mathbf{C}^T\ \mathbf{E} - \mathbf{E}\ ]\ \mathbf{q}^p \tag{2.33}$$

The latter, in force of Eq. (2.31), must be self-equilibrated, i.e., $\mathbf{C}^T\mathbf{s}^r = \mathbf{0}$.

Variations of the load vector **p** are described by means of a finite number of load parameters $\beta_1(t),....,\beta_k(t)$, following Section 2.2. In vector form we can write:

$$\mathbf{p}(t) = \mathbf{Q}\ \beta\ (t) \tag{2.34}$$

where **Q** is a constant matrix collecting the vectors $\mathbf{t}_i$ of unit load configurations. The limits of variation of each of the load schemes are defined by limits between which the corresponding load factor is allowed to vary (see (2.11)). The domain Ω of admissible load variations, defined by Eq. (2.11), is given to a certain common factor ξ by the reference domain Ω_R (2.15).

If a practical application of the Melan's theorem (1.1) is intended for arbitrary varying loads, the elastic stress variable in time should be replaced by its appropriately constructed envelope. In the case of a PWL criterion (2.24) the most convenient way to do so is determining an envelope for projections of the stress vector s^e on the directions of gradients of all the faces of the yield polyhedrons (index j). That forms a reduced elastic envelope for a reference load domain. In the index-summation notation, used for clarity, it is:

$$d_j = \max_{\Omega_R} N_{kj} s^e_k(t) \tag{2.35}$$

In other words the elastic stress envelope vector **d** is:

$$\mathbf{d} = \max \left\{ \mathbf{N}^T \ \mathbf{E} \ \mathbf{C} \ \mathbf{K}^{-1} \ \mathbf{p}(t) \middle| \ \ \mathbf{p}(t) \in \Omega_R \right\} \tag{2.36}$$

Practical computations are done using matrices collecting the vectors of elastic stress due to each unit independent load and their projections on the normal to the j-th face of the yield surface, respectively:

$$\mathbf{T} = \mathbf{E} \ \mathbf{C} \ \mathbf{K}^{-1}\mathbf{Q}, \qquad \mathbf{L} = \mathbf{N}^T\mathbf{T} \tag{2.37}$$

Eq. (2.36) may be rewritten as follows:

$$d_j = L_{ji} \ \alpha_i, \tag{2.38}$$

where α_i determines the extreme values of the i-th loading scheme inside the reference domain γ_i^+ or γ_i^-, when the components L_{ji} are positive or negative, respectively. Using notation from (2.15b) the above may be written as:

$$d_j = L_{ji} \ \gamma_i^o + \left| L_{ji} \right| \ \gamma_i^a \tag{2.39}$$

2.4 Linear programming formulation

Let us consider at first the shakedown problem, because limit analysis and elastic limit problems may be considered as particular cases of the shakedown.

Using Maier's description [70] given in the preceding section, the shakedown problem may be formulated as a dual pair of classical linear programming problems. The LP formulation of the Melan's theorem (1.1) gives:

$$\xi_{SD} = \max \left\{ \xi \mid \xi \mathbf{d} + \mathbf{N}^T \mathbf{s}^r \leq \mathbf{k}, \qquad \mathbf{C}^T \mathbf{s}^r = 0 \right\}, \tag{2.40}$$

Where $\mathbf{s}^r$ is a looked-for time-independent self-equilibrated state of generalized stresses and $\mathbf{d}$ represents an appropriately reduced envelope (2.36) of these variables for the reference load domain Ω_R (2.15). Multipliers ξ satisfying the inequality in (2.40) determine admissible load domains proportional to the reference domain Ω_R. The maximum admissible domain ($\xi = \xi_{SD}$) is the shakedown domain that is looked for.

By a formal construction of the dual problem to (2.40) (see, e.g., [10]), it is obtained:

$$\xi_{SD} = \min \left\{ \mathbf{k}^T \underline{\lambda} \mid \mathbf{d}^T \underline{\lambda} = 1, \quad \mathbf{Cu} = \mathbf{N}\underline{\lambda}, \quad \underline{\lambda} \geq 0 \right\}, \tag{2.41}$$

which represents the Koiter's theorem integrated over a compatible plastic deformation cycle, the integration enabled by PWL form of the yield criterion. If a normalizing constraint is imposed on the term $\mathbf{d}^T \underline{\lambda}$ representing the left-hand side of the Koiter's theorem (1.2) the term $\mathbf{k}^T \underline{\lambda}$ (plastic dissipation) is equal to the multiplier ξ. This multiplier determines inadmissible load domains proportional to the reference domain. The minimum domain and its multiplier ξ_{SD} correspond to the shakedown solution.

It should be noted that the duality holds if in both formulations the yield criterion (2.24) for unconstrained plastic flow of the cross-section was used instead of the local plasticity condition (2.4). It is known that such a substitution is exact neither in the case of Koiter's theorem (see [81]) nor in the Melan's one (see [66]). In the former case this substitution means assuming the "ratcheting surface" [81] identical to the yield surface. In reality, they are very close together (see [108]) and at least the incremental collapse may be correctly dealt with using the yield surface.

On the other hand, the criterion (1.1) is influenced by "pseudoresidual" stresses (self-equilibrated within the cross-section [66]) and cannot be correctly expressed by cross-sectional stress resultants. Fortunately, when applying the criterion (1.3), it can be shown [101] that alternating plasticity is independent from the "permanent" load β_i^o (2.12). Therefore, the problem can be solved, in the absence of residual stress $\mathbf{s}^r$, as the elastic problem (using the elastic-limit criterion) for the reference load domain (2.15a) appropriately reduced ($\gamma_i^o = 0$).

The elastic problem is, as mentioned above, a trivial subproblem of (2.40), in the absence of residual stresses.

The limit analysis problem represents a shakedown case under the reference load domain reduced to points at its vertices. It means that the problem (2.40) is uncoupled for each vertex and residual stresses $\mathbf{s}^r$ may be different for $\mathbf{d}$ at each vertex of the Ω_R.

In practical calculations it is frequently more convenient to use a direct LP formulation of limit analysis theorems (see, e.g., [19], Chapter 5), being mathematically equivalent to the description above:

$$\xi_L = \max\left\{ \xi \;\middle|\; \mathbf{C}^T\mathbf{s} = \xi\mathbf{p}, \quad \mathbf{N}^T\mathbf{s} \le \mathbf{k} \right\}, \tag{2.42}$$

$$\xi_L = \min\left\{ \mathbf{k}^T\dot{\underline{\lambda}} \;\middle|\; \mathbf{p}^T\dot{\mathbf{u}} = 1 , \quad \mathbf{C}\dot{\mathbf{u}} = \mathbf{N}\dot{\underline{\lambda}}, \;\; \dot{\underline{\lambda}} \ge 0 \right\}, \tag{2.43}$$

This formulation was used in all the known papers concerning limit analysis of space frames [25], [26], [39].

2.5 Inadaptation and post-yield analysis

The formulation given in Sections 2.3 and 2.4 concerns a geometrically linear description. To be rigorous, it should be admitted that the history-independence of shakedown and limit analysis solutions (load multipliers and corresponding collapse modes) is strictly valid only in geometrically linear cases. This history-independence is essential for making feasible plastic analysis under arbitrarily varying loads. Therefore, it seems that the only practical way that permits accounting for geometry changes due to deformations is the post-yield approach (see Section 1.2). It consists of sequential solutions of geometrically linear problems done for consecutively modified configuration of the structure. The problem is, thus, linearized step-by-step.

This approach assumes geometry changes due to plastic deformations only. Since the non-compatible plastic deformations are generally small when compared with kinematically admissible ones the displacements may be attributed to the collapse (immediate or incremental) mechanism only. In the lumped-compliance beam model plastic deformations are assumed to be concentrated at nodal cross-sections (generalized plastic hinges). Therefore, the deformed configuration of the structure will consist of rigid-body displacements of straight elements between the hinges. Such a mode of subsequent deformations is shown in Fig. 2.4.

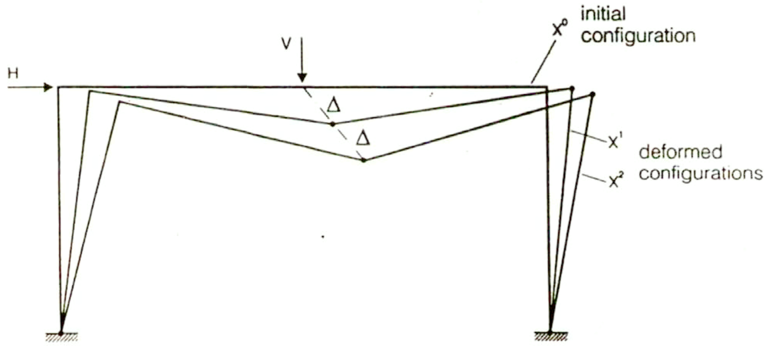


Fig. 2.4 Structure deformed following a collapse

Initial data for the inadaptation or post-yield sequential procedures are furnished by the solution of the problem at undeformed geometry (2.40, 2.41) or (2.42, 2.43), respectively. Vectors of plastic multipliers λ and of nodal displacements **u** are derived from the kinematical formulation or by the formal dualization of the primal problem. However, they are determined up to an arbitrary common factor. To obtain the modified configuration of the structure a step Δ for these modifications has to be chosen. Then, the factor μ^{o} should be determined, normalizing a certain characteristic displacement into the step value Δ. The most reasonable and simple procedure for choosing Δ was proposed by König and Siemaszko [109], [111], who were the first to apply the post-yield approach to the shakedown. It consists of normalizing into Δ the maximum modulus of local displacement. The factor μ^{o}, and factors μ^{δ} (for subsequent configurations) should be determined from the normalizing relation

$$\mu^{\delta} = \frac{\Delta}{\max\left|\mathbf{u}^{\delta}\right|} \qquad \delta = 0,\ 1,\ 2,\ \ldots.. \tag{2.44}$$

For the given collapse mode, with $\max\left|\mathbf{u}^{1}\right| = \Delta$, we can modify the vector $\mathbf{x}^{o}$ describing nodal coordinates of the undeformed structures into the vector $\mathbf{x}^{1}$ and so on:

$$x^{\delta+1} = x^{\delta} + \mu^{\delta}\,\mathbf{u}^{\delta} \qquad \delta = 0,\ 1,\ 2,\ \ldots... \tag{2.45}$$

Then, we can pass to the solution of the next step, etc. At each step the procedure is repeated with the formulation remaining as given in Section 2.4. Of course, the step Δ should be sufficiently small.

The dual formulations for the shakedown and for the limit analysis problems are now, respectively:

$$\xi_{SD}(x^{\delta}) = \max\left\{\xi \mid \xi\mathbf{d}(x^{\delta}) + \mathbf{N}^{T}\mathbf{s}^{r} \le \mathbf{k}, \qquad \mathbf{C}^{T}(x^{\delta})\,\mathbf{s}^{r} = 0\right\}, \tag{2.46}$$

$$\xi_{SD}(x^{\delta}) = \min\left\{\mathbf{k}^{T}\lambda^{\delta} \mid \mathbf{d}^{T}(x^{\delta})\lambda^{\delta} = 1,\ \ \mathbf{C}(x^{\delta})\mathbf{u}^{\delta} = \mathbf{N}\lambda^{\delta},\ \ \lambda^{\delta} \ge 0\right\} \tag{2.47}$$

$$\xi_L(x^\delta) = \max\left\{ \xi \mid \mathbf{C}^T(x^\delta)\mathbf{s} = \xi\mathbf{p}, \qquad \mathbf{N}^T\mathbf{s} \le \mathbf{k} \right\}, \tag{2.48}$$

$$\xi_L(x^\delta) = \min\left\{ \mathbf{k}^T\dot{\underline{\lambda}}^\delta \mid \mathbf{p}^T(x^\delta)\dot{\mathbf{u}}^\delta = 1, \quad \mathbf{C}\dot{\mathbf{u}}^\delta = \mathbf{N}\dot{\underline{\lambda}}^\delta, \quad \dot{\underline{\lambda}}^\delta \ge 0 \right\} \tag{2.49}$$

The above approach is applicable for sufficiently stiff structures, because deformations in the pre-shakedown or (pre-yield) phase are neglected. It may concern, therefore, rather the inadaptation and the post-yielding processes than the preceding phases. In other words, such procedure gives information on the sensitivity of the shakedown and the limit analysis solution to shape distortions induced by collapse-mode deformations or by initial imperfections of the structure.

When regarding Fig. 1.3, it is clear that in situations of geometrical hardening (increasing curve) such a "rigid-plastic" approach has rather different physical sense. At geometrical softening (decreasing curve) no precise information on the maximum admissible "peak-load" may be obtained. However, we can determine the shakedown (or limit) load for the structure assuming a given control (admissible) displacement. This displacement may, e.g., be derived from an incremental analysis of selected processes or fixed following some code requirements, or possibly chosen depending upon the level of exactness of the erection process. At any case such an approach permits at least to determine if the shakedown (or yielding) process is stable or unstable. This topic will be discussed in Chapter 6.

CHAPTER 3

GENERALIZED VARIABLES AND YIELD CRITERIA

3.1 Description of beam element in space

Let us recall general linear static and geometric relations for a straight beam element, the hypotheses of plane cross-section and of unconstrained torsion assumed. The two-node element is loaded at its ends (nodes) only. The local coordinate system is introduced, with x-axis tangent to the element axis and y, z following principal axes of the bi-symmetric cross-section. The element has 12 degree of freedom, six at each nodal point. In the general model the vector of cross-sectional stress resultants $s^{(e)}$ has 12 components (see Fig. 3.1):

$$s^{(e)} = \left\{ n_x^l \; m_x^l \; m_y^l \; m_z^l \; t_y^l \; t_z^l \; n_x^r \; m_x^r \; m_y^r \; m_z^r \; t_y^r \; t_z^r \right\}^T \tag{3.1}$$

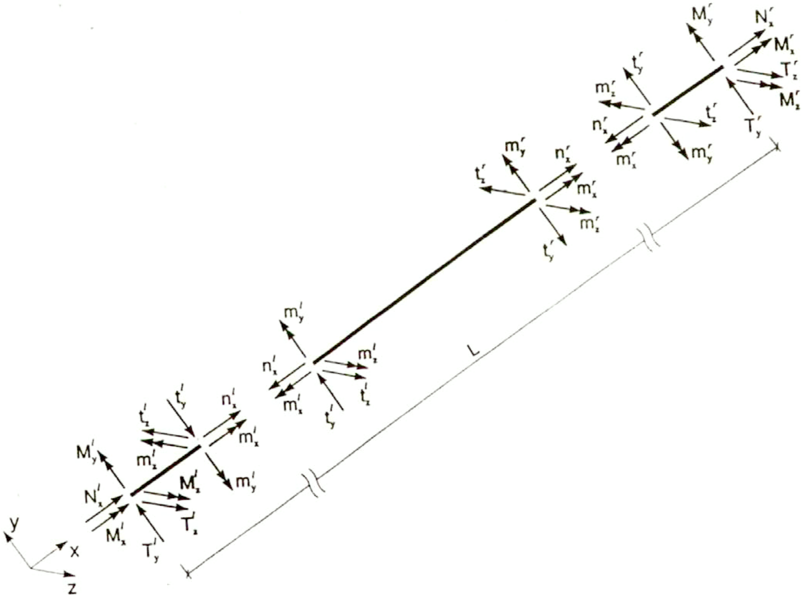


Fig. 3.1 Equilibrium of bar space element

where

n_x	normal force,
m_x	torsional moment,
m_y, m_z	bending moments in oxz & oxy and
t_y, t_z	shearing forces in oxy & oxz

for each left (^{l}) and right (^{r}) node, respectively. The load vector $p^{(e)}$ of corresponding components of external forces applied at left and right nodes, has also 12 components (see Fig. 3.2):

$$p^{(e)} = \left\{ N_x^l \ M_x^l \ M_y^l \ M_z^l \ T_y^l \ T_z^l \ N_x^r \ M_x^r \ M_y^r \ M_z^r \ T_y^r \ T_z^r \right\}^T \tag{3.2}$$

with the same notation as before.

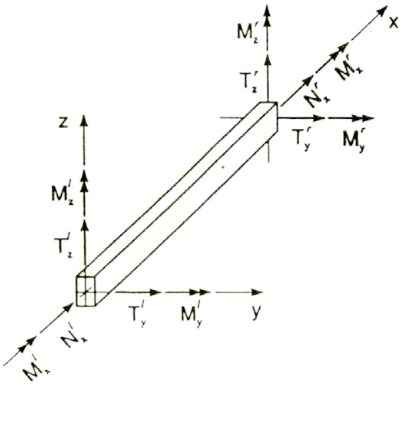


Fig. 3.2 Space beam element: external loads

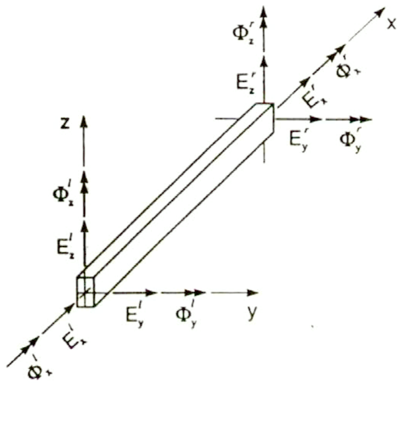


Fig. 3.3 Space beam element: generalized displacements

The vector of stress resultants $\mathbf{s}^{(e)}$ must be in equilibrium with the vector of external loads $\mathbf{p}^{(e)}$ (Fig. 3.1). This condition gives 12 equilibrium equations, which may be written in a matrix form as:

$$\mathbf{p}^{(e)} = \mathbf{C}^{(e)T}\mathbf{s}^{(e)} \tag{3.3}$$

where $\mathbf{s}^{(e)} \in R^{(12)}$, $\mathbf{p}^{(e)} \in R^{(12)}$ and $C^{(e)}$ is a 12x12 compatibility matrix.

Introducing dual variables, we get a vector of deformations of the element axis:

$$\mathbf{q}^{(e)} = \left\{e_x^l\ \varphi_x^l\ \varphi_y^l\ \varphi_z^l\ e_y^l\ e_z^l\ e_x^r\ \varphi_x^r\ \varphi_y^r\ \varphi_z^r\ e_y^r\ e_z^r\right\} \tag{3.4}$$

and a vector of generalized displacements (Fig. 3.3):

$$\mathbf{u}^{(e)} = \left\{E_x^l\ \Phi_x^l\ \Phi_y^l\ \Phi_z^l\ E_y^l\ E_z^l\ E_x^r\ \Phi_x^r\ \Phi_y^r\ \Phi_z^r\ E_y^r\ E_z^r\right\}^T \tag{3.5}$$

Geometrical relations are:

$$\mathbf{q}^{(e)} = \mathbf{C}^{(e)}\ \mathbf{u}^{(e)} \tag{3.6}$$

In this way we obtained the most general relations (statical and geometrical) for a bar in space frames Figs. 3.2, 3.3. However, the number of variables in these relations can be reduced. Because the loads are assumed acting only at nodes, axial force, shear forces, and torsional moment have to be constant along the element. Following the Bernoulli assumption, shear deformations are disregarded and, therefore, shearing forces are no more active variables (generalized stress). They may be expressed by bending moments. Consequently, the influence of shearing forces on yielding of cross-sections has to be disregarded.

Then, putting $m_x=m_x^r=m_x^l$ and $n_x=n_x^r=n_x^l$, we have finally a 6-component vector of generalized stresses, as shown below:

$$\mathbf{s}^{(e)} = \left\{n_x\ m_x\ m_y^l\ m_z^l\ m_y^r\ m_z^r\right\}^T \tag{3.7}$$

The above reduction of generalized stress state induces reduction in dual variables. Using total elongation $e_x = e_x^l - e_x^r$ and total torsion rotation $\varphi_x = \varphi_x^l - \varphi_x^r$, the generalized strains are:

$$\mathbf{q}^{(e)} = \left\{ e_x \;\; \varphi_x \;\; \varphi_y^l \;\; \varphi_z^l \;\; \varphi_y^r \;\; \varphi_z^r \right\}^T \tag{3.8}$$

Finally, the equilibrium equation $\mathbf{p}^{(e)} = \mathbf{C}^{(e)T}\mathbf{s}^{(e)}$ and the geometrical relations $\mathbf{q}^{(e)} = \mathbf{C}^{(e)}\mathbf{u}^{(e)}$ are, for this case, shown on Fig. 3.4.

	N_x^l	T_y^l	T_z^l	M_x^l	M_y^l	M_z^l	N_x^r	T_y^r	T_z^r	M_x^r	M_y^r	M_z^r
n_x	-1						1					
m_x				-1						1		
m_y^l			$-\frac{1}{L}$		1				$\frac{1}{L}$			
m_z^l		$\frac{1}{L}$				1		$-\frac{1}{L}$				
m_y^r			$-\frac{1}{L}$						$\frac{1}{L}$		1	
m_z^r		$\frac{1}{L}$						$-\frac{1}{L}$				1

Fig. 3.4 Equilibrium equations for the used bar model in local coordinates

As it was assumed in Section 2.3 the plastic deformations are concentrated at nodes, so the elastic law is applied for the whole element:

$$\mathbf{s}^{(e)} = \mathbf{E}^{(e)}\mathbf{q}^{(e)}, \tag{3.9}$$

where the elasticity matrix (see Fig. 3.5) depends on the geometry of the element (L - length, A - cross-sectional area and I, J - moments of inertia) and on material proprieties.

	e_x	φ_x	φ_y^l	φ_z^l	φ_y^r	φ_z^r
n_x^r	$\frac{EA}{L}$					
m_x^r		$\frac{GJ}{L}$				
m_y^l			$\frac{4EI_y}{L}$		$\frac{2EI_y}{L}$	
m_z^l				$\frac{4EI_z}{L}$		
m_y^r			$\frac{2EI_y}{L}$		$\frac{4EI_y}{L}$	
m_z^r				$\frac{2EI_z}{L}$		$\frac{4EI_z}{L}$

Fig. 3.5 Elasticity matrix for the used bar model in local coordinates

Before assembling the structure, i.e., before writing global equations for the whole frame starting from equations for an element, we have to fix a global coordinate system and to transform the relations derived before in the local system into the global one OXYZ (Fig. 3.6).

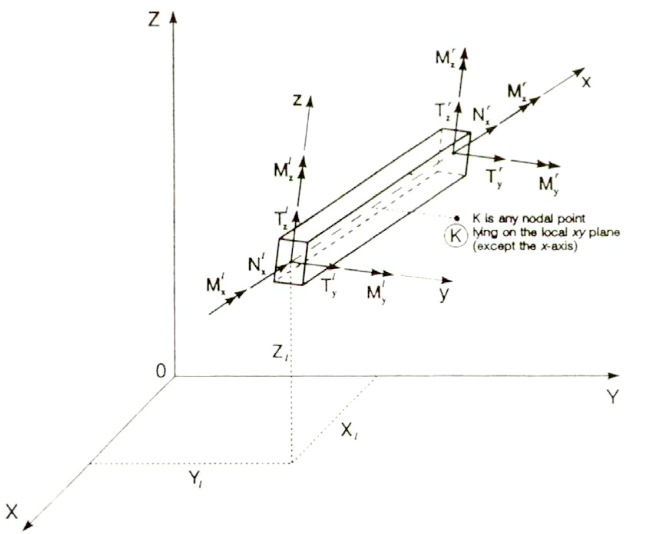


Fig. 3.6 Local and global coordinates

In the general case of vectors $\mathbf{s}^{(e)}$, $\mathbf{q}^{(e)}$, $\mathbf{p}^{(e)}$, $\mathbf{u}^{(e)}$ describing the state of the structure, only the vector of external loads $\mathbf{p}^{(e)}$ and the vector of generalized displacement $\mathbf{u}^{(e)}$ may be transformed.

All the above operations are done in subroutines (ctmat.for) and (emat.for) which were introduced in the POL(SAP) main program for the elastic solution (see Section 4.3).

The relation between both coordinate systems (rotation matrix) is determined from global coordinates of both ends of the element and an external point on one of principal planes (see point K in Fig. 3.6), where this point is any point that lies on the local x-y plane (outside the x-axis).

3.2 Yield criteria

When describing the limit analysis or shakedown problems, the plasticity condition entering into the fundamental theorems (1.1, 1.2) should be formulated in the same way as the whole problem, i.e., in our case - in generalized stresses. This is called the yield criterion (YC), and is represented as a function of cross-sectional active stress-resultants (generalized stress) s^c by several analytical expressions (as in Eq. (2.3)):

$$f_j(\mathbf{s}^c) < \mathbf{K}_j \tag{3.10}$$

with

$$\mathbf{s}^c = \left\{ \mathbf{n}\ \mathbf{m}_x\ \mathbf{m}_y\ \mathbf{m}_z \right\} \tag{3.11}$$

Construction of the yield criterion was for long a central point of plastic analysis, especially for shells (e.g., [106], [121]). Since yielding of cross-sections at different end nodes is independent, constructing the corresponding yield criteria and collecting them in a supervector for the whole structure is trivial and it will be presented for PWL yield criteria later.

Satisfying a yield criterion for a cross-section means that an unconstrained plastic flow may occur at the cross-sectional plane. Following the assumptions of the classical beam theory which determines the set of generalized stresses (3.11) and using the local coordinate system (see Section 3.1), yielding means that some of the strain rates $\dot{\varepsilon}_{xx}$, $\dot{\varepsilon}_{xy}$, $\dot{\varepsilon}_{xz}$ are non-vanishing. It corresponds to non-vanishing curvatures changes, twist and extension of the beam axis, and if the lumped model is adopted, with plastic deformations concentrated in the plastic hinges at nodes, the vector of generalized strain is:

$$\dot{\mathbf{q}}^c = \left\{ \dot{e}\ \dot{\varphi}_x\ \dot{\varphi}_y\ \dot{\varphi}_z \right\} \tag{3.12}$$

The cross-section may be considered as a structure under the set of external loads (3.11). To be more clear, we can imagine, e.g., a cantilever beam loaded at the end by the set (3.11). Generalized stresses in this beam are constant along its axis and equal to the loads (3.11). We should solve the limit analysis problem for such a structure and the limit load surface in Fig. 1.5 becomes the yield surface (YS) representing the yield criterion for the cross-section.

It is convenient to present the respective yield surfaces in nondimensional coordinates

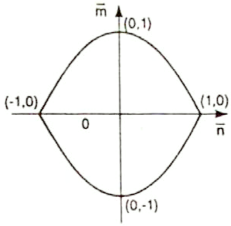
$$\bar{n} = \frac{n}{n_o}, \quad \bar{m}_x = \frac{m_x}{m_{ox}}, \quad \bar{m}_y = \frac{m_y}{m_{oy}}, \quad \bar{m}_z = \frac{m_z}{m_{oz}} \tag{3.13}$$

where the plastic moduli n_o, m_{ox}, m_{oy} and m_{oz} are the yield values under separate action of one variable only. These moduli may be easily obtained (excluding torsion) from commercial catalogues of steel profiles.

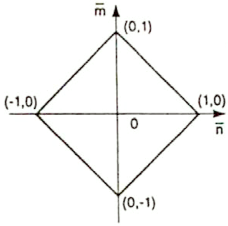
Following the beam assumption, the plasticity condition should be used in the form (2.2). It should be noted, that the linear form of the condition (e.g., in the absence of torsion) does not imply the linearity of the yield criterion.

Plastic flow of the cross-section needs all its surface to be plastified, with eventual exception of the neutral axis. Therefore, in the absence of torsion (only normal stresses remain) the problem becomes elementary. It is reduced to calculation of the resultants for two uniformly stressed ($\sigma_{xx} = \sigma_o$ and $\sigma_{xx} = -\sigma_o$) parts of the cross-section separated by a straight neutral axis. Expressions for all stress resultants (n, m_y, m_z) present a parametric form of the yield criterion, the parameters describing the position of the neutral axis in the plane (y,z). The corresponding procedure and numerous results may be found, e.g., in [17].

In this way, the well known parabola for rectangular cross-section and the rhombus for ideal sandwich may be easily obtained (Fig. 3.7). In the case of lumped-area models of cross-sections, considering the surface area to be concentrated in points or layers (sandwich, "multipoint" cross-sections in [121]), these criteria are PWL.



(a) Rectangular cross-section



(b) Sandwich cross-section

Fig. 3.7 Yield surface for moment-force interaction

Determining the yield criterion in the presence of torsion is a more difficult task. Exact analytical solutions are available only in particular cases and bounding limit analysis techniques have to be used. Numerous old case studies are available [49], [121] and the problem is studied up to now. Finite element linear programming approach (2.42) is now commonly applied [19], [39] and quasi-automatic procedures were proposed [28].

Determination of the yield criteria is essential for the analysis of space frames. However, this topic is outside the scope of our study. Following the description above, we may consider that for a given cross-section the YC is known. In reality, data for practically used steel profiles are rare or lacking, but preparing an appropriate catalogue of this type should be an object of a commercially oriented study.

3.3 Elastic Criteria

When the load multiplier ξ_E concerning the elastic-limit load (Fig. 1.5) is looked for or when we determine the shakedown multiplier ξ_{SD} concerning appearance of the alternating plasticity (see Section 2.4), the elastic criterion (EC) for the cross-section is needed. Similarly to the yield criterion, we can write it in the same variables as (3.11):

$$f_j^E(s^c) \leq k_j^E \tag{3.14}$$

with k_j^E being elastic moduli (elastic-limit value) under a separate action of each stress resultant. Equal sign in (3.14) means appearance of first plasticity in the cross-section. It means that the criterion (2.2):

$$f = (\sigma^2 + c\ \tau^2)^{1/2} = \sigma_o \tag{3.15}$$

is satisfied at some point of the cross-section. Following the classical beam theory adopted here, stresses are linear functions of the variables s^c. Thus, the normal stress is:

$$\sigma = \frac{n}{A} + \frac{m_y \bar{z}}{I_y} + \frac{m_z \bar{y}}{I_z} \tag{3.16}$$

with A, I_y, I_z being the area and moments of inertia of the cross-section and $\bar{y}$, $\bar{z}$ being coordinates of the extreme points of the contour.

For shear stress induced by torsion, let us recall only the simplest formula concerning any open cross-section approximated by a system of thin rectangles:

$$\tau = \frac{m_x\ t}{J}\ \alpha \tag{3.17}$$

with t being the thickness of the cross-section, J the sum of polar moments for sub-elements, and α a correction factor derived from a more sophisticated analysis. In calculation of τ, also stresses due to shearing forces t_y, t_z should be, in principle, taken into consideration, since this analysis needs not to be compatible with the kinematical assumptions. However, these assumptions are valid when the stresses from shearing forces are small. Therefore, they are neglected here to make the approach more consistent.

Verifying equalities (3.14) with the use of the above formulae all over the cross-section means determining equations of the EC corresponding to selected points of the cross-section. The points are extreme corners and centers of the longer sides of rectangular components. If a more refined approach is used, concave angles of the cross-section should be also considered. This procedure is a standard task of structural dimensioning; only its presentation (construction of the elastic-limit surface ES) is adapted to our needs. Some commercial computer codes give subroutines for performing that. However, until recently they covered only the case with absence of torsion. Their recent versions (e.g., ABAQUS version 5.2, WDKM [87]) include subroutines for the general case, even for complex thin-walled cross-sections. Using this subroutines for different configurations of stress variables, the ES may be easily derived.

The elastic moduli k_j^E in (3.14) are simply:

$$n_E = n_o = \sigma_o\ A, \quad m_{Ey} = \frac{\sigma_o\ I_y}{\max|z|}, \quad m_{Ez} = \frac{\sigma_o\ I_z}{\max|y|}, \quad m_{Ex} = \frac{\sigma_o\ J}{\sqrt{3}\ \max\ t\ \alpha} \tag{3.18}$$

The last expression corresponds to the Huber-Mises plasticity condition (c=3 in Eq. 3.14). To represent the EC in the space of generalized stress s^c as a convex elastic-limit surface (ES) the nondimensional coordinates are used as in Section 3.2 (Eq. 3.13). The coordinates of the ES at the axes will be:

$$\bar{n}_E = 1, \quad \bar{m}_{Ex} = \frac{m_{Ex}}{m_{ox}}, \quad \bar{m}_{Ey} = \frac{m_{Ey}}{m_{oy}}, \quad \bar{m}_{Ez} = \frac{m_{Ez}}{m_{oz}} \tag{3.19}$$

Alternative nondimensional coordinates for the ES only may be

$$\bar{n}'=1,\ \bar{m}'_x = \frac{m_x}{m_{Ex}},\quad \bar{m}'_y = \frac{m_y}{m_{Ey}},\quad \bar{m}'_z = \frac{m_z}{m_{Ex}} \tag{3.19a}$$

which gives normalized values on the axis $\bar{m}'_{Ei} = \pm 1$.

3.4 Piecewise Linear Criteria

The linear programming formulation of the limit analysis and shakedown problem needs a PWL yield criterion (2.24). Its segment for a given cross-section is:

$$\mathbf{f} = \mathbf{N}^{cT}\mathbf{s}^{c} - \mathrm{K} \le 0 \tag{3.20}$$

with the matrix **N** composed by column vectors representing gradients of plane faces of the yield surface. The equation (3.20) in the space of four generalized stresses (3.11) may be presented symbolically as in Fig. 3.8(a) or, using nondimensional coordinates (3.13), in Fig. 3.8(b). Such presentation will be useful in assembling the criteria.

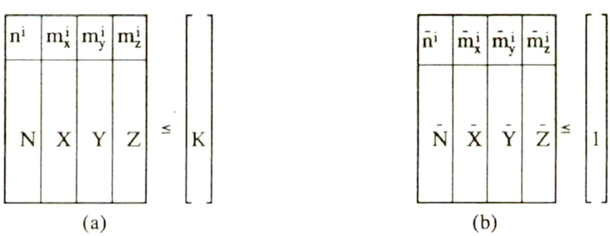


Fig. 3.8 Yield criterion for a cross-section

This linearization may be done either considering a lumped-surface model of the cross-section (sandwich approximation "multipoint" cross-sections [121]), with appropriately linearized plasticity criteria (3.15) or by approximating the non-linear exact YS by a system of hyperplanes as close to it as possible. Such approximations were proposed from the beginning of limit analysis, because they permitted analytical solution. For bounding techniques it is reasonable to use PWL surfaces inscribed in and circumscribed on the exact YS. Because of the convexity condition, the lower and upper bounds for the YS are "generalized sandwich" and "limited interaction" approximations. They are represented by a hyper-pyramid and a hypercube, respectively. (see their shapes in $\bar{n}$-$\bar{m}_z$ space Fig. 3.9(a), (d)). Columns of the corresponding matrices N^T (Fig. 3.8) for the lower and upper-bound approximations are given in Eqs. 3.21 and 3.22, respectively.

$$\bar{N} = \begin{bmatrix} +1\\+1\\+1\\+1\\+1\\+1\\+1\\+1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1 \end{bmatrix} \quad \bar{X} = \begin{bmatrix} +1\\+1\\+1\\+1\\-1\\-1\\-1\\-1\\+1\\+1\\+1\\+1\\-1\\-1\\-1\\-1 \end{bmatrix} \quad \bar{Y} = \begin{bmatrix} +1\\+1\\-1\\-1\\+1\\+1\\-1\\-1\\+1\\+1\\-1\\-1\\+1\\+1\\-1\\-1 \end{bmatrix} \quad \bar{Z} = \begin{bmatrix} +1\\-1\\+1\\-1\\+1\\-1\\+1\\-1\\+1\\-1\\+1\\-1\\+1\\-1\\+1\\-1 \end{bmatrix} \tag{3.21}$$

$$\bar{N} = \begin{bmatrix} +1\\0\\0\\0\\0\\0\\0\\-1 \end{bmatrix} \quad \bar{X} = \begin{bmatrix} 0\\+1\\0\\0\\0\\0\\-1\\0 \end{bmatrix} \quad \bar{Y} = \begin{bmatrix} 0\\0\\+1\\0\\0\\-1\\0\\0 \end{bmatrix} \quad \bar{Z} = \begin{bmatrix} 0\\0\\0\\+1\\-1\\0\\0\\0 \end{bmatrix} \tag{3.22}$$

It should be remarked that these PWL approximations have no physical meaning (no corresponding cross-section model), with exception of the sandwich cross-section for the moment-force interaction.

The size of the shakedown problem and the computation time grow rapidly with the increase of number of inequality constraints in (2.40). Therefore, choosing a PWL approximation of the YS that is simple but sufficiently close to the exact one is very important. We discussed above two extreme PWL forms: the lower bound and the upper bound approximations. They are rarely acceptable, especially the latter. Errors thus introduced may concern not only under - or overestimation of the load multiplier, but also may lead to modeling of the collapse mechanism fairly different than in reality. This fact is well known in the classical limit analysis. As our interest is in the post-yield and inadaptation behaviour, with the configuration changes following the collapse mechanisms, such errors should be avoided.

In order of simplicity, the following approximation, frequently accepted as satisfactory, is the "octant-point" approximation. It consists in determining the YS polyhedron with two vertices on each axis and one in each octant (hyperoctant) of the space. The nondimensional coordinates of the vertices are

$$A(\pm 1,0,0,0),\ \ldots;\ D(\pm d_n,\pm d_x,\pm d_y,\pm d_z) \tag{3.23}$$

with permutation of a non-vanishing coordinate in A and with all the sign combinations in D. The number of vertices will be:

$$2n + 2^n \tag{3.24}$$

n being the dimension of the stress space. The lower and upper-bound approximations are particular case of this type, with some vertices disappearing.

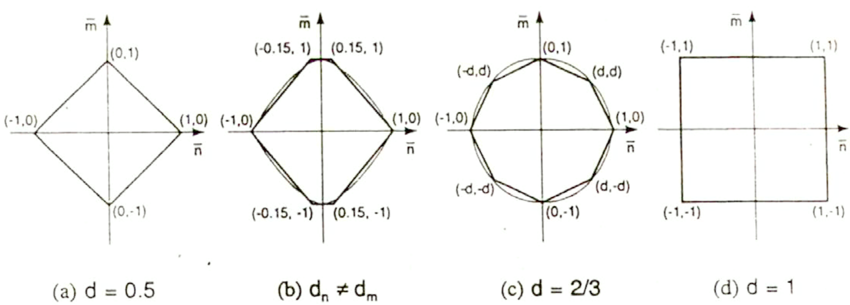


Fig. 3.9 PWL approximation of the moment-axial force interaction

The PWL approximations discussed above are shown in the 2-dimensional case (interaction of bending moment and axial force) in Fig. 3.9. Fig. 3.9(a) shows the lower bound (sandwich) type and the Fig. 3.9(b) illustrates an octant-point approximation, proposed in codes for I-shaped commercial profiles. Fig. 3.9(c) shows a PWL approximation of the curve for rectangular cross-section. It belongs also to the octant-point type but with its particular form: the PWL surface is inscribed into the exact one and the point D is lying on the bisector $d_i=d_j$ (hyper-bisector) of the hyperoctant. This approximation is called, following [39] "d-approximation" and, since it is accepted in the major part of our computations, it will be discussed in detail in the next Section.

When one or several generalized stresses appear to be small in comparison to their moduli, the YS may be assumed to be a parallelepiped with generatrices parallel to its axis. Limited interaction (3.22) is assumed between this variable and the others. We have now a "partial-limited interaction", which may be represented in symbolic form, following Fig. 3.8(b), as below:

$\bar{n}_x$	$\bar{m}_x$	$\bar{m}_y$	$\bar{m}_z$
0	$\bar{X}$	$\bar{Y}$	$\bar{Z}$
1	0	0	0
-1	0	0	0

≤ (1)

(a)

$\bar{n}_x$	$\bar{m}_x$	$\bar{m}_y$	$\bar{m}_z$
$\bar{N}$	0	$\bar{Y}$	$\bar{Z}$
0	1	0	0
0	-1	0	0

≤ (1)

(b)

Fig. 3.10 Yield criterion for slender structures and structures with small torsional rigidities

The cases in Fig. 3.10 correspond to slender structures (a) and to structures with small torsional rigidities (b). Both cases are very frequent in the engineering practice.

When a yield criterion for an element is to be derived, we should compose two criteria for both end cross-sections of the element. Taking into account that n and m_x are equal in both cross-sections, the generalized stress space becomes 6-dimensional (3.7). However, yielding of both cross-sections is independent and it means that the limited-interaction appears between the two sub-surfaces. It is the situation like that presented in Fig. 3.10. We can represent now the YC for an element with an extended matrix N^{eT}, like in Fig. 3.11.

$$
\begin{array}{cccccc}
\bar{n}^e & \bar{m}_x^e & \bar{m}_y^L & \bar{m}_z^L & \bar{m}_y^R & \bar{m}_z^R
\end{array}
$$

$$
\left[\begin{array}{c|c|c|c|c|c}
\bar{N} & \bar{X} & \bar{Y} & \bar{Z} & 0 & 0 \\
\hline
\bar{N} & \bar{X} & 0 & 0 & \bar{Y} & \bar{Z}
\end{array}\right] \leq \left[\begin{array}{c} 1 \\ 1 \end{array}\right]
$$

Fig. 3.11 Yield criterion for an element

Assembling the yield criterion for the whole structure means constructing a hyperdiagonal matrix N^T with the matrices N^{eT} (Fig. 3.11) for all the elements.

All the considerations in this Section may concern also elastic criteria. Linearization is eased in this case, because in the absence of torsion the exact surfaces are PWL, at least for PWL shapes of the cross-sections. It should be noted, that if nondimensional coordinates (3.13) are used, coordinates of the vertices of the ES on axes are now as in (3.19) and not (1 0 0 0, ...; etc.).

3.5 "D-approximation" of the yield criterion

We shall consider here in more detail the most convenient for implementation form of the "octant-point" approximation (3.23). It concerns the polyhedral surface inscribed into the exact YS and with vertices on axes (A-point) and on hyperbisectors (D-points). In (3.23) we should put:

$$d_n = d_x = d_y = d_z = d \tag{3.25}$$

This approximation was proposed in [39] and successfully applied to the limit analysis of space frames [25], [26], [39]. No other applications to the plastic analysis of frames accounting for all the four generalized stresses are known.
Data collected in [39] from case studies for cross-sections of different shapes and under different sets of generalized stresses give idea on the values of d appearing in practice. They are recalled in Tab. 3.1. Some d-values for PWL approximations of some theoretical surfaces are given in Tab. 3.2.

No.	Active Internal Forces	Square	Hollow box	Wide Flange
1	$n_x + m_y$	0.618	0.588	0.545
2	$n_x + m_x$	0.618	0.565	0.678
3	$n_y + m_z$	0.677	0.664	0.646
5	$n_x + m_y + m_z$	0.518	0.500	0.518
6	$m_x + m_y$	0.721	0.717	0.766
7	$m_x + m_y$	0.721	0.719	0.770
8	$m_x + m_y + m_z$	0.571	0.571	0.605
9	$n_x + m_x + m_y + m_z$	0.471	0.461	0.504

Tab. 3.1 Coordinates of d-vertex in positive "octant" of R^2, R^3 and R^4 spaces for different shapes of the cross-sections

Following (3.24) the number of vertices is 24 and the corresponding matrix **V** of their coordinates is:

$$\mathbf{V}=\left\{\begin{matrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & d & -d & d & d & d & -d & -d & -d & d & d & d & -d & -d & d & d & -d \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & d & d & -d & d & d & -d & d & d & -d & -d & d & -d & -d & -d & d & -d \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & d & d & d & -d & d & d & -d & d & -d & d & -d & -d & d & -d & d & -d \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & d & d & d & d & -d & d & d & -d & d & -d & -d & d & -d & -d & d & -d \end{matrix}\right\} \tag{3.26}$$

Selecting square submatrices of (3.26) means selecting hyperplanes determined by such sets of four vertices (hyper-tetragonal faces). Excluding meaningless submatrices (singular or corresponding to planes interior to any vertex), we arrive at 48 sets describing 48 faces of the polyhedron. Details of the corresponding permutation procedure may be found, e.g., in [25].

Equations of the corresponding hyperplanes are obtained, in a standard way, by putting zero as a value ot the extended determinant of the submatrix considered, as below:

$$\det\begin{vmatrix} \bar{n}_x & \bar{m}_x & \bar{m}_y & \bar{m}_z & 1 \\ & & & & 1 \\ & & & & 1 \\ & \text{sub } \mathbf{V}^T & & & 1 \\ & & & & 1 \end{vmatrix} = 0 \tag{3.27}$$

The above describes the j^{th} row in the matrix $\mathbf{N}^{cT}$. It may be easily shown that because of the symmetries of the YS only one configuration of the submatrices may appear for the whole yield polyhedron.
These possible configurations are, following the notation from (3.23):

I: (A,D,D,D), II: (A,A,D,D), III: (A,A,A,D) (3.28)

Let us consider the configuration (A,D,D,D). To select the submatrices in determinant of Eq. (3.27) we should associate with each vector A three of four neighbouring vectors D, e.g.:

A(0, 0, 0, 1), D_1(d, d, d, d), D_2(d, -d, d, d), D_3(d, d, -d, d) (3.28)

Calculating the determinant (3.27) with the coordinates of the above points we obtain the corresponding equation of the hyperplane:

$$(1-d)\,\bar{n} + d\,\bar{m}_z = d \tag{3.29}$$

Other hyperplanes will correspond to meaningful combinations of three points D from the eight neighbouring each point A (six combinations) and the above should be repeated for all eight A-vertices. That finally gives 6x8=48 planes. The columns of the matrix $\mathbf{N}^{cT}$ Fig. 3.8(a) are given in (3.32), in the dimensional form used in practical calculations.

Notations used in (3.32) are:

$$d_i = d\,\alpha_i, \qquad e_i = (1-d)\alpha_i, \qquad i = n,\ x,\ y,\ z \qquad \text{with} \tag{3.30}$$

$$\alpha_n = \frac{m_{oy}}{n_{ox}}, \quad \alpha_x = \frac{m_{oy}}{m_{ox}}, \quad \alpha_y = \frac{m_{oy}}{m_{oy}} = 1, \quad \alpha_z = \frac{m_{oy}}{m_{oz}} \tag{3.31}$$

$$N = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\ d_n\\-d_n\\d_n\\-d_n\\e_n\\-e_n\\e_n\\-e_n\\d_n\\-d_n\\d_n\\-d_n\\e_n\\-e_n\\e_n\\-e_n\\d_n\\-d_n\\d_n\\-d_n\\e_n\\-e_n\\-e_n\\e_n\\ 0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \quad X = \begin{bmatrix} e_x\\-e_x\\-e_x\\e_x\\d_x\\-d_x\\d_x\\-d_x\\e_x\\-e_x\\-e_x\\e_x\\d_x\\-d_x\\d_x\\-d_x\\e_x\\-e_x\\-e_x\\e_x\\d_x\\-d_x\\d_x\\-d_x\\ 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \quad Y = \begin{bmatrix} d_y\\-d_y\\d_y\\-d_y\\e_y\\-e_y\\-e_y\\e_y\\ 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\ e_y\\-e_y\\-e_y\\e_y\\d_y\\-d_y\\d_y\\-d_y\\ 0\\0\\0\\0\\0\\0\\0\\0\\ e_y\\-e_y\\-e_y\\e_y\\d_y\\-d_y\\d_y\\-d_y \end{bmatrix} \quad Z = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\ d_z\\-d_z\\d_z\\-d_z\\e_z\\-e_z\\-e_z\\e_z\\ 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\ e_z\\-e_z\\-e_z\\e_z\\d_z\\-d_z\\d_z\\-d_z\\d_z\\-d_z\\d_z\\-d_z\\e_z\\-e_z\\-e_z\\e_z \end{bmatrix} \quad K = d \cdot \begin{bmatrix} m_{oy}\\m_{oy} \end{bmatrix} \tag{3.32}$$

It is easy to verify that the configuration (ADDD) considered in details above gives a convex surface if d ≥ 0.5. As it can be seen from Tab. 3.1, this is nearly always the case. Some values of d may be slightly less than 0.5. However, as numerical data in Section 5.2 prove, results are only slightly sensitive to small changes in d around d=0.5. Therefore, our main interest will devoted to this configuration. Other cases will be discussed in the 3-dimensional space below.

Although the problem of space frames should be considered in the 4-dimensional stress space, in many practically important cases this space may be reduced to R^3. As it was already mentioned, torsional elastic rigidity of open thin-walled cross-sections is small in comparison with flexural rigidity. Therefore, torsional moments are small ($\bar{m}_x$ << 1) and can be frequently neglected. Another case that may be considered in R^3 is that with axial forces ($\bar{n}$ << 1) neglected. It is possible when horizontal loads are important and vertical loads applied directly to columns are not of primary importance. We have in these two cases $s^c = (n, m_y, m_z)$ or $s^c = (m_x, m_y, m_z)$. To cover both cases we should present the R^3 problem in $s^c = (s_1, s_2, s_3)$.
In the 3-dimensional case, we shall have, following (3.28), two configurations possible:

I: (A, D, D), II: (A, A, D) (3.33)

The first of these cases of the YS (considered above) gives a convex polyhedron for $\frac{1}{2} \le d \le 1$, the second for $\frac{1}{3} \le d \le \frac{1}{2}$. They are shown in Fig. 3.12, together with their extreme case for upper and lower bounds.

Numbers of vertices and hyperplanes are given in Tab. 3.2 for different number of space dimensions.

Type of presentation	R^n		Approx. 1 lower bound		d-Approx. d ≥ 0.5		Approx.3 Upper bound
No of vertices V^e	R^2	4		8		4	
	R^3	6	$2 \times n$	14	$2 \times n + 2^n$	8	2^n
	R^4	8	$d = \frac{1}{n}$	24		16	d=1
NO of Hyperplanes	R^2	4		8		4	
N^T	R^3	8	2^n	24	$4 \times n(n-1)$	6	$2 \times n$
	R^4	16		48		8	

Tab. 3.2 Number of vertices and number of hyperplanes for different yield criteria

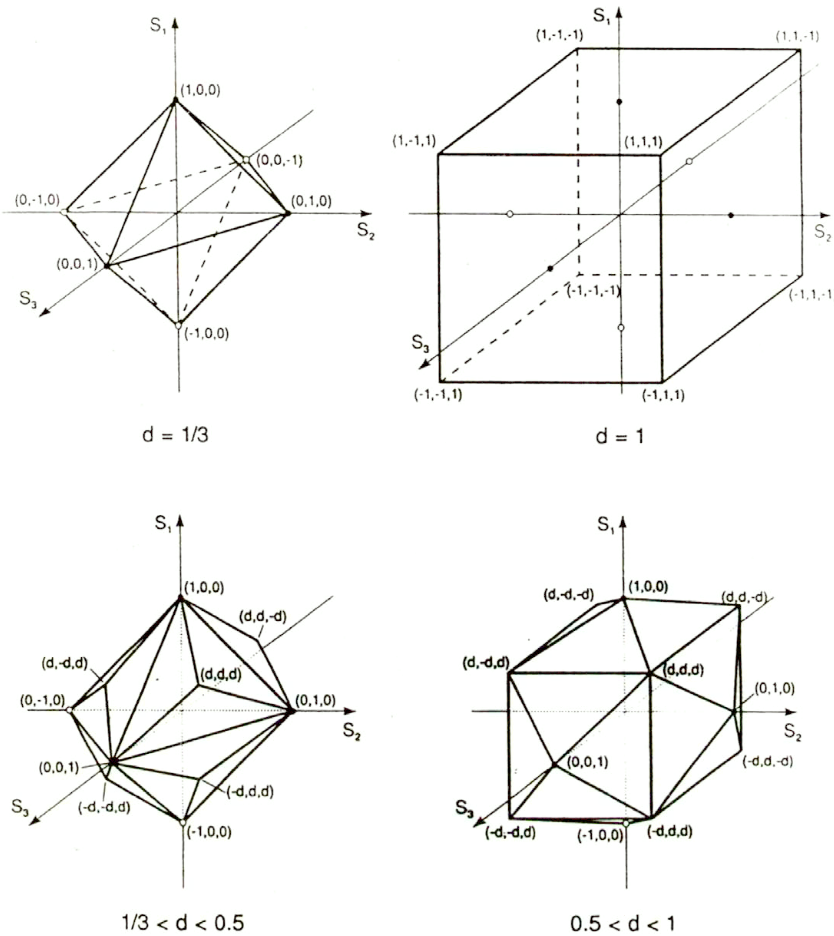


Fig. 3.12 d-polyhedron in R^3

It should be mentioned that the PWL yield criteria may be used in linear programming in a form alternative to the hyperplane description applied here. It is a so-called "vertex description" [120], using directly the vertex matrix (3.26) and proceeding to convex combination of the vertices. Details of such a procedure may be found in [25]. Some time ago this approach was considered as highly promising [25], [75], but these hopes seem now to be exaggerated.

CHAPTER 4

NUMERICAL PROGRAM SDLAS

4.1 Assumptions and methods

The program SDLAS concerns shakedown analysis of space frames and its extreme case of limit analysis. Loads are determined by a finite number of parameters β_i. They may vary arbitrarily inside a hyperrectangular domain in the space of the parameters. The shape of this domain is given up to a load multiplier ξ by the reference domain Ω_R (Fig. 1.5, Eq. (2.16)). The output of the program is the load multiplier for the limit load ξ_L, for the shakedown load ξ_{SD} and for the elastic limit load ξ_E. It is accompanied with the corresponding immediate collapse and incremental collapse mechanisms. Using a FE discretization in straight beam elements, the problem was formulated (in general variables, see Section 2.4) as a linear programming problem. The classical Bernoulli–St. Venant theory of space beams is used, so that contribution of four generalized stresses: axial force n, torsion moment m_x and biaxial bending m_y, m_z is accounted for, both for elastic deformation and for yielding. Two-node beam elements are used, the loads are considered to be applied in nodes only and at shear centers of the cross-sections.

The formulation is based on geometrically linear relations. Taking into account geometry changes due to deformation is assumed by a step-by-step linearized procedure, i.e., by sequential solutions of the linear problem for the structure with a modified configuration. Configuration change is generated by the actual mechanism of incremental collapse (in shakedown) or of immediate collapse (in limit analysis). Details and supplementary assumptions of this approach are given in the next Section.

Yield criteria YC and elastic criteria EC corresponding to full plasticity and to first plasticity of the cross-section, respectively, are represented by PWL approximations. Any approximation can be used, but numerical results are given for a 48-face polyhedral yield surface (or elastic surface), the so called "d-approximation" (see Section 3.5). For comparison an upper-bound PWL approximation (8 faces) and a lower bound PWL approximation (16 faces) are used (Eqs. (3.22) and (3.21), respectively).

The duality of the formulation (Eqs. (2.40), (2.41)) needs the use of the same yield criterion both is statical and kinematical approach. Such approach describes correctly the incremental collapse. The incremental collapse is of main interest for us, because it can contribute to the geometry changes at the inadaptation process. Therefore, the principal module of the program concerns this behaviour; the multiplier ξ_{SD} determines the load domain for which the incremental collapse does not appear. If not specially indicated, "shakedown" means, in the following, shakedown concerning incremental collapse.

The alternating plasticity analysis needs taking into account pseudoresidual stresses self-equilibrated into the cross-section. Following the discussion given in Section 2.4, it may be done by introducing a module of elastic analysis under an appropriately reduced reference load domain (γ_i^o=0 in Eq. (2.15a)).

The linear programming problem is solved using the classical Simplex procedure (see [10]) as used also by the author for optimization problem in frames [6]. The shakedown problem is represented by the Simplex table in Fig. 4.1, obtained directly from the formulation (2.40) and/or (2.41). The limit analysis problem may be treated in the same way with the envelope vector **d** reduced to the vertices of the reference domain Ω_R and with residual stresses $\mathbf{s}^r$ looked for separately for each vertex. However, the form of the formulation (2.40, 2.41) was conceived specially for the shakedown. Therefore, it appears more convenient to treat the limit analysis using the formulation (2.42, 2.43). The corresponding Simplex table is given in Fig. 4.2.

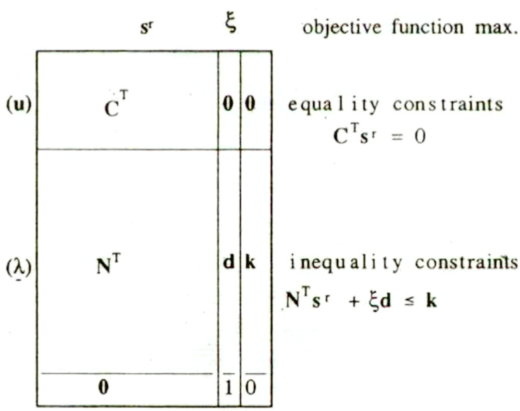


Fig. 4.1 Simplex table for shakedown problem

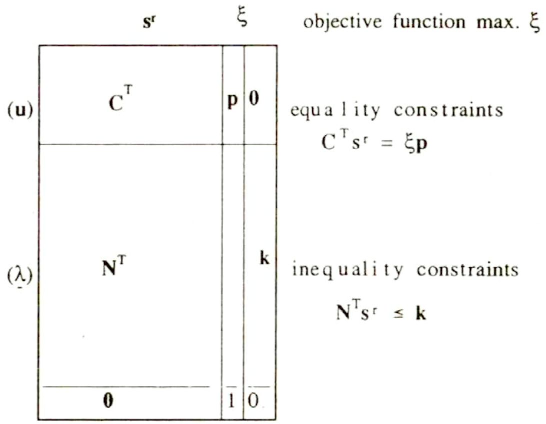


Fig. 4.2 Simplex table for limit analysis problem

Determination of elastic stress distributions s^E from unit loads **T** (Eq. (2.37)) for each loading scheme is done using the POL(SAP) program. Elastic analysis resulting in determination of elastic-limit load multiplier ξ_E represents a subproblem of the shakedown analysis, obtained by putting $s^r = \mathbf{0}$ in the Simplex table in Fig. 4.1.

Alternating plasticity analysis (determination of the corresponding load multiplier ξ_{SDA}) is performed using the module for the elastic subproblem of the shakedown problem, with the reference domain appropriately reduced to $\Omega'_R(\pm\, \gamma_i^a)$, following Eq. (2.15a) with γ_i^o=0. It means that ξ_{SDA} may be determined as:

$$\xi_{SDA}\ (\gamma_i^+, \gamma_i^-) = \xi_E\ (\pm\gamma_i^a) \tag{4.1}$$

The program concerns elastic-perfectly plastic material. However, its extension to piecewise linear hardening, following [72] and taking into account an experience with plane frames [109], should not present serious difficulties.

4.2 Analysis of deformed structures

Following the formulation in Section 2.5, geometry changes induced by plastic deformations are accounted for in a step-by-step linearized manner. It consists of the application of the program to subsequent configurations $\mathbf{x}^{\delta}$ of the structure. Next $\mathbf{x}^{\delta+1}$ configuration is derived from the preceding one by updating nodal coordinates with the values $\mathbf{u}^{\delta}$ corresponding to the actual collapse mechanism of the structure:

$$\mathbf{x}^{\delta+1} = \mathbf{x}^{\delta} + \mu^{\delta}\, \mathbf{u}^{\delta} \tag{4.2}$$

all starting from the solution for the initial configuration $\mathbf{x}^{o}$. Actually the most stringent mechanism $\mathbf{u}^{\delta}$ is used. It is obtained directly as an element of the output from the program on the preceding step. As the collapse mechanisms are determined up to an arbitrary factor, the values $\mathbf{u}^{\delta}$ have to be normalized with respect to the modulus of the maximum deflection:

$$\mu^{\delta} = \frac{\Delta}{\max \left|\mathbf{u}^{\delta}\right|} \tag{4.3}$$

The value Δ is the chosen step for configuration changes. It has to be selected sufficiently small to model the smooth changes appearing in the continuous deformation process. Subsequent configurations differ by a constant value of the step Δ, (Figs. 2.4, 4.3) sometimes corresponding to displacements in different points of the structure. Therefore plotting the curves of the load multiplier ξ_{SD} (or ξ_L) against the displacement should be done using a control displacement component in a characteristic point (u_z^c in Fig. 4.3).

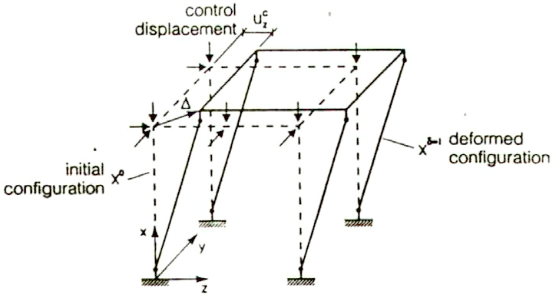


Fig. 4.3 A deformed configuration and control displacement

The existence of a determined collapse mechanism is needed for the program to be able to proceed at consecutive steps. Therefore, this analysis concerns practically incremental collapse and limit analysis. When the alternating plasticity mode (neglecting pseudoresidual stresses) appears, no collapse mechanism is determind and the program stops. However, it may happen that during the deformation process the collapse mechanism changes in such a way that there are no increments of the control displacement at subsequent steps. It will be represented in the load-displacement curve as a final point, similarly as if the alternating plasticity appeared. To distinguish the cases it may be convenient to display the step numbers. Plotting curves for different control displacements in parallel is useful, as it permits the change of the control variable, if needed. Difficulties in the appropriate choice of the control variable appear frequently in nonlinear analysis, when no strictly monotonically increasing variable exists.

The elastic-limit multiplier ξ_E, when calculated for deformed structures, has no physical meaning, since no corresponding mechanism exists. To modify the configuration the shakedown collapse mechanism is used here. This is a subproblem of the shakedown problem and has only an auxiliary character. It may furnish an information of the sensitivity of the elastic-limit load to certain initial imperfections that may be considered to be the most unfavorable.

Moreover, the elastic subproblem with a reduced reference domain (see Eq. (4.1)) constitutes the module for the analysis of alternating plasticity. This module may be used to check at each step of the incremental mode if the alternating plasticity mode is not more restrictive.

4.3 Description of the program SDLAS

The program concerns analysis of elastic-plastic skeletal structures under loads varying arbitrarily in a prescribed domain. All the history-independent types of structural response are considered, i.e., limit analysis, shakedown and elastic limit. Geometry changes due to inadaptation or post-yielding are accounted for with a step-by-step linearized procedure for consecutively modified structure configurations.

For a given shape of the load domain (ratios of extreme values of independently varying loads) the shakedown, the limit-load and the elastic-limit multipliers are obtained and the corresponding collapse mechanisms are determined. The above results are obtained by a sequential procedure for structures deformed in the inadaptation or post-yielding process.

A finite element discretization in two-node spatial beam elements is adopted, loads are also discretized at nodal points. A linear programming formulation is used, with an appropriately piecewise linearized yield criterion. Contribution of axial forces, torsion and biaxial bending to yielding of the cross-section is taken into account. The present version of the program does not account for strain hardening of the material.

Principal components of the program are described below. Its assumptions and methods are discussed in Sections 4.1 and 4.2.

The program consists of the following blocks:

A. The INI PROGRAM containing basic information about the structure and loads which prepares initial data for the main program SDLAS-MAIN for the first step only.

B. The (POL)SAP PROGRAM for elastic analysis. The elastic stress field $\mathbf{s}^e$ is determined (for each of independent unit load systems) for a current structure configuration.

C. The SDLAS-MAIN PROGRAM for limit and shakedown analysis. It determines the envelope vector of generalized elastic stresses **d**; then, the linear programming problem of Eqs. (2.40), (2.41) or (2.42), (2.43) is solved by means of the classical simplex method. In this way the shakedown factor ξ_{SD}, limit analysis factor ξ_{LA} and the corresponding most stringent mechanism of collapse are determined.

D. *SIMPLEX PROCEDURE* using linear programming (LP) for shakedown and limit analysis.

E. The *POST-PROCESSING* determines the current structure configuration after the last step attained and prepares data for the next step.

The input data and data formats, listing of the program INI, listing of the program SDLAS-MAIN, and listing of modified and new subroutines in POL(SAP) are given together with the output results in the Appendices.

PROGRAM SDLAS:

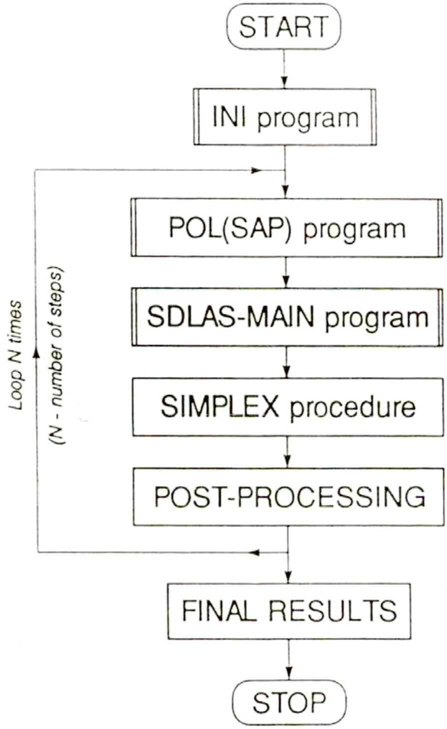


Fig. 4.4 Flow chart of the program SDLAS

A. PROGRAM INI:

The following subroutines prepare input data for the main program SDLAS-MAIN.

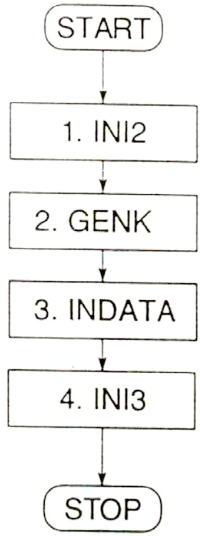


Fig. 4.5 Flow chart of the program INI

where

1. Subroutine INI2 reads the input data.
2. Subroutine GENK generates the vector of yield modulus.
3. Subroutine INDATA writes the initial data.
4. Subroutine INI3 interfaces INI program with SDLAS-MAIN program.

The most important files:

1. Input data (FRAME.DAT)
2. Print the initial data (SSSS.DAT)
3. Data for SDLAS-MAIN program (SDOWN.DAT)

B. PROGRAM POL(SAP):

This program is needed to solve the linear-elastic problems for space frame structure.

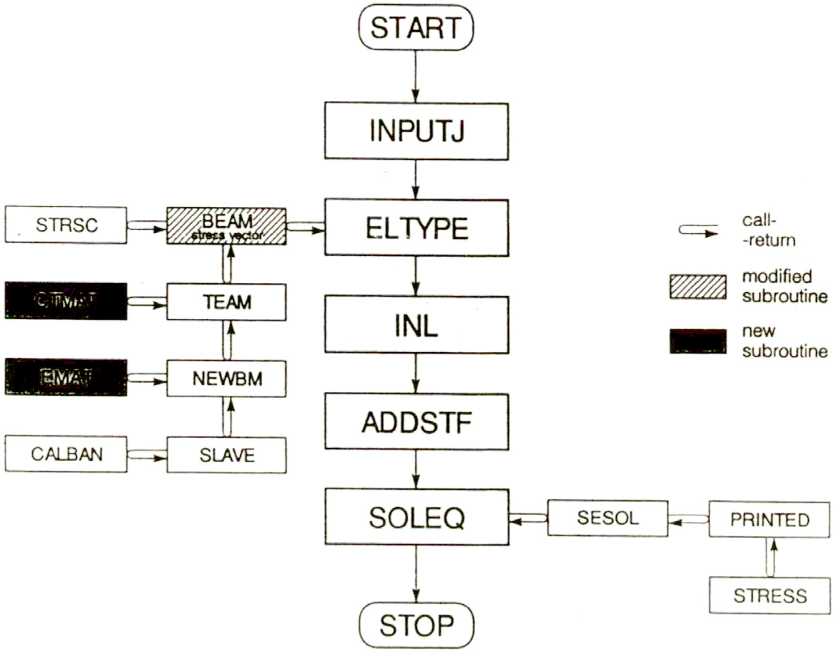


Fig. 4.6 Flow chart of the POL(SAP) program for beams

The most important files:

1. Input data (EL.D)
2. Output results (OEL.D)
3. C^T Matrix (CTMAT.DAT)
4. E Matrix (EMAT.DAT)
5. Stress vector (STVECT.DAT)

C. PROGRAM SDLAS-MAIN:

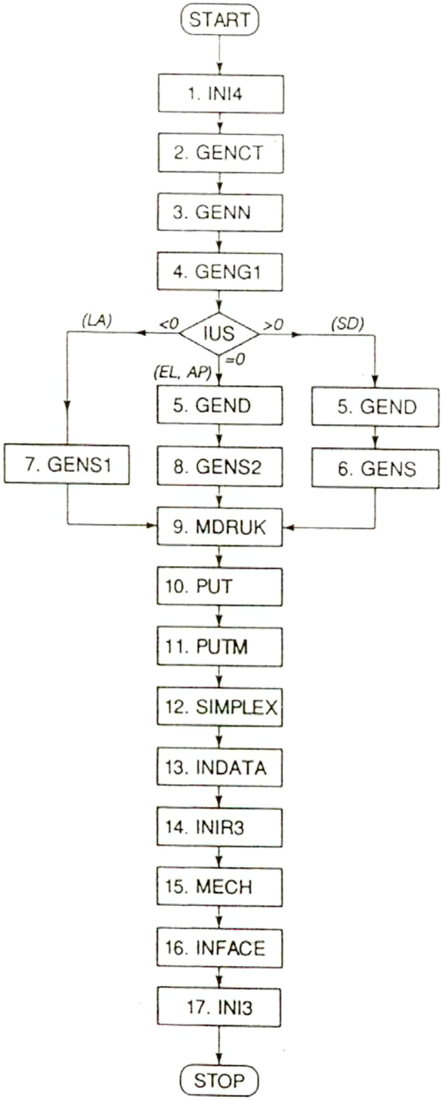


Fig. 4.7 Flow chart of the SDLAS-MAIN program

The functions of these subroutines are as follows:

1. Subroutine INI4 for reading data from INI program (sdwon.dat).
2. Subroutine GENCT for preparing C^T from POL(SAP) program.
3. Subroutine GENN for generation of element gradient matrix.
4. Subroutine GENG1 for generation of the gradients.
5. Subroutine GEND for elastic stresses envelope vector.
6. Subroutine GENS for composing Simplex matrix for SD analysis.
7. Subroutine GENS1 for composing Simplex matrix for LA analysis.
8. Subroutine GENS2 for composing Simplex matrix for EL analysis.
9. Subroutine MDRUK for printing the simplex matrix.
10. Subroutine PUT for putting array in the simplex matrix.
11. Subroutine PUTM for putting matrix C in the simplex matrix.
12. Subroutine SIMPLEX consists of many subroutines (simplex procedure).
13. Subroutine INDATA for printing the initial data.
14. Subroutine INIR3 for writing the results.
15. Subroutine MECH for calculation of mechanism information.
16. Subroutine INFACE for interfacing between SDLAS-MAIN program and POL(SAP) program
17. Subroutine INI3 for writing data for the next step in SDLAS-MAIN program.

The important files:

1. (SSS.DAT) : output results.
2. (STRVECT.DAT) : elastic stress vector from POL(SAP) program.
3. (SDOWN.DAT) : input data for SDLAS-MAIN for the first step only.
4. (SIMP.DAT) : print simplex matrix.
5. (EL.D) : data file for the next step of POL(SAP) program.
6. (PLOT.DAT) : for plotting diagrams of output results.
7. (CTMAT.DAT) : compatibility matrix from POL(SAP) program.

CHAPTER 5

CASE STUDIES

5.1 Selection of examples

Examples considered were selected in order to test the program at different practical situations, to give indications for acceptable approximations of the yield criteria and to furnish an introductory information on some features of spatial response of simple systems. All these aspects were considered both at initial geometry and in the process of growing deformation.

Taking into account the multitude of the parameters influencing response of spatial elastic-plastic frames (geometry, load configuration and domain, elastic rigidities, elastic and plastic strength parameters), even qualitative conclusions need very extensive parametric studies. Therefore, the simplest possible geometries were selected. That concerns a 4-column one-span one-storey rectangular clamped frame ("basic example", Fig. 5.1).

More complex structures were selected in order to test the program and to compare results for limit loads with those obtained by other authors. Unfortunately, only some limit-analysis results for undeformed space frames are known until now [25], [26], [39].

Loading process is described by no more than three independently varying parameters, and load configurations were chosen to permit discretization of the basic structure with no more than 15 elements, and to produce probable sway-type spatial collapse mechanisms. These mechanisms are of special interest if large displacements are considered ("P-Δ effect"). Because of that loads are supposed to vary within such domains that incremental-collapse inadaptation mode is supposed to be the most dangerous. If the alternating plasticity were decisive, no large-displacement study would be of interest.

Experience on the admissible PWL approximations of the yield criterion concerns comparison of results for the d-approximation (Section 3.5) and the upper and lower bounds for the criterion.

Study of the strength parameters concerned comparison of the same structure ("basic example") torsionally compliant (large-flange I-shaped cross-section) in Section 5.2 and torsionally very stiff (box cross-section) in Section 5.3.

Geometric parameters were studied with increasing the height of the structure (Section 5.4) and for spatial interaction of two plane frames (Section 5.6).

Finally a parametric study of eccentrically braced frames (EBF) was presented. These structures are of increasing interest, especially in aseismic engineering, displaying a significant phase of plastic response ("overall ductility") permitting for energy absorption in the case of catastrophic loads.

Studies concerning initial stability of the plastic behaviour and the sensitivity of this process to deformation are gathered in the next Chapter.

5.2 Basic Example I

The space frame in Fig. 5.1 is described in NEL=9 beam-column elements having end sections corresponding to exterior supports, interior joints and at load points along span. The frame has NDF = 30 degrees-of-freedom referenced to the global X-Y-Z axis system indicated in Fig. 5.1, i.e., three displacements and three rotations at each of five free nodes.

All elements have in common an idealized wide-flange cross-section shown in Fig. 5.2. The form of the PWL yield condition for each element end-section is specified as type (1, 2, 3); which corresponds to (nondimensional) PWL yield loci (see Section 3.5): the lower-bound, d=0.504 approximation, and upper-bound, respectively. The value for d was selected following [39], see Tab. 3.1.

Three independently varying sets of loads are applied: vertical P_1 and two horizontal P_2, P_3. Two loading programs are considered with vertical loads oriented either downward or upward. For the undeformed structure results are, of course, identical for both programs. The results for classical limit load multiplier ξ_{LA}, shakedown multiplier ξ_{SD} and elastic limit multiplier ξ_{EL} for different yield conditions are shown in Tab. 5.1.

No.	Yield Criterion (No. of hyperplanes)	L.L ξ_{LA}	S.D ξ_{SD}	E.L ξ_{EL}
1	Lower bound approx. (16hp)	0.12462	0.11427	0.06024
2	"D-approx" d=0.504 (48hp)	0.13274	0.12406	0.06621
3	Upper bound approx. (8 hp)	0.17433	0.15886	0.09984

Tab. 5.1 The values of limit, shakedown and elastic multipliers for different yield conditions (undeformed structure)

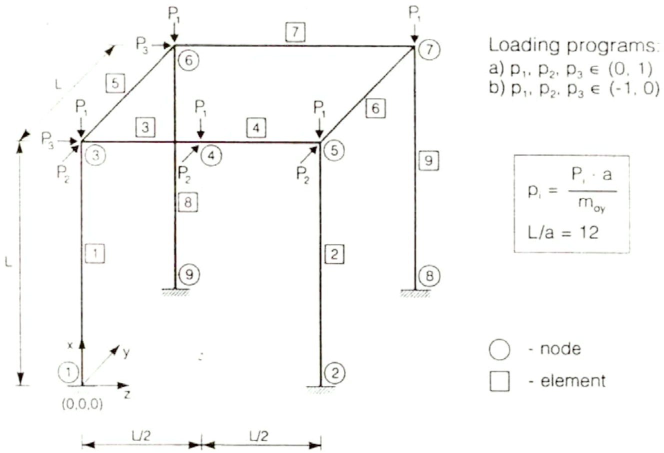


Fig. 5.1 Basic example I

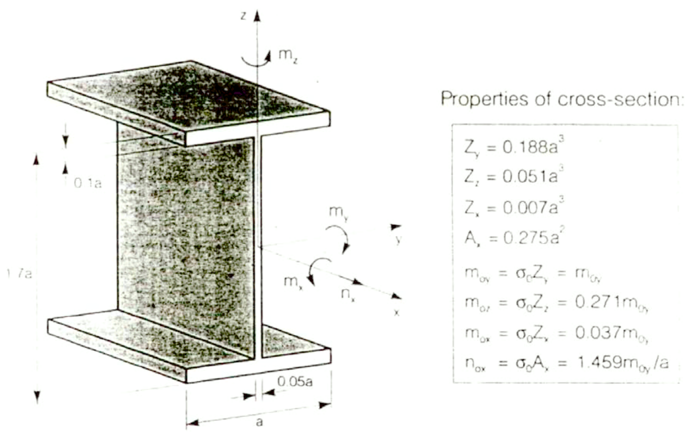


Fig. 5.2 Basic example I (cross-section)

For the deformed structure results will be different for different load programs. To plot the load multiplier-to-displacement curves we choose, following Section 4.2, the step $\Delta=\left|\mathbf{u}_{max}\right|= 1$ cm = (1/600 L) and the control displacement u_{y3}. The curves are plotted in Figs. 5.3(a), (b), (c), for the three yield criterion approximations, respectively. Points on curves correspond to consecutive steps.

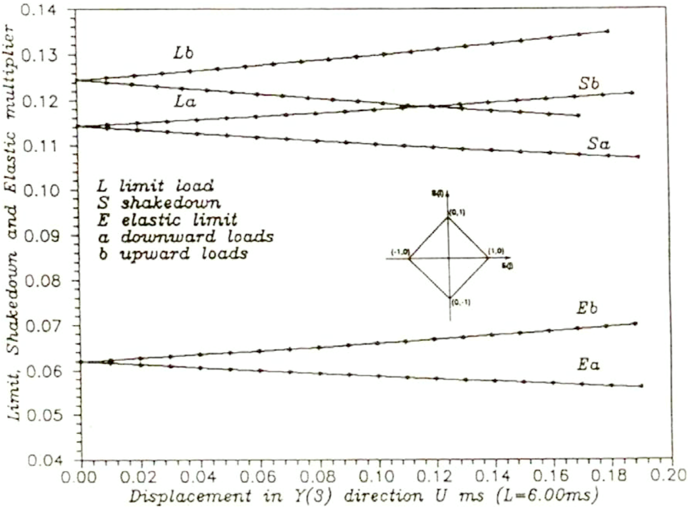


Fig. 5.3 (a) Response to downward and upward loads (d=0.25)

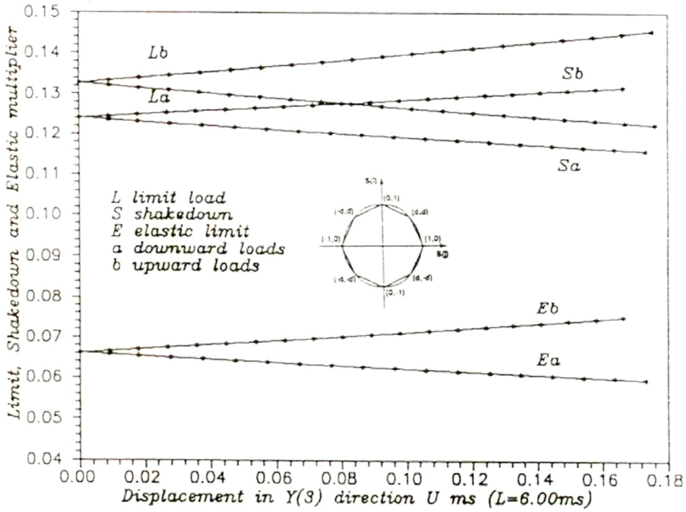


Fig. 5.3 (b) Response to downward and upward loads (d=0.504)

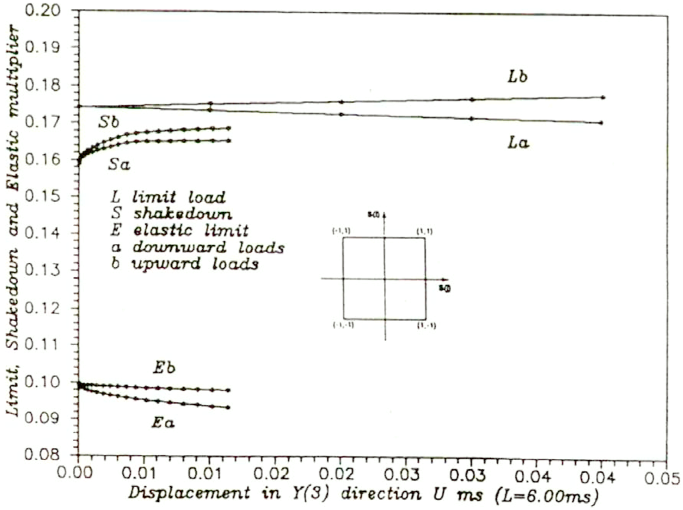


Fig. 5.3 (c) Response to downward and upward loads (d=1.0)

It may be noted that the upper bound approximation (d=1, Fig. 5.3 (c)) differs importantly from the d=0.504 ("correct") approximation, whereas the lower bound (d=0.25) gives results relatively closer to it. The same concerns the overall character of the curves and the collapse modes. It may be seen from the irregular distribution of step points on the curves in Fig. 5.3 (c) that in this case the shakedown mode is quite different than for other approximations.

As it was expected, nearly all types of response are unstable at downward loads and stable at upward loads.

As it follows from the formulation of the problem (Section 2.4), curves for shakedown concern the incremental-collapse mode. To verify if the alternating plasticity is not more restrictive a modified elastic analysis should be applied, following Eqs. (4.1) and (2.15b). As it can be seen from Fig. 5.4, for all the YC approximations considered, the alternating plasticity is not decisive. In other examples this verification is not visualized. More discussion of these results will be given with the next example.

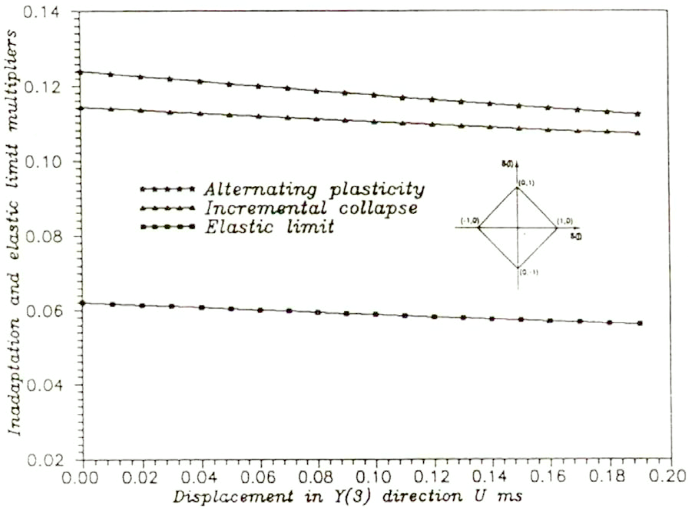


Fig. 5.4 (a) Incremental and alternating mode multipliers for downward load program (d=0.25)

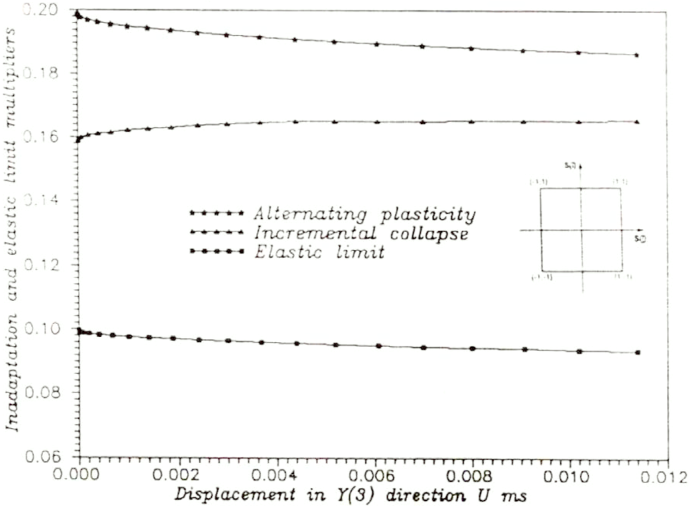


Fig. 5.4 (b) Incremental and alternating mode multipliers for downward load program (d=1.0)

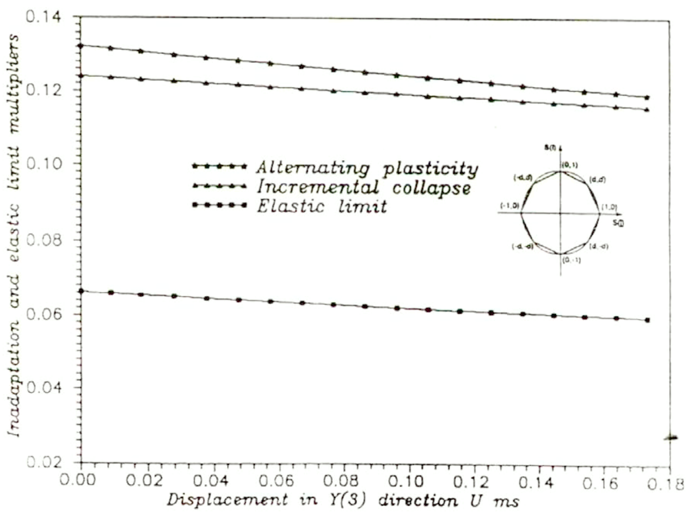


Fig. 5.4 (c) Incremental and alternating mode multipliers for downward load program (d=0.504)

5.3 Basic example II

The structure considered in Section 5.2 was very compliant to torsion, as it is always in the case of thin-walled open cross-sections. Now, the same structure as given in Fig. 5.1 is considered but in a torsionally stiff version. Namely, square hollow-box cross-section is used. The plastic moduli for such a section are as below:

$$m_{0y}=m_{oz}, \quad m_{ox}=0.75m_{oy}, \quad n_o=45m_{oy}/L=3m_{oy}/a \tag{5.1}$$

where L is the span, a is the square size and L/a=15. As can be seen when comparing the above with the data from Fig. 5.1, the closed box cross-section has torsional strength 20 times higher than the I-shaped (at the same maximum bending strength).

Calculations were performed, as before, for two loading programs, with downward and upward vertical forces, respectively. For the undeformed structure both programs give, of course, the same load multipliers. They are listed for different d-approximations in Tab. 5.2. The results confirm observation from the preceding example that the lower-bound approximation (d=0.25) gives results no more than 10% less than the "exact values" (following Tab. 3.1, d≅0.5)

No.	Yield Criterion (No. of hyperplanes)	L.L ξ^o_{LA}	S.D ξ^o_{SD}
1	Lower bound approximation (16hp)	0.19591	0.19115
2	"D-point"approxim. (48hp) d=0.50	0.21493	0.20457
2a	"D-point"approxim. (48hp) d=0.75	0.29090	0.28444
3	Upper bound approximation (8hp) (limited interaction)	0.35555	0.35540

Tab. 5.2 The values of limit and shakedown multipliers for different yield conditions (undeformed structure)

We see that the sensitivity of multipliers to changes of d, when d ≤ 0.5, is small. Therefore, even if the "exact" value for the hollow square cross-section is slightly less than to d=0.5 (Tab. 3.1), the latter value may be considered correct. On the other hand, the upper-bound approximation appears totally inadmissible.

The above conclusion is confirmed also for the deformed structure (Figs. 5.5, 5.6, 5.7). Inaccuracies produced by the upper-bound approximation are additionally increased by appearance of collapse mechanisms sometimes quite distant from reality. That produces supplementary errors in large-displacement results. For example, the stepped limit-load curve in Fig. 5.7 indicates changes of the sway-collapse mode alternatively in Y and Z directions. These changes are not observed in other cases.

Load multipliers are plotted against horizontal displacement of the upper node in (Figs. 5.5, 5.6, 5.7). Computations were continued up to the displacement of about 20% of the span (span=height). The results confirm and strengthen the effect observed already in the preceding example: limit load is more sensitive to displacements than the shakedown load. Therefore, at stable situations (upward loads), Figs. 5.6, 5.7, the two values are divergent, whereas at unstable situation Figs. 5.5, 5.6, they are convergent. This convergence is so quick that at the displacement of about 0.05L the multipliers coincide. Therefore, if the initial multipliers are the same (Fig. 5.7), this identity is maintained at all unstable configurations. That interesting feature will be discussed later.

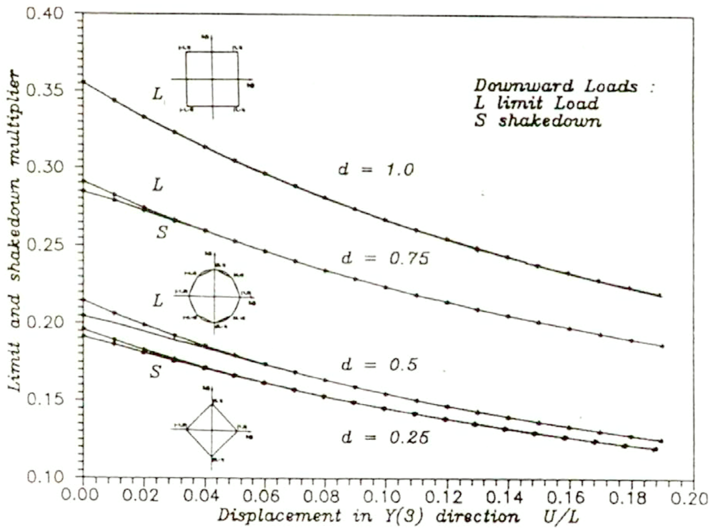


Fig. 5.5 Basic example II: downward loads for different yield criteria

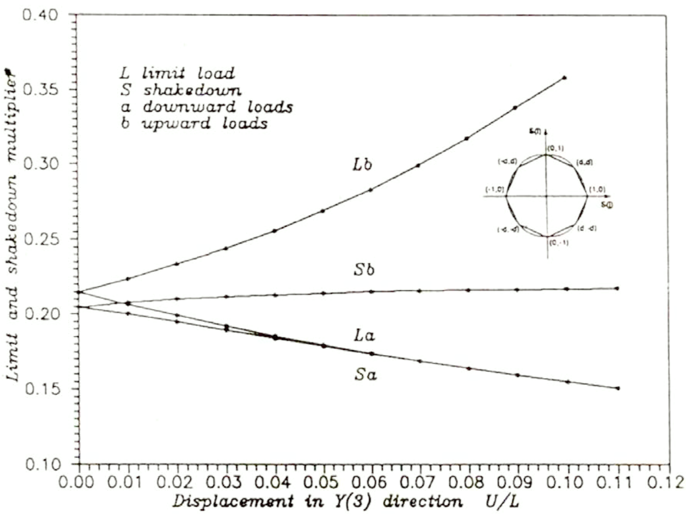


Fig. 5.6 Basic example II: stable and unstable behaviour (d=0.5)

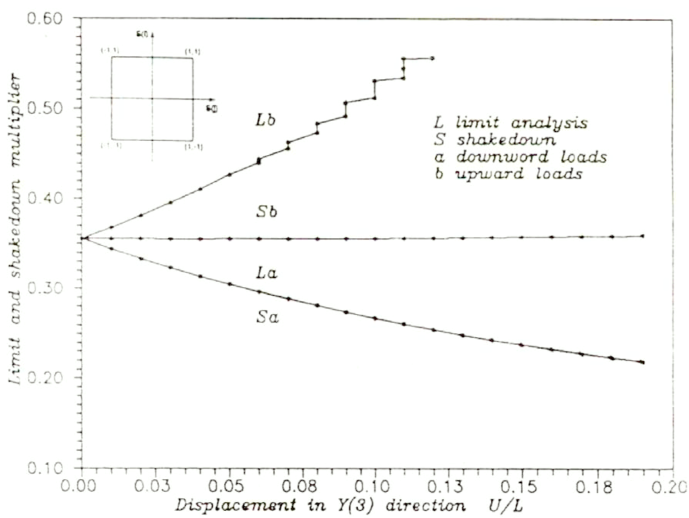


Fig. 5.7 Basic example II: stable and unstable behaviour (d=1.0)

5.4 The height dependence

The Basic Example I (Section 5.2) is used, with the same cross-sectional properties and the same load configuration, but with a variable height equal to λL (Fig. 5.8). Only downward loading program is considered.

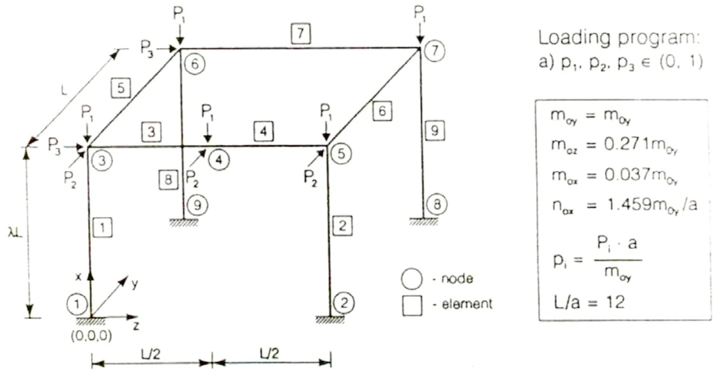


Fig. 5.8 ***Height-to-span*** *case study*

Results for limit load, shakedown load and elastic limit multipliers are listed in Tab. 5.3 and shown in Fig. 5.9 at undeformed geometry for a large range of height-to-span ratios λ. Once more different YC approximations are considered.

d	Y.C = 1 (d=0.25)			Y.C = 2 (d=1.0)			Y.C = 3 (d=0.504)		
λ	L_1	S_1	E_1	L_2	S_2	E_2	L_3	S_3	E_3
0.5	0.15698	0.14578	0.09034	0.18066	0.16887	0.11015	0.15940	0.14802	0.09896
1.0	0.12462	0.11427	0.06024	0.17433	0.15886	0.09984	0.13274	0.12406	0.06621
1.5	0.09285	0.08549	0.04750	0.13105	0.11912	0.06970	0.09805	0.09167	0.04997
2.0	0.0728	0.06641	0.03858	0.10072	0.08905	0.04566	0.07614	0.07089	0.04023
2.5	0.05907	0.05419	0.03165	0.08057	0.07107	0.03310	0.06146	0.05776	0.03193

Tab. 5.3 The values of ***limit, shakedown*** *and elastic limit multipliers for different yield conditions with changing height-to-span ratio λ (undeformed structure)*

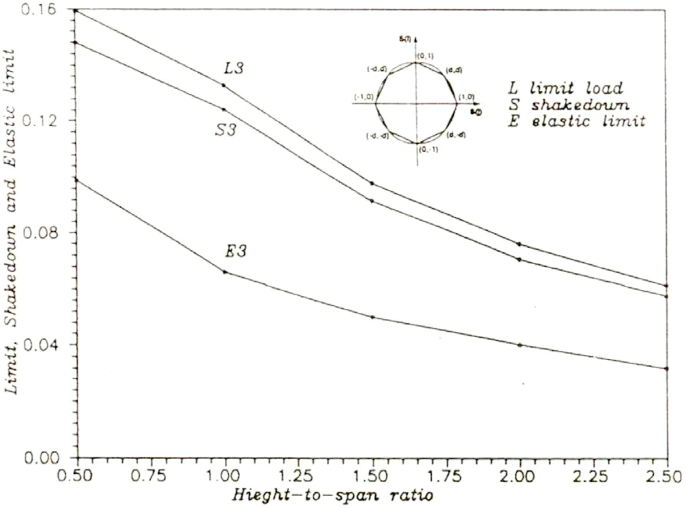


Fig. 5.9 Decreasing load parameters with the (λL) height increase (d=0.504)

In Tab. 5.4 some synthetic comparisons are given. As observed before, the lower bound is well close to the d=0.504 case, whereas the upper bound is more distant. The ratio ξ_{SD}/ξ_{LA} is nearly independent from the frame height and is, for the "exact" value d=0.504 about 93%.

Ratios	λ=0.5	λ=1.0	λ=1.5	λ=2.0	λ=2.5
S_1/L_1%	92.8%	91.7%	92.1%	91.2%	91.7%
S_1/E_1%	161 %	189 %	179 %	172 %	171 %
L_1/E_1%	174 %	207 %	195 %	189 %	187 %
L_1-S_1	0.01119	0.00999	0.00736	0.00638	0.00488
S_1-E_1	0.05237	0.05402	0.03798	0.02783	0.02254
$\frac{L_1-S_1}{S_1-E_1}$%	21.3 %	18.4 %	19.3 %	22.9 %	21.6 %

Tab. 5.4 (a) The ratios between limit, shakedown and elastic limit multipliers for lower bound (YC=1) yield condition

Ratios	λ=0.5	λ=1.0	λ=1.5	λ=2.0	λ=2.5
S_2/L_2%	93.5%	91.1%	90.9%	88.4%	88.2%
S_2/E_2%	153 %	159 %	171 %	195 %	215 %
L_2/E_2%	164 %	175 %	188 %	220 %	243 %
L_2-S_2	0.01179	0.01547	0.01193	0.01167	0.00950
S_2-E_2	0.05872	0.05902	0.04942	0.04339	0.03797
$\frac{L_2-S_2}{S_2-E_2}$%	20.0 %	26.2 %	24.1 %	26.9 %	25.0 %

Tab. 5.4 (b) The ratios between limit, shakedown and elastic limit multipliers for upper bound (YC=2) yield condition

Ratios	λ=0.5	λ=1.0	λ=1.5	λ=2.0	λ=2.5
S_3/L_3%	92.9%	93.5%	93.5%	93.1%	93.9%
S_3/E_3%	150 %	187 %	183 %	176 %	181 %
L_3/E_3%	161 %	200 %	196 %	189 %	192 %
L_3-S_3	0.01138	0.00866	0.00635	0.00525	0.00370
S_3-E_3	0.04906	0.05785	0.04170	0.03066	0.02583
$\frac{L_3-S_3}{S_3-E_3}$%	23.3 %	14.9%	15.2 %	17.1 %	14.3 %

Tab. 5.4 (c) The ratios between limit, shakedown and elastic limit multipliers for D-approx. (YC=3) yield condition

Dependence of the limit load and shakedown multipliers upon the deformation (horizontal displacement at an upper node) is given in Figs. 5.10 and 5.11, respectively, for different values of the height-to-span ratio λ. As we see the curves are slightly descending. The destabilizing P-Δ effect decreases with the height increasing. It may be easily explained by the fact that with the height increasing, the importance of horizontal forces to yielding increases, whereas only the vertical forces are responsible for the destabilizing effects.

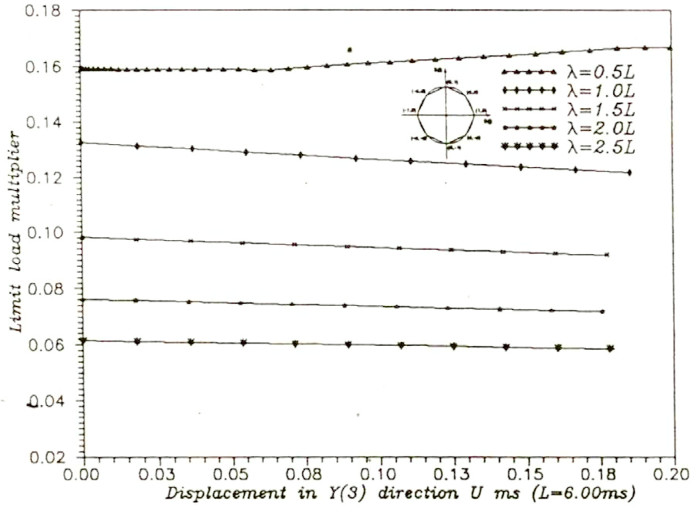


Fig. 5.10 Post-yield behaviour for different height-to-span ratios

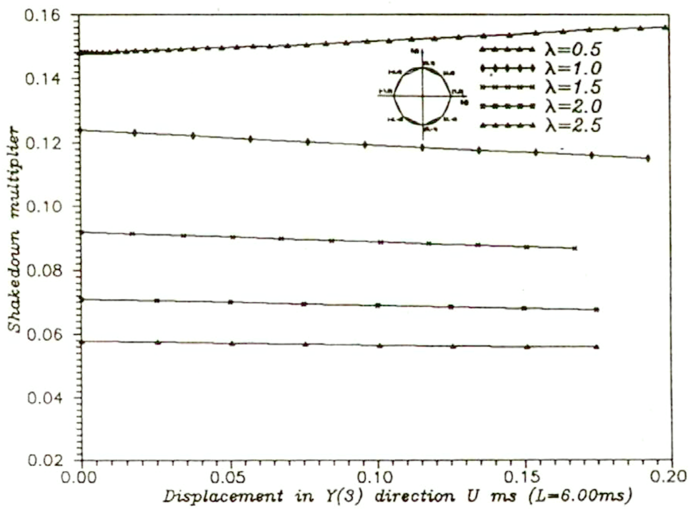


Fig. 5.11 Incremental collapse behaviour for different height-to-span ratios

Only for a very low frame (λ=0.5) some geometrical hardening appears. That is due to the fact that for a low-rise long-span structure the beam-type collapse modes prevail, whereas deformed beams are strengthened by deflections. It is evident that in this case the control point chosen does not correspond to maximum displacement. That is clearly shown by an irregular distribution of the step points on the curve (they correspond to the constant step Δ=2 cm). These curves are shown in more detail in Fig. 5.12. It is clearly seen that at post-yield behaviour at least four distinctly different immediate collapse mechanisms appear during the deformation process. Similar situation happens at incremental collapse. It may be interesting to observe again that the sensitivity to displacement is stronger in limit analysis than in shakedown (see Section 5.3), even in so complicated a situation, as shown in Fig. 5.12.

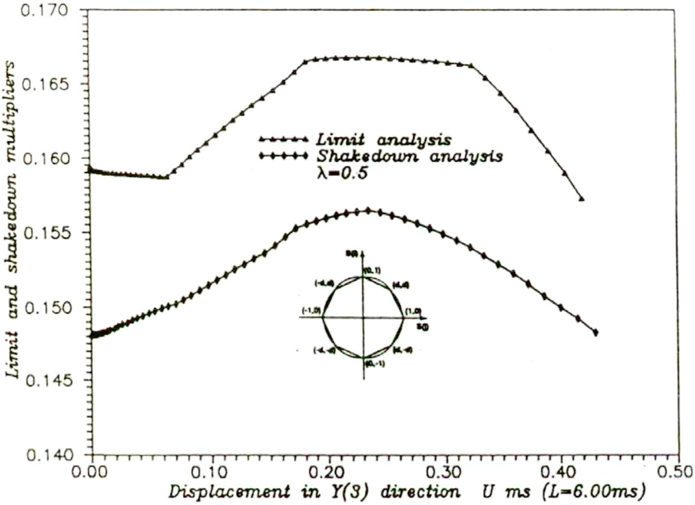


Fig. 5.12 Changing mode at collapse of a low-rise frame

5.5 Multistorey frames

Two examples of multistorey frames were considered, with the loading conditions similar to those for the Basic Example I and with cross-sectional properties identical as in Sections 5.2 and 5.4. In Fig. 5.13, a 2-storey frame is shown, loaded by three sets of independent forces P_1, P_2, P_3 acting in X, Y and Z directions, respectively. The structure is discretized into 18 elements with 60 degrees of freedom. Two loading programs with upward and downward vertical forces are considered.

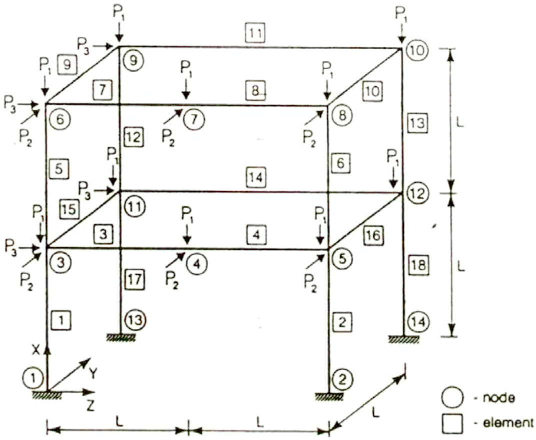


Fig. 5.13 2-storey space frame

Results for different YC approximations in the case of the undeformed structure are given in Tab. 5.5.

No.	Yield Criterion (No. of hyperplanes)	L.L ξ_{LA}	S.D ξ_{SD}
1	lower bound approx. (16hp)	4.5923E-2	4.4195E-2
2	"D-approx" d=0.504 (48hp)	5.0273E-2	4.7733E-2
3	Upper bound approx. (8 hp)	6.3233E-2	6.0380E-2

Tab. 5.5 The values of limit, shakedown and elastic multipliers for different yield conditions (2-storey undeformed structure)

The post-yield and inadaptation curves are plotted in Fig. 5.14 against a control displacement at the frame top. Results are given for the downward and upward vertical forces. The lower-bound approximation was used.

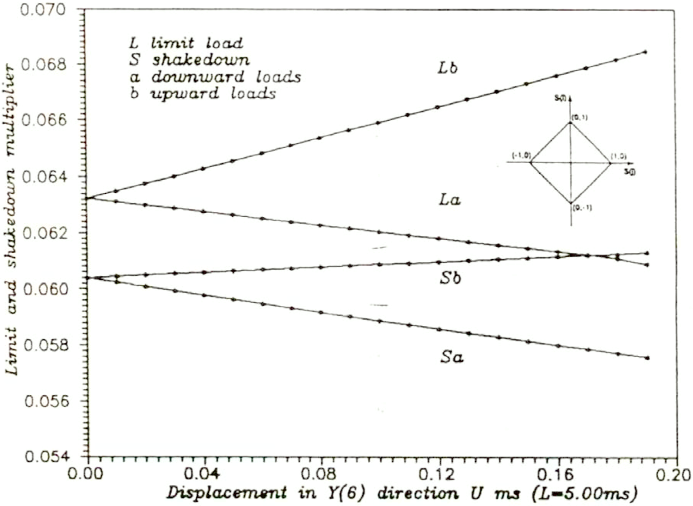


Fig. 5.14 Post-yield and incremental collapse of a 2-storey frame

A 5-storey frame, Fig. 5.15, is loaded at each floor as in the preceding case. Discretization into 45 elements with 174 degrees of freedom was applied. The case was selected for testing the program and compare results with the limit analysis results obtained elsewhere [25]. The results for the undeformed frame are given in Tab. 5.6.

No.	Yield Criterion (No. of hyperplanes)	L.L ξ_{LA}	S.D ξ_{SD}
1	lower bound approx. (16hp)	0.37500	0.13375
2	Upper bound approx. (8 hp)	0.32484	0.10638

Tab. 5.6 The values of limit and shakedown multipliers for different yield conditions (5-storey undeformed structure)

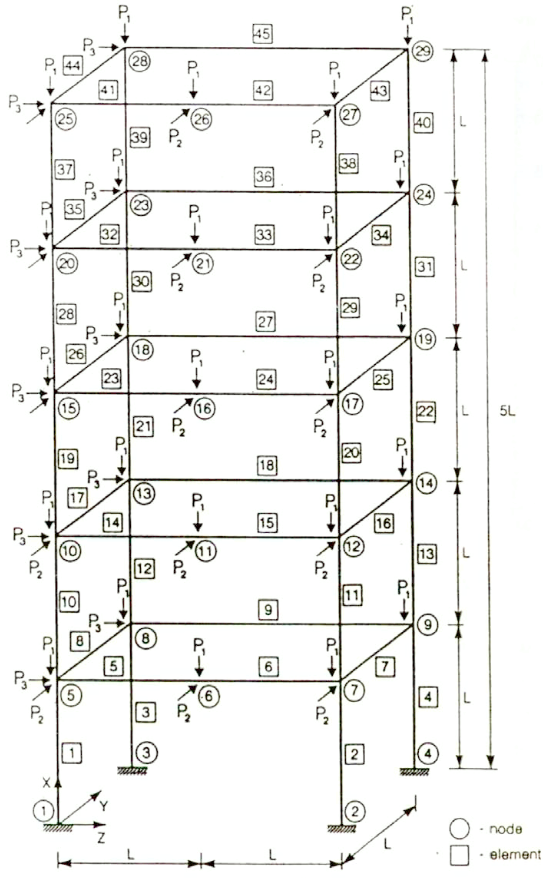


Fig. 5.15 5-storey frame

5.6 Spatial interaction of plane frames

Decomposition of complex structures into simpler components is especially important in the analysis of surface skeletal structures, permitting simple but effective analysis. These structures are attractive because of their service merits (free space inside), but may reveal unexpected behaviour aspects, especially at extreme loading situation (e.g, earthquake). Unfortunately, experience concerning spatial interaction of plane inelastic frames is scarce. Here we perform only a very introductory study of one aspect of such an interaction. The frame like in Basic Example I is considered, Fig. 5.16, with a varying width-to-length ratio β. It can be seen that above a certain width ratio (β≥2), this parameter does not influence the results.

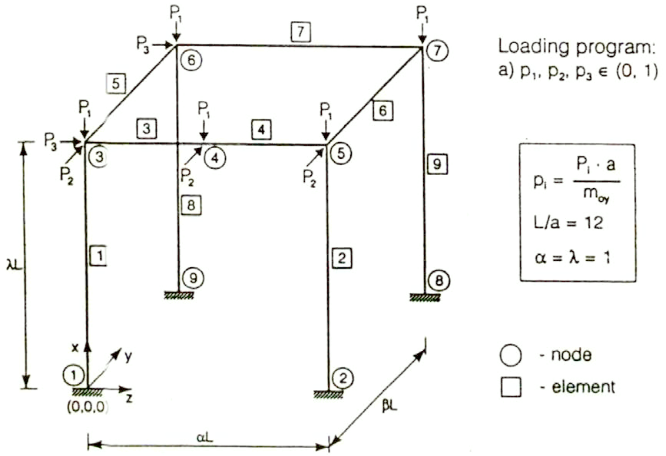


Fig. 5.16 Space frame with a variable width

Such a frame under two independent horizontal load sets may undergo either horizontal translations of the top face or more complex motions: diagonal flattening or torsional rotation of the top face. Under loads as in Fig. 5.16 rather the former case occurs. Results given in Tab. 5.7, obtained for the undeformed structure for a wide range of the frame width βL, are plotted in Fig. 5.17. Because of the introductory character of the study the lower bound approximation of the YC is used. It needs less computing time, whereas, as discussed in Section 5.3, it gives quite satisfactory results, at least for nondeformed configurations.

$\alpha=1$	ξ_{LA}	ξ_{SD}	ξ_{EL}
β =0.2	0.14490	0.12648	0.07490
β =0.3	0.14024	0.12526	0.07798
β =0.4	0.13642	0.12354	0.07867
β =0.5	0.13357	0.12181	0.07432
β =0.6	0.13120	0.12011	0.07145
β =0.7	0.12920	0.11855	0.06852
β =0.8	0.12747	0.11711	0.06600
β =0.9	0.12596	0.11569	0.06386
β =1.0	0.12462	0.11427	0.06247
β =1.5	0.11953	0.10839	0.05644
β =2.0	0.11619	0.10494	0.05396
β =2.5	0.11391	0.10307	0.05273
β =5.0	0.10866	0.10067	0.05100

Tab. 5.7 Limit, shakedown and elastic limit-load multipliers for changing width-to-span ratio β

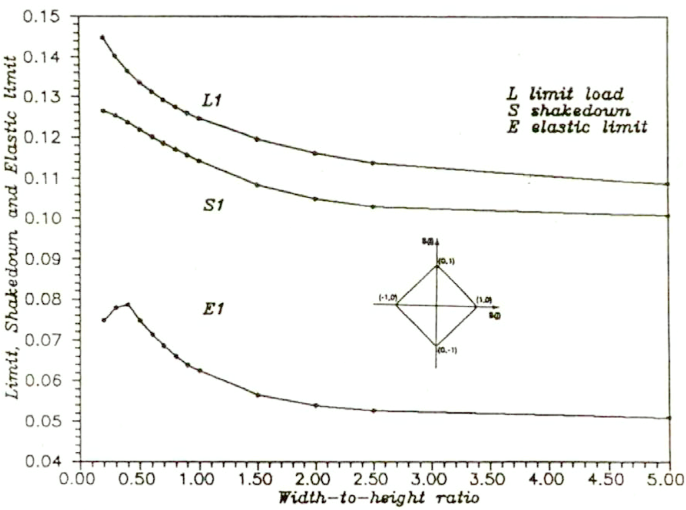


Fig. 5.17 Dependence of the multipliers upon the frame width ratio β

5.7 Eccentrically Braced Frames

(5.7.1) Motivation

Plane eccentrically braced frames are chosen as an example of practical engineering application of our procedures. They possess good properties in dynamic, repeated and cyclic loading environments.

Eccentrically braced frames are among those systems which recently proved to possess inherently higher overall ductility over their braced frames counterparts. In fact, a high energy dissipation rate occurs during the successive formation of plastic hinges due to flexural stresses, (in addition to shear stresses in the portion of the webs of some members, commonly referred to as links, see Fig. 5.18.

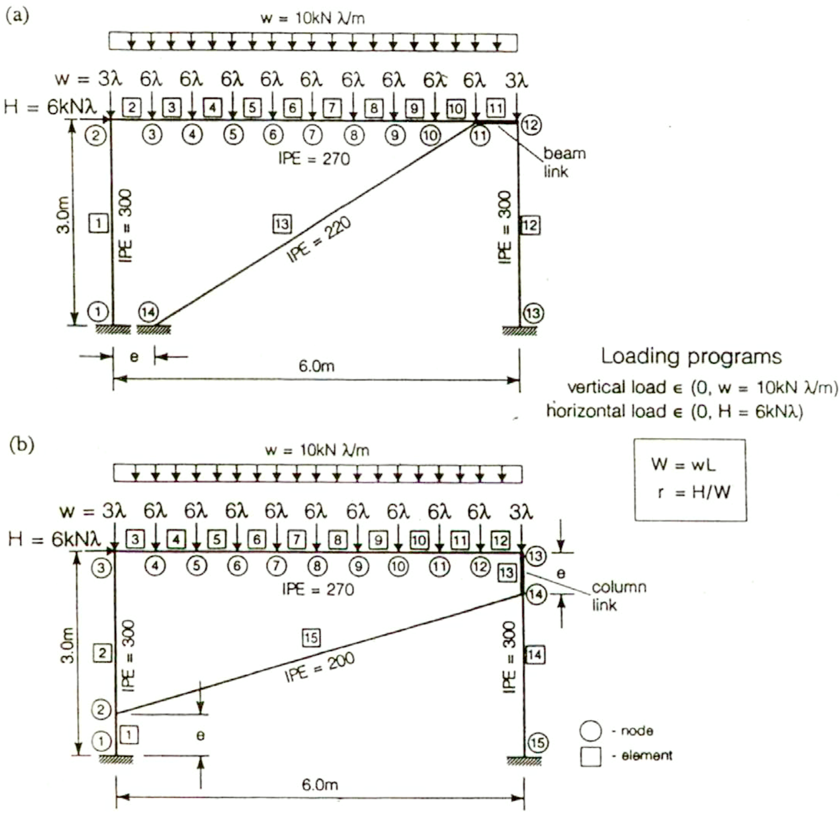


Fig. 5.18 Eccentrically braced frames: a) Beam link, b) Column-link

Because of eccentricities which have the above mentioned favourable effect under exceptional loads, the elastic-limit service load frequently decreases. The collapse load being less sensitive to eccentricities, the difference between limit load and the elastic-limit load becomes more considerable when eccentricity is introduced. In this situation the shakedown analysis is of increased importance. On the other hand, the length of the plastic deformation process (resulting in increasing the "overall ductility" needed) makes more important the post-yield and inadaptation analyses. That is why the EBF appear to be a good practical example for application of the method and the procedure worked out in this thesis.

The presentation of this Section differs somewhat from the preceding case studies. It follows from a more practical orientation of this study. Also the form of figures presented follows requirements of the CSCE, as an extended version of this study has been presented at the CSCE conference [1].

(5.7.2) Structural details

The model used in this study is limited to single-bay, one-storey plane braced frames (Fig. 5.18). Column's section is IPE 300 and height 3.00 m; beam's section is IPE 270 and span 6.00 m. As a special case concentrically braced frames are also analyzed. Position of the IPE 220 brace element is expressed in terms of symmetrically disposed eccentricities. This representation allows for the examination of some limiting cases of frames' geometries such as: double-bay one-storey and single bay two-storey braced or unbraced frame configurations. The boundary between braced or unbraced is herein defined in accordance with the Eurocode 3, stating that: A structure is classified as being braced if it's lateral in-plane rigidity is properly secured by a system of bracing capable enough to give side-sway movement δ_a not to exceed one-fifth of δ_s computed for unbraced systems ($\delta_a \leq \delta_s/5$) .

It should be noticed that choosing between these two structural systems (namely, CBF and EBF) would depend on particular desired building arrangements. In practical situations the eccentricities are intentionally introduced to be of small magnitude. Thus, eccentric brace members rarely impair either function of the building or it's equipment. Moreover, unavoidable eccentricities are often tolerated by designers either for fabrication requirements or for the purpose of details simplification.

(5.7.3) Results of a parametric study

The curves presented in Figs. 5.19 to 5.22 show the limit and shakedown multipliers for undeformed structures, versus the eccentricity **e*** expressed in dimensionless form defined as follows:

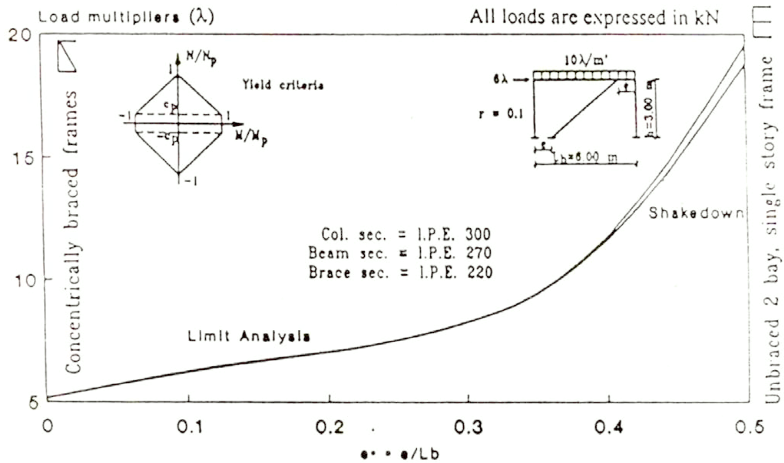


Fig. 5.19 Shakedown and limit-load multipliers for beam links, r=0.1 (assuming geometric linearity)

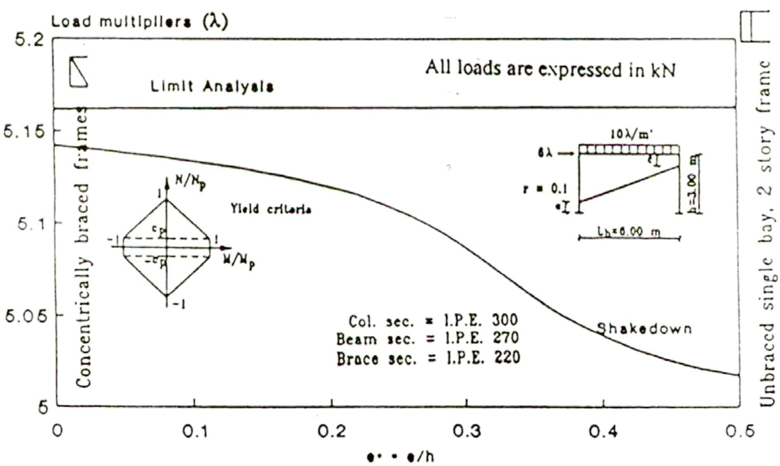


Fig. 5.20 Shakedown and limit-load multipliers for column links, r=0.1 (assuming geometric linearity)

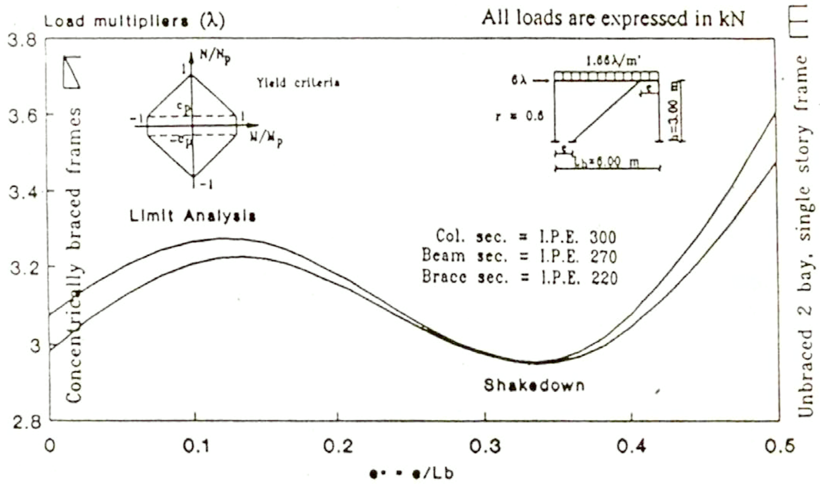


Fig. 5.21 Shakedown and limit-load multipliers for beam links, r=0.6 (assuming geometric linearity)

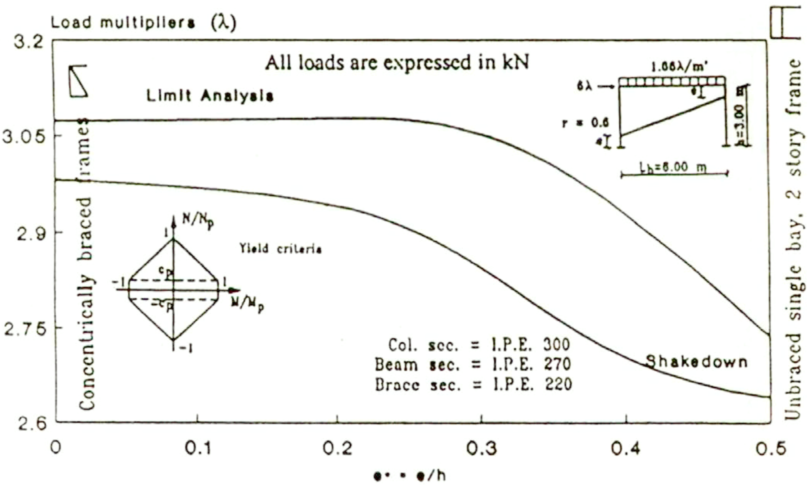


Fig. 5.22 Shakedown and limit-load multipliers for column links, r=0.6 (assuming geometric linearity)

"Link" beams : $e^{*} = e/L_b$ (Fig. 5.18a)

"Link" columns : $e^{*} = e/h$ (Fig. 5.18b)

where L_b is the beam's span and h is the column's height. Two levels of loading are investigated, namely for r=0.6 and r=0.1, consisting of horizontal load H and vertical load W varying arbitrarily and independently from zero to their maximum values. Proportions of these extreme values are determined by the ratio r=H/W, where W denotes the total vertical load. The applied loading program is intended to simulate loading schemes at service limit state conditions. Vertical uniformly distributed load is discretized into 11 forces at nodes.

The yield criteria depending upon two generalized stresses: bending moment and axial force are taken in the "octant" PWL approximation, Fig. 3.9 (b), as required by the Eurocode 3. However, in a future study the influence of shearing forces should be also accounted for and a 3-dimensional generalized-stress space should be considered. Namely, these forces are of considerable magnitude in the "link" elements, as it occurs frequently in truss-like structures with overlapping joints.

Limit load and inadaptation states correspond to the most unfavorable collapse modes. It is known that for a simple one-bay portal frame, these modes may be: beam, sway or a combination of them. In the particular case of EBF, the situation is somewhat more complicated, but generally the modes may be classified in the same way.

The results presented in Fig. 5.19, with the load parameter r=0.1, indicate limit and shakedown multipliers increasing with the increase of the eccentricity e^{*}. The lowest multipliers for both cases are obtained for the structural configuration with $e^{*}=0$ referred to as CBF. The highest multipliers are obtained for the unbraced double-bay, one-storey frame. In the case of r=0.6, Fig. 5.21, the maximum limit and shakedown multipliers are obtained for eccentricities $e^{*}=0.1$ and 0.5, while the lowest values are obtained for $e^{*}\cong 0.35$.

The situation is different for the link-columns Fig. 5.18 (b). At r=0.1 the limit load multiplier ξ_{LA} is constant Fig. 5.20, for all eccentricities e^{*}; for r=0.6, Fig. 5.22, it is constant only for $e^{*}<0.3$, while the shakedown multipliers ξ_{SD} decrease with the increase of the eccentricities.

(5.7.4) Post-yield and inadaptation

Non-linear geometric effects are reflected in Figs. 5.23 to 5.25 for the beam-link type of the structure shown in Fig. 5.18 (a), with $e^{*}=0.1$. For r=0.1 the structure collapses in a beam-like mode and the vertical midspan deflection is chosen in Fig. 5.23 as a control displacement. For r=0.5 a mixed-type sway/beam mechanism appears and either vertical midspan deflection or the horizontal components may be

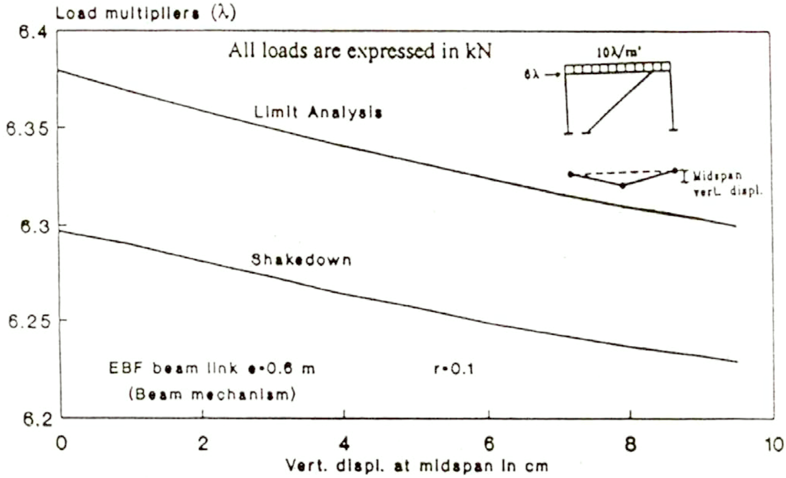


Fig. 5.23 Limit and shakedown multipliers versus vertical displacement, r=0.1

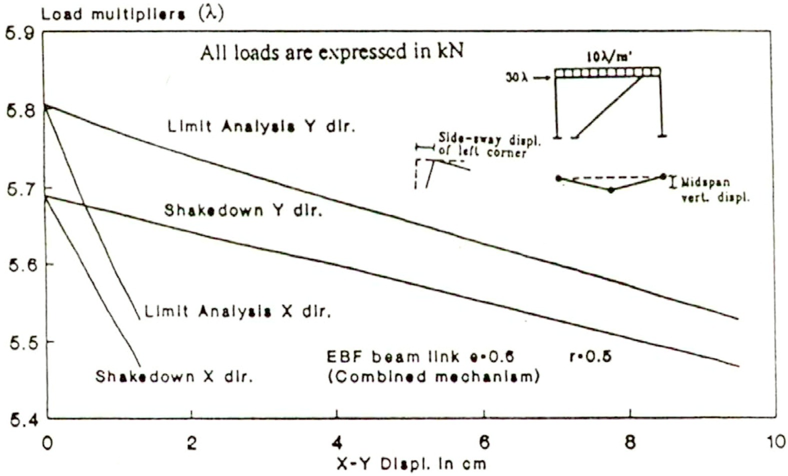


Fig. 5.24 Limit and shakedown multipliers versus midspan, and side-sway displacements, r=0.5

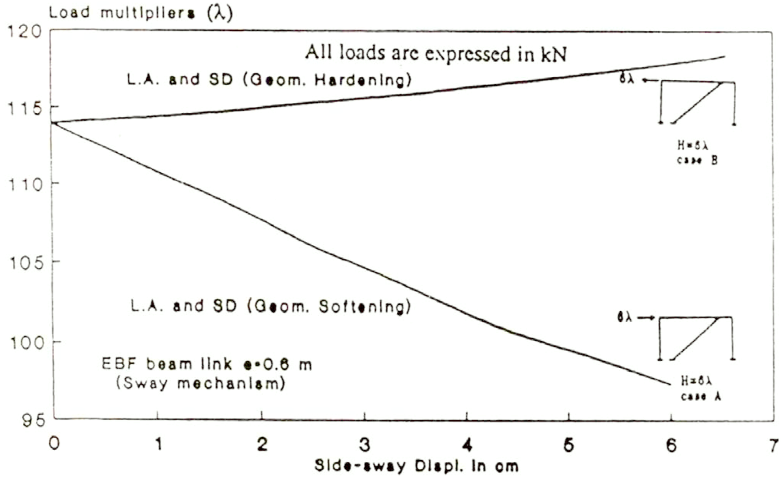


Fig. 5.25 Limit and shakedown multipliers versus side-sway displacement (stable and unstable behaviour)

chosen as a control variable. In Fig. 5.24 the load multipliers are plotted separately against these two variables. The above illustrates the case, discussed in Section 4.2, when using of two or more control variables is recommended.

All the above curves are decreasing with the displacements (geometrical softening). However, in some cases, especially when vertical loads are insignificant, geometrical hardening may appear. Fig. 5.25 illustrates this phenomenon. Vertical load is absent and the frame is loaded with a horizontal force H applied at the left corner of the structure. The horizontal load **H** is oriented either rightward (case A) or leftward (case B) and is allowed to vary between zero and H. As in the majority of cases of one-parameter non-reversible loading, limit and shakedown loads are identical. The load multipliers for the non-deformed structure are almost the same for both load orientations. However, response of deformed structure is different in the two cases. It means that changing the orientations of braces, (braces in compression instead of tension) will obviously alter substantially the structure behaviour; even a change from unstable to stable situation may be obtained.

It is to be mentioned that in many practical applications, structures may suffer from lessened strength attributed to such changes in geometry as: initial imperfections, deviation, inclination, lack of straightness of columns or beams, often occurring and tolerable in steel works. The above may furnish information on the influence of these imperfections on the structure behaviour. Moreover, some general practical conclusions may be derived from the above study.

Namely, recommendations concerning optimal structural configurations derived from the classical limit analysis seem to hold also when a large-deformation or/and shakedown design is considered; in the latter case, it is limited to non-reversible loading programs.

Destabilizing geometrical effects prevail, but column-link structures are more sensitive to deformations than the beam-link ones. In the latter case, Fig. 5.20, an optimal eccentricity level and maximum non-danger level may be discerned.

SENSITIVITY TO GEOMETRICAL EFFECTS AS A DESIGN FACTOR

6.1 Two-level problem

Since our approach to the analysis of the post-yielding and inadaptation consists of a step-by-step linearization of the nonlinear problem, procedures concerning the first two steps correspond, in some manner, to the formulation inherent to the classical sensitivity theory. In this interpretation a measure of the structure response (ξ_L, ξ_{SD}) is studied as a function of another measure of the structure response $\max|\mathbf{u}|$; the latter is treated as an imperfection of the ideal (i.e., initial) configuration. To include this reasoning into the framework of the sensitivity theory, let us recall some notions of the latter.

Generally, formulations for sensitivity analysis can be divided into three groups. The simplest technique is the finite difference method (FDM) for calculation of sensitivities based on successive perturbations of the design variables followed by re-analysis of the system (Green and Haftka [38]). This may be ineffective since the accuracy problems often arise; in such a case analytical methods turn out to be more appropriate. The direct differentiation method (DDM) is based on differentiation of system equations with respect to design variables to obtain sensitivity equations. The adjoint system method (ASM) defines an adjoint system whose solution permits evaluation of sensitivities (Adelman and Haftka [2]). As an extension of the general sensitivity theory (see, e.g., Frank [32]), Haftka and Mróz [41] presented a variational approach based on the initial load method of 1st- and 2nd-order sensitivities of linear and nonlinear systems. Using the fuzzy set theory Kleiber [56] estimated nonlinear structural response sensitivity to imperfections. A comprehensive discussion of the finite element formulation and computational aspects for structural sensitivity problems in statics and dynamics was presented by Hien and Kleiber [46]. The sensitivity of optimal plastic design with respect to geometric imperfections and post-critical deformations was presented by Siemaszko and Mróz [114]. It was shown there that the concept of optimal plastic design should be modified in order to provide a proper safety factor against collapse for a specified range of imperfections and configuration changes. A similar idea is used here: modification of the safety factors required by codes is proposed as a function of an acceptable imperfection level. The imperfections are either present at the beginning or induced by deformations.

To evaluate the sensitivity of the shakedown ξ_{SD} or of the limit load multipliers ξ_L to non-linear geometrical effects by the FDM approach, the shakedown multipliers ξ^o_{SD} and ξ^1_{SD} or limit-analysis multipliers ξ^o_L and ξ^1_L for two structural configurations: the initial undeformed x^o and an admissibly deformed x^1, respectively, have to be determined and compared.

The corresponding procedure follows the sequential formulation , as in Section 2.5. limited to a two-level analysis. The lower-level problem concerns the undeformed structure configuration x^o and is described by Eqs. (2.40), (2.41) or by (2.42), (2.43) for shakedown or limit analysis, respectively. The upper-level problem (2.45-48), is referred to an admissible deformed configuration $\mathbf{x}^1$. This configuration is defined by the superposition of the most stringent plastic increments $\mathbf{u}^o$ given by the lower-level solution onto the initial configuration $\mathbf{x}^o$. To obtain the upper-level configuration x^1, a scaling factor μ should be determined in such a way that the maximal displacements do not exceed some prescribed admissible displacements:

$$\max{}^{*}\left\{\mu\ \mathbf{u}^0\right\} \le \left\{\mathbf{u}_{adm}\right\}^{*} \tag{6.1}$$

Components of the vector $\mathbf{u}_{adm}$ may be defined using admissible characteristic displacements of the structure, e.g., a fraction of the span or of the structure (storey) height. It may be also determined by an incremental analysis of selected prior-to-collapse loading histories or even by an engineering experience. It should be remarked that the formula (6.1) leaves a broad range of liberty in selecting constraints for an appropriate determination of the scaling factor. The asterisk (*) in (6.1) means that in special cases the maximized vector may contain only some components being of special interest. Any representative displacement can be chosen as a control parameter u. The simplest way is to adopt for it the generic parameter of the program $\max|\mathbf{u}|$.

The non-linear geometrical effects stabilize the inadaptation or post-yield process if for the corresponding load multipliers we have $\xi^1_{SD} > \xi^o_{SD}$ or $\xi^1_L > \xi^o_L$. In the opposite case the process is destabilized. Therefore (see Fig. 1.3 and discussion at the end of Section 2.5), in both cases the rules of selecting admissible values will be somewhat different. At geometrical hardening the intended increase of load multipliers corresponds simply to the maximum tolerable deformation. At softening a possible decrease of multipliers will depend on the deformations prior to the limit state and/or on possible initial imperfections of the structure.

6.2 Sensitivity to geometrical effects and safety factors

If the sensitivity is defined as a rate of changes of shakedown or limit load multipliers with increasing control displacement measure u, we get:

$$\frac{d\xi}{du} = \lim_{\Delta u \to 0} \frac{\Delta\xi}{\Delta u} = \lim_{\Delta u \to 0} \frac{\xi(u+\Delta u) - \xi(u)}{\Delta u} \tag{6.2}$$

Since ξ is an implicit function of u, a finite difference scheme has to be used:

$$\frac{d\xi}{du} \simeq \frac{\Delta\xi}{\Delta u} = \nu, \qquad \text{if } \Delta u \text{ is very small,} \tag{6.3}$$

where ν is the shakedown (limit-load) sensitivity factor. It represents a "tangent" sensitivity and may be referred to the initial configuration (u=0), as in Fig. 6.1, or to any deformed one. If the Δu is not required to be small, we have a "secant" sensitivity:

$$\nu = \frac{\xi^i - \xi^{i-1}}{u^i - u^{i-1}} \tag{6.4}$$

where ξ^i means the multiplier calculated at the configuration $\mathbf{x}^i = \mathbf{x}^{i-1} + \mu\mathbf{u}^{i-1}$. Rigorously, $\mathbf{u}^i$ should be a total vector of all $\mu^\delta\mathbf{u}^\delta$ (Eq. 2.45) small steps. However, if we want to apply a simplified 2-step procedure we have to put $\mathbf{u}^i = \mathbf{u}^o\mu^i$ as used in (6.1). To have a initial-secant sensitivity ν, in (6.4) it should be put $\xi^{i-1} = \xi^o$ and $u^{i-1} = 0$.

A structure is always designed with some margin of safety; the working loads are less than the collapse loads. The safety factor for the structure design *S* must be equal or greater than one ($S \geq 1$). If we design the structure to satisfy the safety factor *S* required by codes with respect to the collapse load for the undeformed structure, the real safety level will be different, depending upon the phenomena of geometrical hardening or softening. To maintain the real security on the level required by the code we can either apply the code factor to the collapse load determined for the structure deformed up to a level determined by a certain control value or to perform the analysis for undeformed structure but using a modified safety factor $\bar{S}$. The new safety factor can be decreased or increased depending on geometrical effects as follows

$$\bar{S} = S\,(1 - \beta\,\nu) \tag{6.5}$$

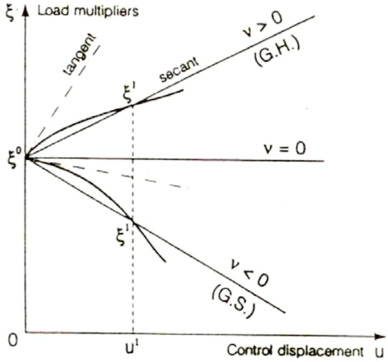


Fig. 6.1 "Secant" and "tangent" sensitivity

where ν is the sensitivity and β is a scalar weighting ratio. We can say that βν is a normalized sensitivity:

$$\beta\nu \;=\; \frac{\xi_1 - \xi_o}{u_1 - u_o}\;\frac{u^{all}}{\xi_o{}^o}\;\alpha \tag{6.6}$$

where α is a "penalty" factor amplifying or attenuating the influence of the geometrical effect on the safety factor. It may depend, e.g., upon the uncertainty level, but will be taken as α=1 in the sequel.
In the simplest situations, when $u^1 = u^{all}$, $u^o=0$, we have:

$$\bar{S} = S\,(1 - \frac{\xi^1 - \xi^o}{\xi^o}\,\alpha) \tag{6.7}$$

It should be noted that it is tacitly assumed that the collapse mode is the same in the whole range $0 \le u < u^1$.

Finally:

If $\nu > 0$ (geometrical hardening) the needed safety factor $\bar{S}$ decreases

$\nu < 0$ (geometrical softening) the needed safety factor $\bar{S}$ increases

$\nu = 0$ (insensitive) $\bar{S} = S$

6.3 Examples

The above modification of the code safety factor S into the really needed $\bar{S}$ is illustrated using the structure frame under loads as in Fig. 6.2, for two loading programs (upward and downward vertical forces).

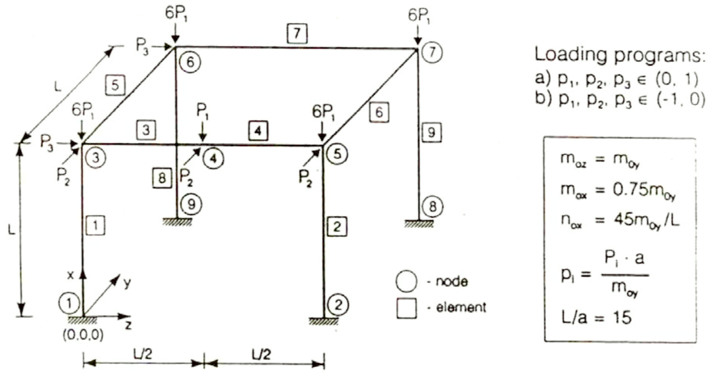


Fig. 6.2 Space frame example

The relationship between the horizontal-displacement at point 3 at upper node of the frame and load multipliers at shakedown or immediate collapse in shown in Fig. 6.3.

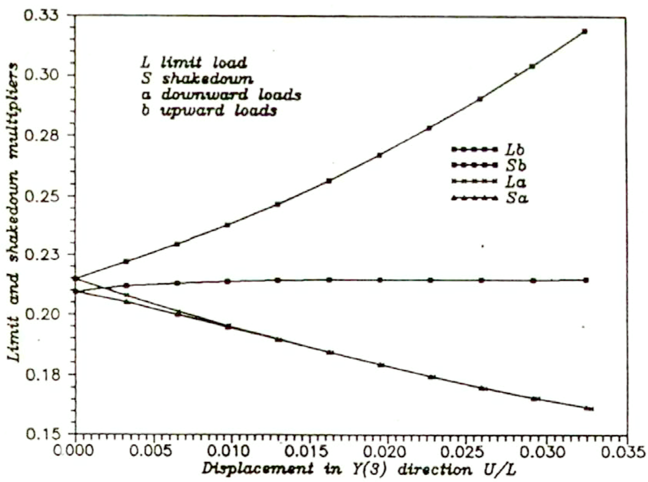


Fig. 6.3 Stable and unstable behaviour

Assuming the allowable displacement at node 3, u_{y3}= L/150, and introducing the corresponding values ξ_L^o, ξ_L^1, ξ_{SD}^o, ξ_{SD}^1 from Fig. 6.3 into Eq. (6.7) we obtain:

* at geometrical hardening (upward loads):

$\bar{S}_L = 0.930\ S$, $\bar{S}_{SD} = 0.983\ S$

for the limit and shakedown, respectively;

* at geometrical softening (downward loads):

$\bar{S}_L = 1.062\ S$, $\bar{S}_{SD} = 1.045\ S$

Modification in the safety factor required is, thus, between +6.2% and -7.0% for limit load and +4.50% to -1.7% for the shakedown load.

Using the safety factor reduced from S into $\bar{S}$ and applying it to linear analysis is more advantageous than a direct analysis of the deformed structure (with unaltered value of S) when a class of different load configurations is encountered. In this case representation using shakedown (limit) loads envelopes is convenient. The envelopes are derived from the analysis of undeformed structures and the level of sensitivity to geometrical effects is indicated. The latter needs, of course, the above second-step analysis but may be done in separate, more general studies and may be introduced as recommended correction for the use of linear analysis. Such a representation by envelopes was used, e.g., in [31] to indicate stability of the collapse modes.

In Fig. 6.4 are given results for limit-load and shakedown multipliers of the frame (as in Section 5.3), loaded only by in-plane forces. Envelopes concern all the domains Ω_R (Section 2.2), i.e:

$$\gamma_1^+ = \gamma_1^{max}, \qquad 0 \le \gamma_2^+ \le \gamma_2^{max}$$

$$0 \le \gamma_1^+ \le \gamma_1^{max}, \qquad \gamma_2^+ = \gamma_2^{max} \tag{6.8}$$

$$\gamma_1^- = \gamma_2^- = 0$$

Besides the sides of the envelope polygons the corresponding immediate collapse and incremental collapse mechanisms are shown. Corrections of the code-required safety factor S into $\bar{S}$ derived from the above discussed 2-step analysis (following Eq. (6.7)) are proportional to the ordinates of the diagrams traced along the sides of the envelopes.

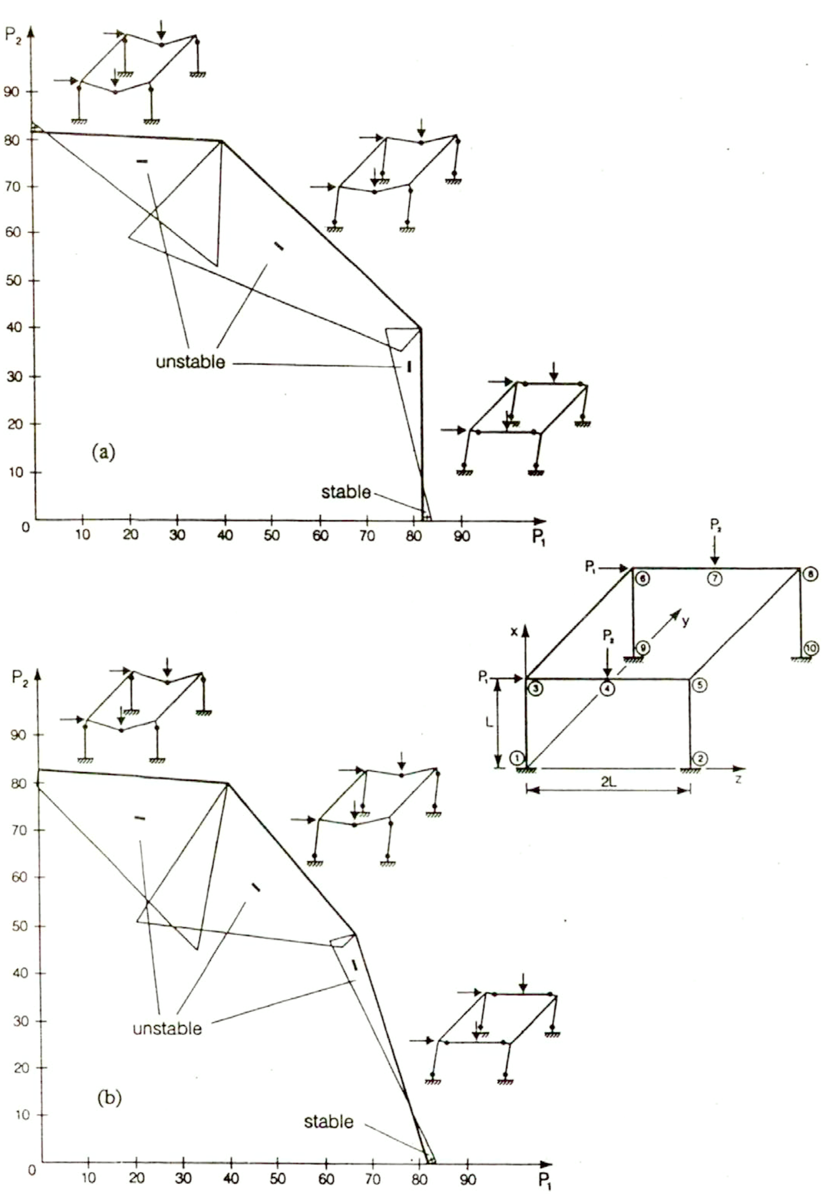


Fig. 6.4 Envelopes for: (a) limit load multiplier, (b) shakedown load multiplier

CHAPTER 7

FINAL REMARKS

7.1 Overview of the results

- A numerical finite element program SDLAS (ShakeDown and Limit Analysis of Space skeletal structures) has been worked out, permitting for a unified analysis of all the history-independent classes of elastic-perfectly plastic response of space skeletal structures. When dealing with these classes incremental historical elastic-plastic analysis may be avoided. Therefore, the analysis is feasible even if the loading history is unknown (which is the most common case) and when loads may vary in an unspecified way within a given range.

  Loads are described by a finite number of parameters β_i arbitrarily and independently varying in time, with a given reference load domain. The "shape" of the domain is determined by ratios of the extreme values of the load parameters. Load multipliers corresponding to limit states of immediate collapse (*limit analysis*), incremental collapse or alternating plasticity (*shakedown analysis*) as well as of first plastic deformation (*elastic-limit load*) are automatically calculated and the reference domain expanded by these multipliers describes domains safe against the above limit states (see Fig. 1.5).
- The linear programming formulation due to Maier [70] was used in its classical version, with assumptions of perfect plasticity and linear geometry, together with a lumped-compliance beam FE discretization. This is the most commonly accepted approach and is applied intentionally, to avoid more sophisticated and challenging but disputable proposals. That is why in the present version, e.g., the material hardening is not taken into account.

  Following the above formulation, analysis of all the classes may be performed using directly elastic solutions (with appropriately chosen distortions at nodes), furnished by any commercial code whose source version is available. The POL(SAP) program has been used as a tool, but in the limit analysis a generic procedure appeared more effective. Loads are discretized in concentrated forces at nodes and are considered applied, as in (POL)SAP, at shear centres of the cross-sections.

- The problem is formulated in generalized variables. Following the classical beam theory, four cross-sectional stress resultants (*axial force, torque and two bending moments*) are generalized stresses contributing to both elastic deformation and to yielding of cross-sections. Therefore, yield criteria (YC, *for fully plastic cross-sections*) and elastic criteria (EC, *for first plasticity*) are expressed in these variables. Determination of particular forms of the criteria is outside the scope of this study. It may be done by standard methods of limit analysis and the elastic beam theory, and here we use results taken from known case studies. Only the "moduli" (limit values under separate action of one stress-resultant) should be determined.

  Any PWL approximation of the above criteria may be used, but practical calculation were performed using the so-called "d-approximation" (see, Section 3.5) described by a 48-faces polyhedron in a 4-dimensional stress space. The above concerns the case of materials with the same response to tension and compression. Therefore criteria are symmetrical with respect to each axis. Since the computational time is strongly dependent upon the number of faces, comparison studies were performed with a lower-bound (16 faces) and an upper-bound (8 faces) approximations. The former gives results close to the d-approximation in majority of the cases, the latter is rather inacceptable.
- The formulation used is geometrically linear. As mentioned above, it was intentionally chosen, because nonlinear formulations do not insure rigorously the history-independence of the response, which is essential here. Therefore, to account for geometry changes appearing in the plastic deformation process the so-called post-yield approach was adopted. This approach when applied to the shakedown may be reported as König-Siemaszko formulation [64], [111], and consists of a step-by-step linearization of the problem. The Maier [70] problem is solved for consecutively modified configurations of the structure. The modifications are determined by the collapse mechanism of the preceding step (see, Section 2.5). The collapse mechanism is automatically determined by duality of the LP formulation. On the output, a dependence of the limit load and shakedown load multipliers upon a control displacement are established. The shakedown procedure concerns first of all incremental collapse. If alternating plasticity becomes more restrictive, there is no more collapse mechanism and the program stops. Therefore, loading programs studied in the examples were chosen to correspond presumably to the incremental collapse, with checking if the alternating plasticity is not more restrictive. Both geometrical hardening and softening situations were studied.

- Because of multitude of parameters involved, majority of the examples concerned only a one-storey one-span space frame (Fig. 5.1) under three sets of independently varying loads. Calculation of examples was conducted at increasing deformation, up to 10%-20% of the smallest dimension of the structure. Case studies considered were intended to furnish information on possible approximations of the yield criteria and on the influence of some strength/geometry parameters. The former aspect has been already discussed above; it may be only added that conclusions derived for undeformed structures are generally valid also at increasing deformation.

  In the studies of cross-sectional properties, torsionally very compliant and very stiff structures are compared (open and closed thin-walled cross-section, respectively). Study of the influence of geometric parameters concerned variation of the height-to-length and width-to-length ratios. As a practical engineering example a study of eccentrically braced frames made of IPE commercial profiles was presented. A more extended version of this study was presented by the author at the CSCE conference [1].
- Data concerning the dependence of limit (or shakedown) load upon the deformation furnished a basis for a proposal concerning modification of safety factors required by codes as a function of sensitivity to the deformation of load multipliers. A simplified (two-step) analysis determines the sensitivity and if the yielding (or inadaptation) process is stable (geometrical hardening) the required safety factor applicable to the results of geometrically linear analysis may be appropriately reduced. In unstable situations (geometrical softening) this factor should be increased. Examples of such modifications of the safety factors either at fixed load configuration or for variable configurations that need construction of the load envelopes are given. This approach was presented by the author in more details at the Euromech 298 colloquium [7].

- Partial results and aspects of the above study were presented at the conferences:
  * Inelastic Behaviour of Structures under Variable Repeated Loads, Euromech 298 Colloquium, Warsaw September 1992 [7].
  * XI Polish Conference on Computational Mechanics, Cedzyna, May 1993 [8].
  * Annual Conference of Canadian Society of Civil Engineers, Fredricton, June 1993 [1].
  * Computer Aided Design - Present and Future, Tempus Workshop at T.U. Budapest-Miskolc, June 1993 [9].

7.2 Discussion and Conclusions

- The program has been conceived to permit a unified analysis of all the history-independent responses of complex structures, since the limit analysis and elastic analysis may be considered as particular cases of the shakedown analysis. It permits also an approximate but simple analysis of inadaptation and post-yielding. That concerns also cases when an incremental approach is not feasible because the loading history is unknown. It permits to deal even with very large displacements, since no second-order approximations are necessary and continuous changes of the collapse mechanism are accounted for.
- The geometry changes taken into account are due exclusively to compatible plastic deformations (collapse mechanism). Therefore the approach describes rather the inadaptation or post-yielding than the prior-to-collapse process. It means that structures rather stiff elastically may be considered for analysis. For elastically more compliant systems our method should be in some way combined with proposals aiming at elastic pre-collapse deformation, e.g., [72], [119].
- Since the procedure permits current modification of the structure configuration, it may be also used to assess the safety of deformed/damaged structures, introducing the vector of configuration changes obtained from in-site measurements.
- The d-approximation of the yield criteria depending upon four stress-resultants appears to be a versatile way do describe different shapes of the criteria met in practice. The case which should be considered has always d>0.5. Sometimes a better approximation may be obtained by taking d larger than the value for inscribed polyhedron. If needed, changing the type of PWL approximation may easily be done. In some structural configurations, as, e.g., eccentrically braced frames or any structures with overlapping joints, using yield criteria accounting for shearing forces is recommended. Some difficulties consist in the scarcity of data concerning the yield criteria for real-shaped cross-sections, especially when torsion is present. A directly practice-oriented study aimed at preparation of a catalogue containing these data is surely needed.
- Discretized loads applied at nodes are considered, as in POL(SAP), to act at shear centres of the cross-sections. If non-symmetric cross-sections were considered, it would need in an elastic analysis only the appropriate way of introducing initial data. The above will concern also plastic analysis under assumption that the position of the shear centre remains constant.

- The case studies revealed an interesting feature concerning plastic behaviour of structures at large displacements. The limit load multipliers appeared to be more sensitive to geometry changes than the shakedown load multipliers. The latter are smaller than the former and, therefore, at geometrical hardening the two corresponding curves are divergent, whereas at softening they converge. In many cases (e.g., Figs. 5.5, 6.3) the shakedown curve soon joins the limit-load curve. This effect seems to be the more visible, the more the structure is torsion resistant and complex. For simple plane frames this effect is less frequent. We can interpret this phenomenon as an ability of the structure to improve its geometry if the complexity of the configuration leaves some space for it. This problem was more extensively discussed at the CAD workshop in Budapest [9].
- The two-step analysis permitting for determination of a "secant" sensitivity of the load multipliers to geometry changes seems a more rational way for estimation of safety than that proposed elsewhere [31], [71], and consisting of verification of the initial stability of the considered process. Sometimes a process initially instable may stabilize quickly and vice-versa. In our proposal an average sensitivity depending on the allowable displacement chosen is found and it determines the correction in the estimation of the structure safety level.

- The advantages of the proposed approach may be summarized as below:
    1. It furnishes load multipliers for all history-independent limit states of skeletal structures, determining load variation domains safe against these limit states.
    2. Mechanisms of immediate collapse as well as incremental collapse (if this mode of inadaptation is decisive) are automatically derived.
    3. The same procedure gives also information on post-yielding and inadaptation processes at large displacements; sensitivity of the corresponding multipliers to deformation may be studied and eventual corrections to the code safety factors may be proposed.
    4. All the above may be done using standard FE procedures and data from commercial codes.

7.3 Future development

Below some research topics that seem to be needed for extension and updating of the actual study, as well as for its easy practical implementation are listed. The author either started or intends to start in the future the work on at least of these topics.

- Accounting for material strain hardening will permit not only a more realistic determination of the load-displacement curves, but will permit also to follow the hardening (softening) due to alternating plasticity. It is feasible and is hoped to be done soon by the author.
- Collecting a data-base concerning plastic properties of real cold-rolled steel profiles and presenting them in a catalogue useful for determination of yield criteria. Such a catalogue would find large application, by far larger than this study. The above concerns first of all the contribution of shear stresses (torsion, shearing forces) to yielding.
- Modules of commercial design programs concerning verification of maximal stresses in elastic cross-sections should and may be rearranged in the form useful for easy derivation of elastic criteria. Similarly to the case of plastic behaviour, the above concerns first of all the contribution of shear stresses.
- To permit study of the prior-to-limit (shakedown) state deformations which is needed, e.g., for estimation of the real peak load at geometrical softening, a study of shakedown by selected incremental processes (see Borkowski, Kleiber, [11]) is intended and implementation of Maier's proposals [72] will be tried.
- Case studies intended concern first of all the influence of torsional rigidity and strength on the shakedown and yielding and on the spatial interaction of interconnected plane frames. The above concerns both initial behaviour and response for large displacements. Among the structural configurations, eccentrically braced frames producing exterior shells of the buildings merit special attention, because of the reasons given in Section 5.7.1.
- Studies in shakedown optimization of steel frames and in the sensitivity of optimum solutions to design parameters are also intended.

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APPENDICES

A.1

```
                                              23-07-93 19:26:04            Page 1

************** FORMAT OF DATA FOR BEAM ELEMENT (SAP IV)****************
-----------------------------------------------------------------------
(1) MAIN---------------------------------------------------------------

      READ (5,100,END=990) HED,NUMNP,NELTYP,LL,NF,NDYN,MODEX,NAD,
     *                     KEQB,N10SV
  100 FORMAT (12A6/9I5)
HED      (1-72)      Head Line
NUMNP    (1-5)       Number of Nodal Points
NELTYP   (6-10)      Number of Element Eype
LL       (11-15)     Nnumber of Load Cases
LLK      (16-20)     Number of Kinematical Loads
NF       (21-25)     Number of Freguences
NDYN     (26-30)     Analysis Code 0-Static
MODEX    (31-35)     Solution Mode 0-Execution, 1-Data Check
NAD      (36-40)     Iteration Vector
KEQB     (41-45)     Equation per Block
N10SV    (46-50)     Tape 10 Save Flag

-----------------------------------------------------------------------
(2) INPUTJ (Record of Nodal Data)--------------------------------------

      READ   (5,1000) IT,N,JPR,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)
 1000 FORMAT (2(A1,I4),5I5,3F10.0,I5,F10.0)

IT       (1)         Coordinate System C-cylindrical
N        (2-4)       Subsequent Number of Nodal Points
JPR      (5)         Code of Print
ID       (6-10)
         (11-15)
         (16-20)      Boundary Conditions Code -1 loss in whole structure
         (21-25)                                1 loss in this direction
         (26-30)                                0 free direction
         (31-35)
X        (36-45)
Y        (46-55)     Nodal Points Coordinate
Z        (56-65)
KN       (66-70)     Increament for Generation 0-without
TN       (71-80)     Nodal Point Temperature

-----------------------------------------------------------------------
(3) MAIN (Data of element type) ---------------------------------------
    Control Card

      READ  (5,1001) NPAR
 1001 FORMAT (14I5)
   wirte format from team but read from main pragram
1  NTYPE=2; (1-5)     Type of Beam Element
2  NUMEL    (6-10)    Number of Beam Element
3  NUMETP   (11-15)   Number of Geometric Property Sets
4  NUMFIX   (16-20)   Number of Fixed end Forces
5  NUMMAT   (21-25)   Number of Materials
6  ????     (26-30)   Number of Continuous Loads

-----------------------------------------------------------------------
(4) TEAM (Material data) ----------------------------------------------

    Material Property Cards
a)  material identyfication card

      READ (5,1000) N,E(N),G(N),RO(N),WGHT(N)
 1000 FORMAT (I5,4F10.0)

NUMAT    (1-5)       Material Identyfication Number
E        (6-15)      Young's Modulus
G        (16-25)     Poisson's Ratio
RO       (26-35)     Mass Density
WGHT     (36-45)     Weight Density
-----------------------------------------------------------------------
(5)TEAM (Data of Cross-Section)----------------------------------------

      READ  (5,1002)N,(COPROP(N,J),J=1,6)
 1002 FORMAT(I5,6F10.0)

N              (1-5)   Section Number
COPROP(N,1)    (6-15)  Axial Area  x
COPROP(N,2)    (16-25) Shear Area  y
COPROP(N,3)    (26-35) Shear Area  z
COPROP(N,4)    (36-45) Torsion        xx
COPROP(N,5)    (46-55) Inertia Moment yy
COPROP(N,6)    (56-65) Inertia Moment zz
-----------------------------------------------------------------------
```

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     Path: E:\SAPP
     File: BEAM1     MAT     6,100  a    23-07-93 19:26:04          Page 2

------------------------------------------------------------------------
(6)TEAM (Data of Gravity Loads Multipliers)-----------------------------

     READ  (5,1006) ((EMUL(I,J),J=1,4),I=1,3)
 1030 FORMAT (4F10.0)

EMUL    (1-10)    case A
        (11-20)   case B      Fraction of X-Direction Gravity
        (21-30)   case C
        (31-40)   case D
Card 2   Y-Direction Gravity
Card 3   Z-Direction Gravity

------------------------------------------------------------------------
(7)TEAM (Element Data)--------------------------------------------------

     READ(5,3000) INEL,INI,INJ,INK,IMAT,IMEL,ILC,INELKI,INELKJ,INC,
3000  FORMAT(10I5,2I6,I8)

INEL         (1-5)     Beam Number
INI          (6-10)    Node i
INJ          (11-15)   Node j
INK          (16-20)   Node k
IMAT         (21-25)   Material Number
IMEL         (26-30)   Element Property Number
ILC(1)       (26- )    case A
ILC(2)       (     )   case B  Fixed End Forces Identifications
ILC(3)       (     )   case C
ILC(4)       (  -40)   case D
INELKI       (41-45)   End Release ode at Node i
INELKJ       (46-50)                           j
INC          (51-56)   Increment - Usefull by Generation of Element

------------------------------------------------------------------------
(8) INL (Data of External Loads)----------------------------------------

  300 READ (5,1001) N,L,R
 1001 FORMAT (2I5,7F10.4)

N     (1-5)     Nodal Point Number
L     (6-10)    Structure Load case
R     (11-20)   X-Direction Force
      (21-30)   Y
      (31-40)   Z
      (41-50)   X-Axis Moment
      (51-60)   Y-Axis Moment
      (61-70)   Z-Axis Moment

------------------------------------------------------------------------
(9) ADDSTF -------------------------------------------------------------

     READ  (5,1002)   (STR(I,L),I=1,4)
 1002 FORMAT (4F10.0)

STR     (1-10)   Multiplier for Element Case  A
        (11-20)                               B
        (21-30)                               C
        (31-40)                               D

------------------------------------------------------------------------
(10)TEAM (Data of Fixed End Forces In Local Coordinates)----------------

      READ (5,1005) N,(SFT(N,J),J=1,12)
1005  FORMAT(I5,6F10.0 / F15.0,5F10.0)

N             (1-5)      Number
SFT(N,1)      (6-15)     Force x
SFT(N,2)      (16-25)    Force y
SFT(N,3)      (26-35)    Force z
SFT(N,4)      (36-45)    Moment xx    NODE I
SFT(N,5)      (46-55)    Moment yy
SFT(N,6)      (56-65)    Moment zz

SFT(N,7)      (1-15)     Force x
SFT(N,8)      (16-25)    Force y
SFT(N,9)      (26-35)    Force z
SFT(N,10)     (36-45)    Moment xx    NODE J
SFT(N,11)     (46-55)    Moment yy
SFT(N,12)     (56-65)    Moment zz

------------------------------------------------------------------------
```

```
        Path: E:\SAPP
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INPUT DATA FOR FIRST STEP (UNDEFORMED CONFIGURATION)
(1)-----------------------------------------------------------------
SPACE.FRAME.3.cases.of.load.(9.Element)
    9    1    3    0    0    0    0   42    0
(2)-----------------------------------------------------------------
    1    1    1    1    1    1    1   .00000     .00000     .00000
    2    1    1    1    1    1    1  6.00000     .00000     .00000
    3    0    0    0    0    0    0   .00000     .00000    6.00000
    4    0    0    0    0    0    0  3.00000     .00000    6.00000
    5    0    0    0    0    0    0  6.00000     .00000    6.00000
    6    0    0    0    0    0    0   .00000    6.00000    6.00000
    7    0    0    0    0    0    0  6.00000    6.00000    6.00000
    8    1    1    1    1    1    1   .00000    6.00000     .00000
    9    1    1    1    1    1    1  6.00000    6.00000     .00000
(3)-----------------------------------------------------------------
    2    9    1    0    1
(4)-----------------------------------------------------------------
    121000000.0       .3        0.        0.
(5)-----------------------------------------------------------------
    1   .068750       0.        0. .00005875  .0088930  .0010425
(6)-----------------------------------------------------------------
        0.        0.        0.        0.
        0.        0.        0.        0.
        0.        0.        0.        0.
(7)-----------------------------------------------------------------
    1    1    3    2    1    1    0    0    0    0    0    0     0
    2    2    5    1    1    1    0    0    0    0    0    0     0
    3    3    4    1    1    1    0    0    0    0    0    0     0
    4    4    5    1    1    1    0    0    0    0    0    0     0
    5    3    6    1    1    1    0    0    0    0    0    0     0
    6    5    7    2    1    1    0    0    0    0    0    0     0
    7    6    7    9    1    1    0    0    0    0    0    0     0
    8    6    8    3    1    1    0    0    0    0    0    0     0
    9    7    9    5    1    1    0    0    0    0    0    0     0
(8)-----------------------------------------------------------------
    3    1   1.0000     .0000     .0000     .0000     .0000     .0000
    3    2    .0000    1.0000     .0000     .0000     .0000     .0000
    3    3    .0000     .0000   -2.0000     .0000     .0000     .0000
    4    2    .0000    1.0000     .0000     .0000     .0000     .0000
    4    3    .0000     .0000   -2.0000     .0000     .0000     .0000
    5    2    .0000    1.0000     .0000     .0000     .0000     .0000
    5    3    .0000     .0000   -2.0000     .0000     .0000     .0000
    6    1   1.0000     .0000     .0000     .0000     .0000     .0000
    6    3    .0000     .0000   -2.0000     .0000     .0000     .0000
    7    3    .0000     .0000   -2.0000     .0000     .0000     .0000
 (9)----------------------------------------------------------------
        1.        0.        0.        0.
        1.        0.        0.        0.
        1.        0.        0.        0.
--------------------------------------------------------------------
```

```
        Path: E:\SAPP
        File: EL1      DOC    3,284  a   20-07-93 23:40:30        Page 1

INPUT DATA FOR SECOND STEP (DEFORMED CONFIGURATION)

(1)--------------------------------------------------------------
SPACE.FRAME.3.cases.of.load.(9.Element)
    9    1    3    0    0    0    0   42    0
(2)--------------------------------------------------------------
    1    1    1    1    1    1    1    .00000    .00000    .00000
    2    1    1    1    1    1    1   6.00000    .00000    .00000
    3    0    0    0    0    0    0    .02000    .00165   6.00031
    4    0    0    0    0    0    0   3.01949    .00993   5.99031
    5    0    0    0    0    0    0   6.01942    .01677   5.99913
    6    0    0    0    0    0    0    .00375   6.00162   5.99979
    7    0    0    0    0    0    0   6.00381   6.01662   5.99951
    8    1    1    1    1    1    1    .00000   6.00000    .00000
    9    1    1    1    1    1    1   6.00000   6.00000    .00000
(3)--------------------------------------------------------------
    2    9    1    0    1
(4)--------------------------------------------------------------
    121000000.0        .3        .0        .0
(5)--------------------------------------------------------------
    1 .06875000 .00000000 .00000000 .00005875 .00889300 .00104250
(6)--------------------------------------------------------------
        0.        0.        0.        0.
        0.        0.        0.        0.
        0.        0.        0.        0.
(7)--------------------------------------------------------------
    1    1    3    2    1    1    0    0    0    0    0    0    0
    2    2    5    1    1    1    0    0    0    0    0    0    0
    3    3    4    1    1    1    0    0    0    0    0    0    0
    4    4    5    1    1    1    0    0    0    0    0    0    0
    5    3    6    1    1    1    0    0    0    0    0    0    0
    6    5    7    2    1    1    0    0    0    0    0    0    0
    7    6    7    9    1    1    0    0    0    0    0    0    0
    8    6    8    3    1    1    0    0    0    0    0    0    0
    9    7    9    5    1    1    0    0    0    0    0    0    0
(8)--------------------------------------------------------------
    3    1    1.0000     .0000      .0000     .0000    .0000    .0000
    3    2     .0000    1.0000      .0000     .0000    .0000    .0000
    3    3     .0000     .0000    -2.0000     .0000    .0000    .0000
    4    2     .0000    1.0000      .0000     .0000    .0000    .0000
    4    3     .0000     .0000    -2.0000     .0000    .0000    .0000
    5    2     .0000    1.0000      .0000     .0000    .0000    .0000
    5    3     .0000     .0000    -2.0000     .0000    .0000    .0000
    6    1    1.0000     .0000      .0000     .0000    .0000    .0000
    6    3     .0000     .0000    -2.0000     .0000    .0000    .0000
    7    3     .0000     .0000    -2.0000     .0000    .0000    .0000
 (9)-------------------------------------------------------------
        1.        0.        0.        0.
        1.        0.        0.        0.
        1.        0.        0.        0.
-----------------------------------------------------------------
```

```
                                                              Path: E:\SAPP
     File: INI2     MAT     3,152  a    23-07-93 19:21:44          Page 1

     INPUT DATA FOR SDLA (SUB. INI2) PROGRAM INI
------------------------------------------------------------------------
(1)  READ (1,*) NEL,NND,ISC,NCLA,MM,NN----------------------------------
     FREE FORMAT
NEL  : NUMBER OF ELEMENT
NND  : NUMBER OF NODAL POINTS
ISC  : NUMBRE OF LOAD CASES
NCLA : NUMBER OF CLASS OF ELEMENT (TYPE)
MM   : NUMBER OF UNKNOWS FOR ELEMENT (6*NEL)
NN   : NUMBER OF DEGREE OF FREEDOM   (6*FREE NODES)
------------------------------------------------------------------------
(2)  READ (1,*) (XW(I),YW(I),ZW(I),I=1,NND) ----------------------------
     FREE FORMAT
XW   : NODAL POINT COORDINATE IN X-DIRECTION
YW   : NODAL POINT COORDINATE IN Y-DIRECTION
ZW   : NODAL POINT COORDINATE IN Z-DIRECTION
------------------------------------------------------------------------
(3)  READ (1,*) (NL(I),NR(I),I=1,NEL) ----------------------------------
NL   :  LEFT  NODE NUMBER
NR   :  RIGHT NODE NUMBER
------------------------------------------------------------------------
(4)  READ (1,*) (ID(I),I=1,6*NND) -------------------------------------
ID   : NUMBER OF DEGREE OF FREEDOM
------------------------------------------------------------------------
(5)  READ (1,*) (NCL(I),I=1,NEL) --------------------------------------
NCL  : MATER IDENTYFICATION NUMBER
------------------------------------------------------------------------
(6)  READ (1,*) (A(I),Ix(I),Iy(I),Iz(I),NO(I),Mxo(I),Myo(I),Mzo(I)
    *,I=1,NCLA)
A    :  AXIAL AREA
Ix   :  TORSION xx
Iy   :  INERTIA YY
Iz   :  INERTIA ZZ
NO   :  YIELDS IN EXTENSION
Mxo  :  YIELDS IN TORSION  MOMENT X
Myo  :  YIELDS IN UNIAXIAL MOMENT Y
Mzo  :  YIELDS IN UNIAXIAL MOMENT Z
------------------------------------------------------------------------
(7)  READ (1,*) (SC(I),I=1,ISC*NN) ------------------------------------
SC   : LOAD VECTOR IN SHAKEDOWN ANALYSIS
ISC  : NUMBER OF LOAD CASES
------------------------------------------------------------------------
(8)  READ (1,*) (P(I),I=1,NN) -----------------------------------------
P    : LOAD VECTOR IN LIMIT ANALYSIS
************************************************************************
INPUT  DATA DURING RUNNING THE PROGRAM

(1)     READ(*,'(A30)')  TYT
TYT     : NAME OF THE TASK

(2)     READ(*,*)  IUS
IUS       = 0   ELASTIC LIMIT
          =-1   LIMIT ANALYSIS
          = 1   SHAKEDOWN ANALYSIS

(3)     READ(*,*)  (RVV(I),RV(I),I=1,ISC)
 RVV RV : LIMIT OF LOADS VARATION
 ISC    : NUMBER OF LOAD CASES

(4)     READ (*,*)  NYC
NYC     : YIELD CONDITION =1  FOR LOWER BOUND APPROX. LB=1
                          =2  FOR UPPER BOUND APPROX. UB=2
                          =3  FOR "D-APPROX."          BS=3
(5)     READ(*,*)  EPSD
EPSD    : ACCURACY

(6)     READ(*,*)  IPRI
IPRI    : LEVEL OF OUT PUT OF PLASTICITY

(7)     READ(*,*)  IPRINT
IPRINT  : LEVEL OF OUT PUT OF SIMPLEX'

(8)     READ(*,*)  KWY
KWY     : STEP OF OUTPUT

(9)     READ(*,*)  AL
AL      : STEP OF CALCULATION
------------------------------------------------------------------------
```

```
      Path: E:\SAPP
      File: FRAME    DOC    1,503  a   23-07-93 19:16:18          Page 1

INPUT DATA FOR SDLA PROGRAM (INI PROG.)FIRST STEP ONLY
(1)-------------------------------------------------------------------
9  9  3  1  54  30
(2)-------------------------------------------------------------------
0.  0.  0.  6.  0.  0.  0.  0.  6.
3   0.  6.  6.  0.  6.  0.  6.  6.
6.  6.  6.  0.  6.  0.  6.  6.  0.
(3)-------------------------------------------------------------------
1  3  2  5  3  4  4  5  3  6  5  7  6  7  6  8  7  9
(4)-------------------------------------------------------------------
1 1 1 1 1 1
1 1 1 1 1 1
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
1 1 1 1 1 1
1 1 1 1 1 1
(5)-------------------------------------------------------------------
1 1 1 1 1 1 1 1 1
(6)-------------------------------------------------------------------
.06875  5875E-8  8893E-6  10425E-7  2.918  .037  1.  .271
(7)-------------------------------------------------------------------
1  0  0  0  0  0
0  0  0  0  0  0
0  0  0  0  0  0
1  0  0  0  0  0
0  0  0  0  0  0
0  1  0  0  0  0
0  1  0  0  0  0
0  1  0  0  0  0
0  0  0  0  0  0
0  0  0  0  0  0
0  0 -2  0  0  0
0  0 -2  0  0  0
0  0 -2  0  0  0
0  0 -2  0  0  0
0  0 -2  0  0  0
(8)-------------------------------------------------------------------
1  1 -2  0  0  0
0  1 -2  0  0  0
0  1 -2  0  0  0
1  0 -2  0  0  0
0  0 -2  0  0  0
----------------------------------------------------------------------
```

A.2

I-PROGRAM INI (INPUT DATA)

```
$save
      PROGRAM INI
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
c----------------------------------------------------------------------
c     This program to prepare the input  data for the main program
c     SDLA for the first step only
c----------------------------------------------------------------------
      DOUBLE PRECISION XW,YW,ZW,Ix,Iy,Iz,N0,Mxo,Myo,Mzo,SC,A,RVV,RV,EPSD
     *,EPS,AL,AA,EY,S0,YNM,VYM,P

      CHARACTER*50 HED
      CHARACTER*12 CDAT
      CHARACTER*30 TYT
      CHARACTER*1 IQ

      DIMENSION XW(12),YW(12),ZW(12),NL(12),NR(12),ID(54),A(4),Ix(4),
     *Iy(4),Iz(4),N0(4),Mxo(4),Myo(4),Mzo(4),SC(90),RVV(6),RV(6),
     *ist(12),NCL(9),YNM(4),VYM(864),P(30)

      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

       DATA  NCL / 9*1 /
c      DATA  YNM /90.D0,1.5D0,2.D0,2.D0/
       DATA  YNM /2918D-3,37D-3,1.D0,271D-3/
c      DATA  YNM /3285D-3,56D-3,1.D0,199D-3/
       DATA ID /12*1,30*0,12*1/

      WRITE (*,*)'THE NAME OF DATA FILE '
      READ  (*,*) CDAT
      OPEN (1,FILE = CDAT )
      OPEN (2,FILE='SSSS.DAT')
      OPEN (4,FILE='SDWON.DAT')

      WRITE(*,*) CDAT

C     ************* SUBROUTINE INI2 FOR INPUT DATA **********************
      CALL INI2(NCL,ID,NL,NR,A,Ix,Iy,Iz,N0,Mxo,Myo,Mzo,SC,ist,RVV,RV,
     *TYT,HED,XW,YW,ZW,YNM,VYM,P)

C     ************* SUBROUTINE INDATD IO WRITE INILIAL DATA***************
      CALL INDATA(TYT,XW,YW,ZW,A,IX,IY,IZ,N0,MXO,MYO,MZO,RVV,RV,SC)
      IT=1

c     **************SUBROUTINE INI3 FOR WRITE DATA FOR SDOWN**************
C     ------------------ interface between ini and sdown------------------
      CALL INI3(NCL,ID,NL,NR,A,Ix,Iy,Iz,N0,Mxo,Myo,Mzo,SC,ist,RVV,RV,TYT
     *,HED,XW,YW,ZW,YNM,VYM,P)

       STOP
       END
***********************************************************************


      SUBROUTINE INI2(NCL,ID,NL,NR,A,Ix,Iy,Iz,N0,Mxo,Myo,Mzo,SC,ist,
     *RVV,RV,TYT,HED,XW,YW,ZW,YNM,VYM,P)
      DOUBLE PRECISION XW,YW,ZW,Ix,Iy,Iz,N0,Mxo,Myo,Mzo,SC,A,RVV,RV,EPSD
     *,EPS,AL,AA,EY,S0,YNM,VYM,P

      CHARACTER*50 HED
      CHARACTER*30 TYT
      CHARACTER*1 IQ

      DIMENSION XW(1),YW(1),ZW(1),NL(1),NR(1),ID(1),NCL(1),
     *A(1),Ix(1),Iy(1),Iz(1),N0(1),Mxo(1),Myo(1),Mzo(1),SC(1),RVV(1),
     * RV(1),ist(1),YNM(1),VYM(1),P(1)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      WRITE(*,*)'*************** SD analysis ****************'
```

```
      WRITE(*,*)'TITLE OF PROBLEM '
      READ (*,'(A50)') HED
      WRITE(2,*) HED
      READ (1,*) NEL,NND,ISC,NCLA,MM,NN
      READ (1,*) (XW(I),YW(I),ZW(I),I=1,NND)
      READ (1,*) (NL(I),NR(I),I=1,NEL)
      READ (1,*) (ID(I),I=1,6*NND)
      READ (1,*) (NCL(I),I=1,NEL)
      READ (1,*) (A(I),Ix(I),Iy(I),Iz(I),NO(I),Mxo(I),Myo(I),Mzo(I)
     *,I=1,NCLA)
      READ (1,*) (SC(I),I=1,ISC*NN)
      READ (1,*) (P(I),I=1,NN)
 4    WRITE(*,*)'NAME OF TASK'
      READ(*,'(A30)')  TYT
      WRITE(*,*)'TYPE OF TASK(LSP) OPT:2,MULTI:1,NORM:0'
      READ(*,*)  LSP
      WRITE(*,*)'TYPE OF TASK(IUS) EL:0 LA:-1 SD:1 *NUMBER OF STEPS'
      READ(*,*)  IUS
      WRITE(*,*)'LIMITS OF VARATION OF LOADS (RVV,RV)*ISC'
      READ(*,*)  (RVV(I),RV(I),I=1,ISC)
      WRITE(*,*) 'YIELD CONDITION NYC:, LB=1: UB=2 : BS=3'
      READ (*,*)  NYC
      WRITE(*,*)'ACCUARCY'
      READ(*,*)  EPSD
      WRITE(*,*)'LEVEL OF OUT PUT OF PLASTICITY'
      READ(*,*)  IPRI
      WRITE(*,*)'LEVEL OF OUT PUT OF SIMPLEX'
      READ(*,*)  IPRINT
C     IF(IUS.LT.-1.OR.IUS.GT.1)THEN
C     WRITE(*,*)'STEP OF OUTPUT'
C     READ(*,*)  KWY
C     ENDIF
      IF(ABS(IUS).GT.1.AND.LSP.LE.1)THEN
      WRITE(*,*)'STEP OF CALCULATIONS'
      READ(*,*)  AL
C     ENDIF
      WRITE(*,*)'O.K.?   Y/N'
      READ(*,'(A1)') IQ
      IF(IQ.EQ.'N'.OR.IQ.EQ.'n')GOTO 4

C     THIS BLOCK FOR LIMIT ANALYSIS
C     (FOR ELASTIC AND L.A SC IS MULTIPLIED BY RV SC(NN))

      IF(IUS.GE.1)GOTO 14
      DO 13 I=1,ISC
      DO 12 J=1,NN
      IF(I.EQ.1)GOTO 15
      SC(J)=SC(J)+SC((I-1)*NN+J)*RV(I)
      GOTO 12
 15   SC(J)=SC(J)*RV(1)
 12   CONTINUE
 13   CONTINUE
 14   CONTINUE

C     ****** CALCULATION OF NV********
      IF(NYC.EQ.1)THEN
      KT=8
      ENDIF
      IF(NYC.EQ.2)THEN
      KT=4
      ENDIF
      IF(NYC.EQ.3)THEN
      KT=24
      ENDIF
      KTT=KT+KT
      KTTT=KTT+KTT
      NV=NEL*KTTT
      NS=NN+NV+1
      NO=NS-1
      NZ=MM+1
      MS=NZ+1
      NSMS=NS*MS
      EPSO=1.D-8
      KWY=1
C     CC=.0
```

```
C      IWY=0
c      IPRI=1
c      IPRINT=0

       CALL GENK(NCL,YNM,VYM)

       DO 18,J=1,NEL
 18    IST(J)=0
C      EPS FOR SIMPLEXA
       EPS=EPSD
       RETURN
       END
*************************************************************************


       SUBROUTINE GENK (NCL,YNM,VYM)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       DIMENSION NCL(1),YNM(1),VYM(1)
       COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
      *,IUS,NCLA,LSP,IPRT,KWY,IPRINT,IT,NYC
       COMMON/RD/EPSD,EPS,AL,AA

c      DATA YNM / 800.,60.,80.,40./
C      YNM      :      YIELD NORMAL FORCE AND MOMENTS
C                       (No,Mxo,Myo,Mzo)
C      NCL(J)   :      NUMBER OF CLASS OF ELMENTS J
C      VYM      :      VECTOR OF YILED MOUDULY

C      KTTT=2*KTT
C       d=.471
c       d=.461
        d=.504
c       d=.750

       JJ=0
       DO  47 J=1,NEL
       NCL(J) = 1
       JK=(NCL(J)-1)*4+3
       DO  47 I=1,KTTT
        JJ=JJ+1
       if(nyc.eq.3)then
c       WRITE(*,*)'D=',D
        vym(jj)=d*ynm(jk)
       else
        VYM(JJ)=YNM(JK)
       endif
 47    CONTINUE

       IF (JJ.NE.NV) WRITE(*,*)'ERROR GENK',JJ,NV

       RETURN
       END
*************************************************************************


       SUBROUTINE INDATA(TYT,XW,YW,ZW,A,IX,IY,IZ,NO,MXO,MYO,MZO,RVV,RV
      *,SC)
       DOUBLE PRECISION XW,YW,ZW,SC,A,IX,IY,IZ,NO,MXO,MYO,MZO,RVV,RV,
      *EPSD,EPS,AL,AA

       CHARACTER*30 TYT
       DIMENSION XW(1),YW(1),ZW(1),A(1),IX(1),IY(1),IZ(1),NO(1),MXO(1),
      *MYO(1),MZO(1),SC(1),RVV(1),RV(1),IST(1)

       COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
      *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
       COMMON/RD/EPSD,EPS,AL,AA

       WRITE(2,*)TYT
       IF(IUS.EQ.0)WRITE(2,330)
       IF(IUS.GT.0)WRITE(2,334)
       IF(IUS.LT.0)WRITE(2,335)
c      IF(IHA.GT.0)WRITE(2,336)
       WRITE(2,333)(XW(I),YW(I),ZW(I),I=1,NND)
       WRITE(2,337)
```

```
      WRITE(2,338)(I,A(I),IX(I),IY(I),IZ(I),NO(I),MXO(I),MYO(I),MZO(I),
     *I=1,NCLA)
      IF(IUS.GT.0)WRITE(2,339)(RVV(I),RV(I),I=1,ISC)
      IF(IUS.LE.0)WRITE(2,340)(SC(I),I=1,NN)
      IF(IUS.GT.0)WRITE(2,340)(SC(I),I=1,NN*ISC)
      WRITE(2,341)EPSD,AL

330       FORMAT(1H+,35X,'ELASTIC ANALYSIS')
331       FORMAT(3H    ,I2.2,1H:,I2.2,1H:,I2.2)
332       FORMAT(2X,10A4)
333       FORMAT(1H ,'CO-ORDINATES',6F11.6)
334       FORMAT(1H+,35X,'****SHAKE DOWN****')
335       FORMAT(1H+,35X,'LIMIT ANALYSIS')
336       FORMAT(1H+,55X,'HARDENING')
337       FORMAT(1H ,34X,'***MATERIAL DATA***'/1X,'CLASS',8X,'AERA',11X,
     *    'Ix',13X,'Iy',13X,'Iz',14X,'No',12X,'Mxo',11X,'Myo',12X,'Mzo')
338       FORMAT(1H ,I4,8D15.4)
339       FORMAT(1H ,'RVV',6F8.3)
340       FORMAT(1H ,'SC(I)',12F8.3)
341       FORMAT(1H ,'ACCURACY',1D15.4,'      STEP',1D15.4)

          RETURN
      END
****************************************************************************


      SUBROUTINE INI3(NCL,ID,NL,NR,A,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,
     *SC,ist,RVV,RV,TYT,HED,XW,YW,ZW,YNM,VYM,P)

      DOUBLE PRECISION XW,YW,ZW,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,SC,A,RVV,RV,EPSD
     *,EPS,AL,AA,EY,SO,YNM,VYM,P
      CHARACTER*50 HED
      CHARACTER*30 TYT
      CHARACTER*1 IQ
      DIMENSION XW(1),YW(1),ZW(1),NL(1),NR(1),ID(1),NCL(1),
     * A(1),Ix(1),Iy(1),Iz(1),NO(1),Mxo(1),Myo(1),Mzo(1), SC(1),RVV(1),
     * RV(1),ist(1),YNM(1),VYM(1),P(1)

      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      WRITE(4,*) HED
      write(4,'(A30)')  TYT
      WRITE(4,*) NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      WRITE(4,*) (XW(I),YW(I),ZW(I),I=1,NND)
      WRITE(4,*) (NL(I),NR(I),I=1,NEL)
      WRITE(4,*) (ID(I),I=1,6*NND)
      WRITE(4,*) (NCL(I),I=1,NEL)
      WRITE(4,*) (A(I),Ix(I),Iy(I),Iz(I),NO(I),Mxo(I),Myo(I),Mzo(I)
     *,I=1,NCLA)
      WRITE(4,*) (SC(I),I=1,ISC*NN)
      WRITE(4,*) (RVV(I),RV(I),I=1,ISC)
      WRITE(4,*) (YNM(I),I=1,4)
      WRITE(4,*) (VYM(I),I=1,NV)
      WRITE(4,*) (IST(I),I=1,NEL)
      WRITE(4,*) EPSD,EPS,AL
      write(4,*) (p(i),i=1,nn)

      RETURN
      END

****************************************************************************
```

A.3

```
III-PROGRAM SDLA( SDAKEDOWN, LIMIT AND ELASTIC LIMIT ANALYSES
    OF SPATIAL STRUCTURAL SYSTEMS
$save

      PROGRAM SDLA
c*******
c     This porgram for space structure (shakedwon,limit analysis
c     and elastic limit)
c*******
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION Ix,Iy,Iz,NO,Mxo,Myo,Mzo

      CHARACTER*50 HED
      CHARACTER*12 CDAT
      CHARACTER*30 TYT
      CHARACTER*1 IQ

      DIMENSION XW(9),YW(9),ZW(9),NL(9),NR(9),ID(54),A(4),Ix(4),Iy(4),
     *Iz(4),NO(4),Mxo(4),Myo(4),Mzo(4),SC(96),RVV(4),RV(4),ist(9),NCL(9)
     *,YNM(4),VYM(864),P(30),AAA(20),BBB(20),CCC(20),DDD(20),EEE(20)
      DIMENSION S(50120),CC(54,54),C(1620),CT(1620),GNT(576),EMM(3*54),
     *EM(3*54),ESED(864),GAMMA(1,4),XMATDAT(1,8),xmatdat1(1,8),
     *xmatdat2(1,8),X(900),Y(60)
      DIMENSION IID(9,6),X1(9),Y1(9),Z1(9),E(9),G(9),RO(9),WGHT(9),
     *COPROP(9,6),EMUL(3,4),STR(4,3),INI(9),INJ(9),INK(9),NNN(10),
     *NNL(10),R(10,6)

      COMMON /PAR1/ NF,NDYN,MODEX,NAD,KEQB,N10SV,KN,MTYPE,NUMEL,IMAT,
     *IMEL,ILC(4),INELKI,INELKJ,INC
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA
C     COMMON/MEC/ IME,AMECH

c*****XMATDAT1 for elastic limit, XMATDAT2 for plastic analysis (SD+LA)
c     DATA XMATDAT /4E-3,1E-6,1E-6,2E-6,90., 1.5, 2., 2. /
C*****DATA  MATDAT/      A,    Ix,     Iy,       Iz,    NO,    Mxo,Myo,Mzo /
      DATA XMATDAT1 /.06875,5875E-8,8893E-6,10425E-7,3.285,.056,1.,.199/
      DATA XMATDAT2 /.06875,5875E-8,8893E-6,10425E-7,2.918,.037,1.,.271/

      OPEN (1,FILE='VECTX.DAT')
      OPEN (2,FILE='SSS.DAT',status='unknown',form='formatted')
      OPEN (3,FILE='STRVECT.DAT',STATUS='OLD')
      OPEN (4,FILE='SDWON.DAT')
      OPEN (7,FILE='SIMP.DAT')
      OPEN (8,FILE='VECTY.DAT')
c     open (9,file='EL.D',status='old')
      open (9,file='EL.D')
      open (10,file='plot.dat',form='formatted')
C     OPEN (10,FILE='PLOT.DAT',FORM='FORMATTED'
c     *,ACCESS='DIRECT',RECL=16)
c     open (10,file='plot.dat',status='unknown',form='formatted'
c     *,access='direct',recl=16)
      open (11,file='plote.dat',form='formatted')
      OPEN (14,FILE='CMATG.DAT',STATUS='OLD')

c     **********ITERATION(MAINLOOP)****************
C     SUBROTINUE INI4 TO READ DATA FORM INI PROGRAM
      CALL INI4(NCL,ID,NL,NR,A,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,
     *SC,ist,RVV,RV,TYT,HED,XW,YW,ZW,YNM,VYM,P)

      if(ius.eq.0)then
       do 20 i=1,8
        xmatdat(1,i)=xmatdat1(1,i)
  20   continue
      else
       do 30 i=1,8
        xmatdat(1,i)=xmatdat2(1,i)
  30   continue
      endif

c***  READ (3,*) EMM
c***  CALL RSTR(EMM,EM,GAMMA,XMATDAT,NCL)
      READ (3,*) EM
      CALL GENCT(C,CT,ID)
```

```
      CALL GENN(GNT,xmatdat)
c***** IUS = 0 for elastic limit ******
c*****It is possible to run the program for SD ans EL at the same time
      if (ius.eq.0) then
        CALL GEND(EM,ESED,RVV,RV,XMATDAT)
        CALL GENS2(S,ESED,VYM,gnt)
        CALL SIMPLEX(S,X,Y,Nv,0,1,0,IBL,EPS,IPRINT)
        CALL INIR3e(X,Y,XW,YW,ZW)
        do 32 i=1,8
        xmatdat(1,i)=xmatdat2(1,i)
  32    continue
        CALL GENN(GNT,xmatdat)
        CALL GEND(EM,ESED,RVV,RV,XMATDAT)
        CALL GENS(CT,S,ESED,VYM,GNT)
      endif
c       CALL SIMPLEX(S,X,Y,NO,NN,NZ,MM,IBL,EPS,IPRINT)
c       GO TO 999

c***** IUS = 1 or ius > 1 for shakedwon ******
      if(ius.ge.1) then
        CALL GEND(EM,ESED,RVV,RV,xmatdat)
        CALL GENS(CT,S,ESED,VYM,GNT)
      endif

c***** IUS < 0 for limit analysis *******
      if (ius.lt.0)  CALL GENS1(CT,S,VYM,GNT,p)
      write(*,*) 'number of steps',IT
      CALL SIMPLEX(S,X,Y,NO,NN,NZ,MM,IBL,EPS,IPRINT)
c     write(1,'(8E16.8)') X
c     write(8,'(8F16.8)') Y
      CALL INDATA(TYT,XW,YW,ZW,A,IX,IY,IZ,NO,MXO,MYO,MZO,RVV,RV,SC)
      CALL INIR3(X,Y,XW,YW,ZW)
      CALL MECH(X,XW,YW,ZW,ID)
      CALL INface(IID,XW,YW,ZW,NUMNP,NUMETP,NUMFIX,NUMMAT,
     *E,G,RO,COPROP,WGHT,LL,INI,INJ,INK,NNN,NNL)
      REWIND 4
c     ******* END OF FIRST ITERATION **********
      IT=IT+1

C     to write data for sdwon for the next step
      CALL INI3(NCL,ID,NL,NR,A,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,SC,ist,RVV,RV,
     *TYT,HED,XW,YW,ZW,YNM,VYM,P)
c 999  continue

c     if you would like to print simplex matrix please call sub. MDRUK
c     CALL MDRUK(S,MS,NS,12,'MSIMPLEXXX')
c     write(*,*) 'number of steps',IT
      STOP 'This step finished o.k'
      END
cccccc*****************************************************************

      SUBROUTINE INI4(NCL,ID,NL,NR,A,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,
     *SC,ist,RVV,RV,TYT,HED,XW,YW,ZW,YNM,VYM,P)

C*****TO READ DATD FROM INI PORGRAM

      DOUBLE PRECISION XW,YW,ZW,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,SC,A,RVV,RV,EPSD
     *,EPS,AL,AA,EY,SO,YNM,VYM,p

      CHARACTER*50 HED
      CHARACTER*30 TYT
      CHARACTER*1 IQ

      DIMENSION XW(1),YW(1),ZW(1),NL(1),NR(1),ID(1),NCL(1),
     * A(1),Ix(1),Iy(1),Iz(1),NO(1),Mxo(1),Myo(1),Mzo(1), SC(1),RVV(1),
     * RV(1),ist(1),YNM(1),VYM(1),p(1)

      COMMON /PAR/NEL,NNO,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      READ(4,'(A50)') HED
      READ(4,'(A30)')  TYT
```

```
      READ(4,*) NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      READ(4,*) (XW(I),YW(I),ZW(I),I=1,NND)
      READ(4,*) (NL(I),NR(I),I=1,NEL)
      READ(4,*) (ID(I),I=1,6*NND)
      READ(4,*) (NCL(I),I=1,NEL)
      READ(4,*) (A(I),Ix(I),Iy(I),Iz(I),N0(I),Mxo(I),Myo(I),Mzo(I)
     *,I=1,NCLA)
      READ(4,*) (SC(I),I=1,ISC*NN)
      READ(4,*) (RVV(I),RV(I),I=1,ISC)
      READ(4,*) (YNM(I),I=1,4)
      READ(4,*) (VYM(I),I=1,NV)
      READ(4,*) (IST(I),I=1,NEL)
      READ(4,*)  EPSD,EPS,AL
      read(4,*) (P(i),i=1,nn)

      RETURN
      END
cccccc*****************************************************************


      SUBROUTINE RSTR (EMM,EM,GAMMA,XMATDAT,NCL)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION EMM(1),EM(1),GAMMA(1,4),XMATDAT(1,8),NCL(1)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

C     REDUCES STERSS    EM
c     INPUT    ELASTIC STRESSES FORM SAP   EMM
c     OUTPUT   REDUCES VALUES OF STRESSES  EM
C     EM = EMM*GAMMA

C     MATDAT(SECTION CLASS/(A,Ix,Iy,Iz,N0,Mxo,Myo,Mzo))
C     GAMMA (SECTION CLASS/(Myo/N0,Myo/Mxo,1,Myo/Mzo))

      DO 18 I=1,NCLA
      GAMMA(I,1) =(XMATDAT(I,7)/XMATDAT(I,5))
      GAMMA(I,2) =(XMATDAT(I,7)/XMATDAT(I,6))
      GAMMA(I,3) =1.
C     GAMMA(I,3) =(XMATDAT(I,7)/XMATDAT(I,7))
 18   GAMMA(I,4) =(XMATDAT(I,7)/XMATDAT(I,8))

      II=1
      DO 22  J=1,ISC
      DO 22  I=1,NEL
      EM(II)   = EMM(II)    *GAMMA(NCL(I),1)
      EM(II+1)= EMM(II+1) *GAMMA(NCL(I),2)
      EM(II+2)= EMM(II+2)  *1.
      EM(II+3)= EMM(II+3) *GAMMA(NCL(I),4)
      EM(II+4)= EMM(II+4) *1.
      EM(II+5)= EMM(II+5) *GAMMA(NCL(I),4)
 22   II=II+6
      RETURN
      END
cccccc*****************************************************************

      SUBROUTINE GENCT(C,CT,ID)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Q(6,12),CC(54,54),C(1),CT(1),ID(1)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC

C     INPUT    CC = CONSISTENT MATRIX C
C     OUTPUT   CT = (CCtranspose)
C     ini- number of the first node of the element
c     inj-                  second
c     c    local matrix c
c     iy1  column of location of the first part of matrix c
c     iy2                             second
c     q    transformed matrix c to the global coordinate

c     OPEN (11,FILE='P8CC.DAT')
c     This matrix from sap program
```

```
c     (not including boundary condition in consideration)
c     OPEN (12,FILE='PRC.DAT')
c     OPEN (13,FILE='PRCT.DAT')


      DO 17 I=1,MM
      DO 17 J=1,MM
 17   CC(I,J)=0.0
      DO 55 K=1,NEL
               READ(14,*)iy1,nc,((q(i,j),i=1,nc),j=1,nc),
     1                   iy2,((q(i,j),i=1,nc),j=nc+1,2*nc)
C             WRITE(*,*)iy1,nc,((q(i,j),i=1,nc),j=1,nc),
C    1                   iy2,((q(i,j),i=1,nc),j=nc+1,2*nc)
C        ********INPUT TO THE CC MATRIX********
      CALL PUTM (Q,1,6,6, CC,IY1+1,(k-1)*6+1,MM,6*NND)
      CALL PUTM (Q,7,12,6,CC,IY2+1,(k-1)*6+1,MM,6*NND)
 55   CONTINUE
c     WRITE(11,'(24F5.2)')CC

C     ******** ELIMINATION OF FIXED DEGREES OF FREEDOM*********
c     **here start taking boundary condition in consideration**
      II=1
      DO 99 I=1,NND*6
      IF (ID(I).EQ.0) THEN
      DO 98 J=1,MM
      C(II) = CC(J,I)
 98   II= II+1
      ENDIF
 99   CONTINUE
c     WRITE(12,'(24F5.2)') C
      IF (II-1 .NE. MM*NN) WRITE (*,*) 'ERROR CC',II-1,MM*NN

C     ************** TRANSPOTION ***************
c     ******** transpose C matrix to CT*********
      DO 170 I=1,NN
      DO 170 J=1,MM
 170  CT((J-1)*NN+I) = C((I-1)*MM+J)
C      WRITE(13,'(48F5.2)') CT
c      WRITE(13,*) CT
       RETURN
       END
cccccc*******************************************************************

      SUBROUTINE GENN (GNT,xmatdat)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION GNT(1),GRAD1(4),S(1),xmatdat(1,8)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA
C****
C     GENERATION OF ELEMENT GRADIENT MATRIX NT
C     INPUT INTO S MATRIX
c****
      DO 28 I=1,KTT
      CALL GENG1(I,GRAD1,xmatdat)
      GNT(I)    =GRAD1(1)
      GNT(I+KTTT)=GRAD1(2)
      GNT(I+KTTT+KTTT) = GRAD1(3)
      GNT(I+KTTT+KTTT+KTTT) =GRAD1(4)
      GNT(I+4*KTTT) = 0.D0
 28   GNT(I+5*KTTT) = 0.D0
C
      DO 29 I=KTT+1,KTTT
      CALL GENG1(I,GRAD1,xmatdat)
      GNT(I) =GRAD1(1)
      GNT(I+KTTT)=GRAD1(2)
      GNT(I+2*KTTT)=0.D0
      GNT(I+3*KTTT)=0.D0
      GNT(I+4*KTTT)=GRAD1(3)
 29   GNT(I+5*KTTT)=GRAD1(4)
C
       RETURN
       END
cccccc*******************************************************************
```

```
      SUBROUTINE GENG(NYP,GRAD)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION GRAD(4)
      DIMENSION GAMMA(1,4)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
C*****
C      GENERATION OF GRADIENTS FOR NYC=1
C      FOR LOWER BOUND
C*****
      GO TO(51,53),NYC
 51   NYC=1

       DO 11 I=1,4
  11   GRAD(I)=1.D0

        X=(NYP-1)/2.
      IF((NYP-1)/2.NE.NINT(X))  GRAD(4)=-1.D0
        X=(NYP-1)/4.
      IF((NYP-1)/4.NE.NINT(X))  GRAD(3)=-1.D0
        X=(NYP-1)/8.
      IF((NYP-1)/8.NE.NINT(X))  GRAD(2)=-1.D0
        X=(NYP-1)/16.
      IF((NYP-1)/16.NE.NINT(X)) GRAD(1)=-1.D0
        GO TO 77
c****
c      GENERATION OF GRADIENTS FOR NYC=2
C      FOR UPPER BOUND
c****
 53   NYC=2
      DO 12 I=1,4
 12   GRAD(I)=0.D0
      IF (NYP.EQ.1.OR.NYP.EQ.9 ) GRAD(1) = 1.D0
      IF (NYP.EQ.2.OR.NYP.EQ.10) GRAD(2) = 1.D0
      IF (NYP.EQ.3.OR.NYP.EQ.11) GRAD(3) = 1.D0
      IF (NYP.EQ.4.OR.NYP.EQ.12) GRAD(4) = 1.D0
      IF (NYP.EQ.5.OR.NYP.EQ.13) GRAD(4) =-1.D0
      IF (NYP.EQ.6.OR.NYP.EQ.14) GRAD(3) =-1.D0
      IF (NYP.EQ.7.OR.NYP.EQ.15) GRAD(2) =-1.D0
      IF (NYP.EQ.8.OR.NYP.EQ.16) GRAD(1) =-1.D0
 77   CONTINUE
      RETURN
      END
ccccccc*****************************************************************


      SUBROUTINE GENG1(NYP1,GRAD1,xmatdat)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION GRAD1(4)
      DIMENSION GAMMA(1,4),xmatdat(1,8)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      common /  /d,e,en,dn,ex,dx,ey,dy,ez,dz
C*****
C      GENERATION OF GRADIENTS FOR NYC=1
C      FOR LOWER BOUND
C*****
      GO TO(52,54,56),NYC
 52   NYC=1

      DO 13 I=1,NCLA
      GAMMA(I,1) =(XMATDAT(I,7)/XMATDAT(I,5))
      GAMMA(I,2) =(XMATDAT(I,7)/XMATDAT(I,6))
c     GAMMA(I,3) =1.
      GAMMA(I,3) =(XMATDAT(I,7)/XMATDAT(I,7))
 13   GAMMA(I,4) =(XMATDAT(I,7)/XMATDAT(I,8))

       GRAD1(4)=GAMMA(1,4)
       GRAD1(3)=1.D0
       GRAD1(2)=GAMMA(1,2)
       GRAD1(1)=GAMMA(1,1)

c       GRAD1(1)=(2./90.)
```

```
c        GRAD1(2)=(2./1.5)
c        GRAD1(3)=1.D0
c        GRAD1(4)=1.d0

        X=(NYP1-1)/2.
       IF((NYP1-1)/2.NE.NINT(X))  GRAD1(4) = -GRAD1(4)
        X=(NYP1-1)/4.
       IF((NYP1-1)/4.NE.NINT(X))  GRAD1(3) = -GRAD1(3)
        X=(NYP1-1)/8.
       IF((NYP1-1)/8.NE.NINT(X))  GRAD1(2) = -GRAD1(2)
        X=(NYP1-1)/16.
       IF((NYP1-1)/16.NE.NINT(X)) GRAD1(1) = -GRAD1(1)
       go to 78
c****
c       GENERATION OF GRADIENTS FOR NYC=2
C       FOR UPPER BOUND
c*****
 54    NYC=2
c      write(*,*)' xmatdat',xmatdat
      DO 15 I=1,NCLA
      GAMMA(I,1) =(XMATDAT(I,7)/XMATDAT(I,5))
      GAMMA(I,2) =(XMATDAT(I,7)/XMATDAT(I,6))
c     GAMMA(I,3) =1.
      GAMMA(I,3) =(XMATDAT(I,7)/XMATDAT(I,7))
 15   GAMMA(I,4) =(XMATDAT(I,7)/XMATDAT(I,8))

c       GRAD1(4)=GAMMA(1,4)
c       GRAD1(3)=1.D0
c       GRAD1(2)=GAMMA(1,2)
c       GRAD1(1)=GAMMA(1,1)

      DO 14 I=1,4
 14   GRAD1(I)=0.D0

      IF (NYP1.EQ.1.OR.NYP1.EQ.9 ) GRAD1(1) =  gamma(1,1)
      IF (NYP1.EQ.2.OR.NYP1.EQ.10) GRAD1(2) =  gamma(1,2)
      IF (NYP1.EQ.3.OR.NYP1.EQ.11) GRAD1(3) =  1.d0
      IF (NYP1.EQ.4.OR.NYP1.EQ.12) GRAD1(4) =  gamma(1,4)
      IF (NYP1.EQ.5.OR.NYP1.EQ.13) GRAD1(4) = -gamma(1,4)
      IF (NYP1.EQ.6.OR.NYP1.EQ.14) GRAD1(3) = -1.d0
      IF (NYP1.EQ.7.OR.NYP1.EQ.15) GRAD1(2) = -gamma(1,2)
      IF (NYP1.EQ.8.OR.NYP1.EQ.16) GRAD1(1) = -gamma(1,1)

      GO TO 78

c******
c       GENERATION OF GRADIENTS FOR NYC = 3
C       FOR D APROXIMATION
c******

 56   NYC=3

      DO 16 I=1,NCLA
      GAMMA(I,1) =(XMATDAT(I,7)/XMATDAT(I,5))
      GAMMA(I,2) =(XMATDAT(I,7)/XMATDAT(I,6))
      GAMMA(I,3) =1.
C     GAMMA(I,3) =(XMATDAT(I,7)/XMATDAT(I,7))
 16   GAMMA(I,4) =(XMATDAT(I,7)/XMATDAT(I,8))
c     d=.408
      d=.504
c     D=.75
      E=1.- D
      EN=E*GAMMA(1,1)
      DN=D*GAMMA(1,1)
      EX=E*GAMMA(1,2)
      DX=D*GAMMA(1,2)
      EY=E
      DY=D
      EZ=E*GAMMA(1,4)
      DZ=D*GAMMA(1,4)

      DO 19 I=1,4
 19   GRAD1(I)=0.d0

      IF (NYP1.EQ.1.OR.NYP1.EQ.49 )then
```

```
    GRAD1(2) = EX
    GRAD1(3) = D
  endif
  IF (NYP1.EQ.2.OR.NYP1.EQ.50 )then
    GRAD1(2) = DX
    GRAD1(3) = E
  endif
  IF (NYP1.EQ.3.OR.NYP1.EQ.51 )then
     GRAD1(2) = EX
     GRAD1(4) = DZ
  endif
  IF (NYP1.EQ.4.OR.NYP1.EQ.52 )then
    GRAD1(2) = DX
    GRAD1(4) = EZ
  endif
  IF (NYP1.EQ.5.OR.NYP1.EQ.53 )then
    GRAD1(1) = DN
    GRAD1(2) = EX
  endif
  IF (NYP1.EQ.6.OR.NYP1.EQ.54 )then
     GRAD1(1) = EN
     GRAD1(2) = DX
  endif
  IF (NYP1.EQ.7. OR.NYP1.EQ.55 )then
    GRAD1(1) = DN
    GRAD1(3) = E
  endif
  IF (NYP1.EQ.8. OR.NYP1.EQ.56 )then
    GRAD1(1) = EN
    GRAD1(3) = D
  endif
  IF (NYP1.EQ.9. OR.NYP1.EQ.57 )then
    GRAD1(1) = DN
   GRAD1(4) = EZ
  endif
  IF (NYP1.EQ.10.OR.NYP1.EQ.58 )then
    GRAD1(1) = EN
    GRAD1(4) = DZ
  endif
  IF (NYP1.EQ.11.OR.NYP1.EQ.59 )then
    GRAD1(3) = E
    GRAD1(4) = DZ
  endif
  IF (NYP1.EQ.12.OR.NYP1.EQ.60 )then
    GRAD1(3) = D
    GRAD1(4) = EZ
  endif
  IF (NYP1.EQ.13.OR.NYP1.EQ.61 )then
    GRAD1(2) =-EX
    GRAD1(3) = D
  endif
  IF (NYP1.EQ.14.OR.NYP1.EQ.62 )then
    GRAD1(2) = DX
    GRAD1(3) =-E
  endif
  IF (NYP1.EQ.15.OR.NYP1.EQ.63 )then
    GRAD1(2) =-EX
    GRAD1(4) = DZ
  endif
  IF (NYP1.EQ.16.OR.NYP1.EQ.64 )then
    GRAD1(2) = DX
    GRAD1(4) =-EZ
  endif
  IF (NYP1.EQ.17.OR.NYP1.EQ.65 )then
    GRAD1(1) = DN
    GRAD1(2) =-EX
  endif
  IF (NYP1.EQ.18.OR.NYP1.EQ.66 )then
    GRAD1(1) =-EN
    GRAD1(2) = DX
  endif
  IF (NYP1.EQ.19.OR.NYP1.EQ.67 )then
    GRAD1(1) = DN
    GRAD1(3) =-E
  endif
```

```
IF (NYP1.EQ.20.OR.NYP1.EQ.68 )then
  GRAD1(1) =-EN
  GRAD1(3) = D
endif
IF (NYP1.EQ.21.OR.NYP1.EQ.69 )then
  GRAD1(1) = DN
  GRAD1(4) =-EZ
endif
IF (NYP1.EQ.22.OR.NYP1.EQ.70 )then
  GRAD1(1) =-EN
  GRAD1(4) = DZ
endif
IF (NYP1.EQ.23.OR.NYP1.EQ.71 )then
  GRAD1(3) =-E
  GRAD1(4) = DZ
endif
IF (NYP1.EQ.24.OR.NYP1.EQ.72 )then
  GRAD1(3) = D
  GRAD1(4) =-EZ
endif
IF (NYP1.EQ.25.OR.NYP1.EQ.73 )then
   GRAD1(3) =-D
   GRAD1(4) = EZ
endif
IF (NYP1.EQ.26.OR.NYP1.EQ.74 )then
   GRAD1(3) = E
   GRAD1(4) =-DZ
endif
IF (NYP1.EQ.27.OR.NYP1.EQ.75 )then
   GRAD1(1) = EN
   GRAD1(4) =-DZ
endif
IF (NYP1.EQ.28.OR.NYP1.EQ.76 )then
   GRAD1(1) =-DN
   GRAD1(4) = EZ
endif
IF (NYP1.EQ.29.OR.NYP1.EQ.77 )then
   GRAD1(1) = EN
   GRAD1(3) =-D
endif
IF (NYP1.EQ.30.OR.NYP1.EQ.78 )then
   GRAD1(1) =-DN
   GRAD1(3) = E
endif
IF (NYP1.EQ.31.OR.NYP1.EQ.79 )then
   GRAD1(1) = EN
   GRAD1(2) =-DX
endif
IF (NYP1.EQ.32.OR.NYP1.EQ.80 )then
   GRAD1(1) =-DN
   GRAD1(2) = EX
endif
IF (NYP1.EQ.33.OR.NYP1.EQ.81 )then
   GRAD1(2) =-DX
   GRAD1(4) = EZ
endif
IF (NYP1.EQ.34.OR.NYP1.EQ.82 )then
   GRAD1(2) = EX
   GRAD1(4) =-DZ
endif
IF (NYP1.EQ.35.OR.NYP1.EQ.83 )then
   GRAD1(2) =-DX
   GRAD1(3) = E
endif
IF (NYP1.EQ.36.OR.NYP1.EQ.84 )then
   GRAD1(2) = EX
   GRAD1(3) =-D
endif
IF (NYP1.EQ.37.OR.NYP1.EQ.85 )then
   GRAD1(3) =-D
  GRAD1(4) =-EZ
endif
IF (NYP1.EQ.38.OR.NYP1.EQ.86 )then
   GRAD1(3) =-E
   GRAD1(4) =-DZ
```

```
      endif
      IF (NYP1.EQ.39.OR.NYP1.EQ.87 )then
         GRAD1(1) =-EN
         GRAD1(4) =-DZ
      endif
      IF (NYP1.EQ.40.OR.NYP1.EQ.88 )then
         GRAD1(1) =-DN
         GRAD1(4) =-EZ
      endif
      IF (NYP1.EQ.41.OR.NYP1.EQ.89 )then
         GRAD1(1) =-EN
         GRAD1(3) =-D
      endif
      IF (NYP1.EQ.42.OR.NYP1.EQ.90 )then
         GRAD1(1) =-DN
         GRAD1(3) =-E
      endif
      IF (NYP1.EQ.43.OR.NYP1.EQ.91 )then
         GRAD1(1) =-EN
         GRAD1(2) =-DX
      endif
      IF (NYP1.EQ.44.OR.NYP1.EQ.92 )then
         GRAD1(1) =-DN
         GRAD1(2) =-EX
      endif
      IF (NYP1.EQ.45.OR.NYP1.EQ.93 )then
         GRAD1(2) =-DX
         GRAD1(4) =-EZ
      endif
      IF (NYP1.EQ.46.OR.NYP1.EQ.94 )then
         GRAD1(2) =-EX
         GRAD1(4) =-DZ
      endif
      IF (NYP1.EQ.47.OR.NYP1.EQ.95 )then
         GRAD1(2) =-DX
         GRAD1(3) =-E
      endif
      IF (NYP1.EQ.48.OR.NYP1.EQ.96 )then
         GRAD1(2) =-EX
         GRAD1(3) =-D
      endif
 78   CONTINUE

      RETURN
      END
cccccc*******************************************************************

       SUBROUTINE GEND(EM,ESED,RVV,RV,xmatdat)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       DIMENSION EM(1),ESED(1),RVV(1),RV(1),xmatdat(1,8)
       COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
      *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
       COMMON/RD/EPSD,EPS,AL,AA

C      GENRATION OF ELASTIC STRESSES ENVELOPE VCECTOR D
C      INPUT : EM(I)(I=1,ISC*MM) ELASTIC STRESSES FOR
C                                 ISC LOAD SCHEMES.
C                 [N,Mx(tor),My,Mz]
C      OUTPUT: ESED(nv) ELASTIC STRESS ENVELOPE D
C      VECTOR  d GENERATION

       DO 18 I=1,NV
  18   ESED(I)=0.D0

       NZP=0
       NC=1
       DO 200 J=1,NEL
       KN=NC
       KMX=NC+1
       KMY=NC+2
       KMZ=NC+3
       KB=NZP+1
       KE=NZP+KTT
       kkb=1
```

```
      kke=ktt
      CALL GED (EM,ESED,KN,KMX,KMY,KMZ,KB,KE,kkb,kke,RVV,RV,xmatdat)

      KMY=NC+4
      KMZ=NC+5
      KB=NZP+1+KTT
      KE=NZP+KTT+KTT
      kkb=1+ktt
      kke=ktt+ktt
      CALL GED (EM,ESED,KN,KMX,KMY,KMZ,KB,KE,kkb,kke,RVV,RV,xmatdat)

      NC=NC+6
      NZP=NZP+KTT+KTT
  200 CONTINUE

      IF(NV.NE.NZP)WRITE(*,*)'ERROR GEN3',NV,NZP
      IF(NC.NE.NZ) WRITE(*,*)'ERROR GEN4',NC,NZ

      RETURN
      END
cccccc*****************************************************************

       SUBROUTINE GED(EM,ESED,KN,KMX,KMY,KMZ,KBB,KEE,kkb,kke,RVV,RV,
      *xmatdat)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)

       DIMENSION EM(1),GRAD1(4),ESED(1),RVV(1),RV(1),xmatdat(1,8)
       COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
      *,IUS,NCLA,LSP,IPRT,KWY,IPRINT,IT,NYC
       COMMON/RD/EPSD,EPS,AL,AA

C      KN      = ACTUAL COLUMN FOR N
C      KMX     = ACTUAL COLUMN FOR Mx
C      KMY         = ACTUAL COLUMN FOR My
C      KMZ         = ACTUAL COLUMN FOR Mz
C      kKB(nyb)    = ROW FOR ACTUAL   YIELD PLANE
C      kKE(nye)    = ROW FOR OPPOSITE YIELD PLANE
C      KTT         = NUMBER OF PLANES IN YIELD CONDITION
C      KT          = KTT/2
C      kb,ke       = using in counter esed=nv
C
       DO 100  J=1,ISC
       nyb=kkb
       nye=kke
       KB =KBB
       KE =KEE
       LN =KN +(J-1)*MM
       LMX=KMX+(J-1)*MM
       LMY=KMY+(J-1)*MM
       LMZ=KMZ+(J-1)*MM

       DO 28 I=1,KT
c      call geng(kkb,grad)
       CALL GENG1(nyb,GRAD1,xmatdat)
c      A=EM(LN)*GRAD(1)+EM(LMX)*GRAD(2)+EM(LMY)*GRAD(3)
c     *+EM(LMZ)*GRAD(4)

       A=EM(LN)*grad1(1)+EM(LMX)*grad1(2)+EM(LMY)*grad1(3)
      *+EM(LMZ)*grad1(4)
       IF(A.GT.0.D0)THEN
        ESED(KB)=ESED(KB)+A*RV(J)
        ESED(KE)=ESED(KE)-A*RVV(J)
       ELSE
        ESED(KB)=ESED(KB)+A*RVV(J)
        ESED(KE)=ESED(KE)-A*RV(J)
       ENDIF


       KB=KB+1
       KE=KE-1
       nyb=nyb+1
       nye=nye-1
   28  CONTINUE
  100  CONTINUE
```

```
      RETURN
      END
ccccccc*****************************************************************


C***** ELASTIC LIMIT******************
      SUBROUTINE GENS2(S,ESED,VYM,gnt)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION S(1),ESED(1),VYM(1),gnt(1),GRAD1(4)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRT,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA
c**** S = 2*(nv+1)

      isme=2*(nv+1)
      write(*,*)'SME =',isme
      DO 16 J=1,isme
 16   S(J) = 0.D0

c     write(*,*)'esed',(esed(i),i=1,nv)
      CALL PUT (ESED,1,NV,S,1,1,nv+1,1)
c     write(*,*)'VYM',(VYM(i),i=1,nv)
      CALL PUT (VYM,1,NV,S,2,1,nv+1,1)
      CALL MDRUK(S,2,nv+1,2,'MSIMPLEXXX')

      RETURN
      END
cccccc*****************************************************************


C*****LIMIT ANALYSIS*******************
      SUBROUTINE GENS1(CT,S,VYM,GNT,p)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CT(1),S(1),VYM(1),GNT(1),GRAD1(4),p(1)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRT,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      DO 16 J=1,NSMS
 16    S(J) = 0.D0

      CALL PUT (CT,MM,NN,S,1,1,NS,-1)
c**** external load vector for limit analysis
      call put (p,1,nn,s,nz,1,ns,1)

      CALL PUT (VYM,1,NV,S,MS,NN+1,NS,1)
C****
C     GENERATION OF ELEMENT GRADIENT MATRIX NT
C     INPUT INTO S MATRIX
C     KTTT=2*KTT
C****
      NZP=NN
      NC=1
      DO 100 J=1,NEL
      CALL PUT (GNT,6,KTTT,S,NC,NZP+1,NS,1)
      NZP=NZP+KTTT
      NC=NC+6
 100  CONTINUE

C    ****** OBJECTIVE FUNCTION ******
      S(NS*NZ)=-1.D0
C     CALL MDRUK(S,MS,NS,12,'MSIMPLEXXX')

      RETURN
      END
cccccc*****************************************************************


C***** SHAKEDWON ANALYSIS **************
      SUBROUTINE GENS(CT,S,ESED,VYM,GNT)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CT(1),S(1),ESED(1),VYM(1),GNT(1),GRAD1(4)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRT,KWY,IPRINT,IT,NYC
```

```
      COMMON/RD/EPSD,EPS,AL,AA

      write(*,*)'nsms',nsms
      DO 16 J=1,NSMS
 16   S(J) = 0.D0

c     write(2,*)'esed',(esed(i),i=1,nv)
      CALL PUT (CT,MM,NN,S,1,1,NS, 1)
      CALL PUT (ESED,1,NV,S,NZ,NN+1,NS,1)
      CALL PUT (VYM,1,NV,S,MS,NN+1,NS,1)
C****
C     GENERATION OF ELEMENT GRADIENT MATRIX NT
C     INPUT INTO S MATRIX
C     KTTT=2*KTT
c****
      NZP=NN
      NC=1
      DO 100 J=1,NEL
      CALL PUT (GNT,6,KTTT,S,NC,NZP+1,NS,1)
      NZP=NZP+KTTT
      NC=NC+6
 100  CONTINUE

C    ****** OBJECTIVE FUNCTION ******
      S(NS*NZ)=-1.D0
C     CALL MDRUK(S,MS,NS,12,'MSIMPLEXXX')

      RETURN
      END
cccccc*******************************************************************

      SUBROUTINE MECH(X,XW,YW,ZW,ID)
      DOUBLE PRECISION EPSD,EPS,AL,AA,X,XW,YW,ZW
      DIMENSION X(1),XW(1),YW(1),ZW(1),ID(1)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      AA=.0D0
       DO 11 I=1,NN
      IF(DABS(X(I)).LE.AA)GOTO 11
      AA=DABS(X(I))
 11    CONTINUE
      AA=AL/AA

C         AA-SCALE FACTOR
C         AL-STEP OF CALCULATION /MAX DISPLACEMENT./

      JZ=0
      DO 30 J=1,NND
      JP=(J-1)*6+1
      IF(ID(JP).EQ.1)GOTO 25
      JZ=JZ+1
      XW(J)=XW(J)+X(JZ)*AA
 25      CONTINUE
      IF(ID(JP+1).EQ.1)GOTO 26
      JZ=JZ+1
      YW(J)=YW(J)+X(JZ)*AA
 26      CONTINUE
      IF(ID(JP+2).EQ.1)GOTO 27
      JZ=JZ+1
      ZW(J)=ZW(J)+X(JZ)*AA
 27      CONTINUE
      IF(ID(JP+3).EQ.1)GOTO 28
      JZ=JZ+1
 28      CONTINUE
      IF(ID(JP+4).EQ.1)GOTO 29
      JZ=JZ+1
 29      CONTINUE
      IF(ID(JP+5).EQ.1)GOTO 30
      JZ=JZ+1
 30      CONTINUE

      IF(JZ.NE.NN)WRITE(*,*)'ERROR MECH',JZ,NN
```

```
C
      RETURN
      END
cccccc*****************************************************************


      SUBROUTINE INIR3E(X,Y,XW,YW,ZW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(1),Y(1),XW(1),YW(1),ZW(1),
     *aaa(20),bbb(20),CCC(20),DDD(20),eee(20)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
c      COMMON/RD/EPSD,EPS,AL,AA
c      COMMON/MEC/ IME,AMECH
C
      If(ius.eq.0)write(2,*)'ELASTIC MULTPLIER',Y(1)
      If(ius.eq.0)write(*,*)'ELASTIC MULTPLIER',Y(1)

        do 117 i=1,it-1
        read(11,*) aaa(i),bbb(i),ccc(i),ddd(i),eee(i)
 117    continue
        rewind 11
 116    aaa(it)=xw(3)
        bbb(it)=yw(3)
        ccc(it)=zw(3)
c       ddd(it)=y(nz)
        eee(it)=y(1)
        do 118 i=1,it
        write(11,*) aaa(i),bbb(i),ccc(i),ddd(i),eee(i)
 118    continue


      RETURN
      END
cccccc*****************************************************************

      SUBROUTINE INIR3(X,Y,XW,YW,ZW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(1),Y(1),XW(1),YW(1),ZW(1),IMECH(30),AMECH(30),
     *aaa(20),bbb(20),CCC(20),DDD(20),eee(20)
      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA
      COMMON/MEC/ IME,AMECH
C
      IF(LSP.LE.1)THEN
      DO 14 I=1,30
   14 IMECH(I)=0
      IME=0
      J=0
      DO 41 I=NN+1,NO
      J=J+1
      IF(X(I).LT.EPS)GOTO 41
      IME=IME+1
      IMECH(IME)=J
      AMECH(IME)=X(I)
 41      CONTINUE
      IF(IPRI.GT.0.OR.LSP.EQ.1)WRITE(*,'(15I4)')(IMECH(I),I=1,IME)
      RKWY=IT/KWY
      IF(IT.GT.1.AND.INT(IT/KWY).NE.NINT(RKWY+.499))GOTO 15
      write(2,*)'***MECHANISM INFORMATION***'
      WRITE(2,'(15I4)')(IMECH(I),I=1,IME)
      write(2,*)'**PLASTIC MULIPLIER**'
      WRITE(2,'(5D10.4)')(AMECH(I),I=1,IME)
   15 CONTINUE
      ENDIF
C
      GOTO(47,44,42),IPRI+1
C
 42    WRITE(2,487)(Y(I),I=1,NZ-1)
       write(2,*)'DISPACEMENT VALUE=NN'
 44    WRITE(2,486)(X(I),I=1,NN)

 47    IF(IT.EQ.11.OR.IT.EQ.1.OR.IT.EQ.21.OR.IT.EQ.111.OR.IT.EQ.121.OR.
```

```
     *IT.EQ.31.OR.IT.EQ.41.OR.IT.EQ.51.OR.
     *IT.EQ.61.OR.IT.EQ.71.OR.IT.EQ.81.OR.IT.EQ.91.OR.IT.EQ.101)THEN
      WRITE(2,*)'EP',EP
      IF(LSP.LE.1.AND.ABS(IUS).GT.1)WRITE(2,488)(XW(I),YW(I),ZW(I)
     *,I=1,NND)
      ENDIF

       IF(IPRI.GT.0)WRITE(*,*)'MULTIPLIER',Y(NZ)
c      If(ius.eq.0)write(2,*)'ELASTIC MULTPLIER',Y(1)
c      If(ius.eq.0)write(*,*)'ELASTIC MULTPLIER',Y(1)
       IF(IUS.GT.0)WRITE(2,*)'SHAKEDOWN MULTPLIER',Y(NZ)
       IF(IUS.LT.1)WRITE(2,*)'LIMIT ANALYSIS MULTPLIER',Y(NZ)
c      CLOSE(2,STATUS='KEEP')

c       displacement from simplex method (displ = value of nn)
 486    FORMAT(1H ,'DISPL',6E16.8)
 487    FORMAT(1H ,'FORCE R',6F11.7)
 488    FORMAT(1H ,'CO-ORDINATES',6F11.6)
        CLOSE(2,STATUS='KEEP')

C       CHOOSE NODE
        if(it.eq.1) go to 116
        do 117 i=1,it-1
        read(10,*) aaa(i),bbb(i),ccc(i),ddd(i),eee(i)
 117    continue
        rewind 10
 116    aaa(it)=xw(3)
        bbb(it)=yw(3)
        ccc(it)=zw(3)
        ddd(it)=y(nz)
c       eee(it)=y(1)
        do 118 i=1,it
        write(10,*) aaa(i),bbb(i),ccc(i),ddd(i),eee(i)
 118    continue

      RETURN
      END
cccccc*******************************************************************

      SUBROUTINE PUT(A,NCC,NRR,S,KC,KR,NS,NSC)
      DOUBLE PRECISION A,S
      DIMENSION S(1),A(1)
C
C     NSC-SCALE FACTOR
C     MATRIX A(NRR,NCC) IS INPUTED TO S(NS,?)
C     BEGINNING WITH ELEMENT (KR,KC)
C
      IF(NSC.NE.1)THEN
      DO 50 I=1,NCC
      KV=(KC-1)*NS+KR-1+(I-1)*NS
      KVV=(I-1)*NRR
      DO 50 J=1,NRR
   50 S(KV+J)=A(KVV+J)*NSC
      ELSE
      DO 60 I=1,NCC
      KV=(KC-1)*NS+KR-1+(I-1)*NS
      KVV=(I-1)*NRR
      DO 60 J=1,NRR
   60 S(KV+J)=A(KVV+J)
      ENDIF
      RETURN
      END
cccccc*******************************************************************

      SUBROUTINE MDRUK(S,NC,NR,IDR,NAME)
      DOUBLE PRECISION S
      DIMENSION S(1)
      CHARACTER*10 NAME
C
C     PRINTING S(NR,NC)
C
C     NC- NUMBER OF COLUMNS
C     NR- NUMBER OF ROWS
```

```
C     IDR-NUMBER OF COLUMNS FOR PAGE(FORMAT 999)
C
      KCC=INT(NC/IDR)+1
      DO 99 KC=1,KCC
      WRITE (7,*) NAME,'          PART',KC
      JDR=IDR
      IF(KC.EQ.KCC)JDR=NC-(KC-1)*IDR
      DO 88 K=1,NR
      JC=(KC-1)*IDR*NR+K
      JCC=JC+(JDR-1)*NR
      WRITE(7,999)(S(I),I=JC,JCC,NR)
   88 CONTINUE
   99 CONTINUE

  999 FORMAT(15E10.3)
      RETURN
      END
cccccc*****************************************************************


      SUBROUTINE PUTM(A,nfc,nlc,NRR,S,KC,KR,NS,MS)
      DOUBLE PRECISION A,S
      DIMENSION S(NS,MS),A(NRR,nlc)
C     MATRIX A(NRR,NCC) IS INPUTED TO S(NS,MS)
C     BEGINNING WITH ELEMENT (KR,KC)
c     nfc : number of frist column
c     nlc : number of last  column
      DO 50 I=KC,KC+nlc-nfc
      DO 50 J=KR,KR+NRR-1
 50   S(J,I)=A(J-KR+1,I-(kc-nfc))
      RETURN
      END
cccccc*****************************************************************


      SUBROUTINE INFACE(IID,X1,Y1,Z1,NUMNP,NUMETP,NUMFIX,NUMMAT,
     1E,G,RO,COPROP,WGHT,LL,INI,INJ,INK,NNN,NNL)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)

      CHARACTER*6 HED(12)

      DIMENSION  IID(9,6),X1(1),Y1(1),Z1(1),E(1),G(1),RO(1),WGHT(1),
     *COPROP(9,6),EMUL(3,4),STR(4,3),
     *INI(1),INJ(1),INK(1),NNN(1),NNL(1),r(10,6)

      COMMON /PAR1/ NF,NDYN,MODEX,NAD,KEQB,N10SV,
     *KN,MTYPE,NUMEL,IMAT,IMEL,ILC(4),INELKI,
     *INELKJ,INC

C-----------------------------------------------------------------------
C-----(1) INTIAL DATD SAP-----------------------------------------------
      READ (9,100) HED,NUMNP,NELTYP,LL,NF,NDYN,MODEX,NAD,KEQB,N10SV
  100 FORMAT (12A6/9I5)
C-----------------------------------------------------------------------
C----(2) READ OR  GENERATE NODAL POINT DATA-----------------------------
      do 222  J=1,numnp
c     READ  (9,1000) N,(IID(N,I),I=1,6),X1(N),Y1(N),Z1(N),KN
      READ  (9,1000) N,(IID(N,I),I=1,6)
 222  CONTINUE
 1000 format (7I5,3F10.0,I5,F10.0)
C----------------------------------------------------------------------
c-----(3)--------------------------------------------------------------
      READ  (9,1001)  MTYPE,NUMEL,NUMETP,NUMFIX,NUMMAT
 1001 FORMAT (5I5)
C-----------------------------------------------------------------------
C----(4) READ AND PRINT MATERIAL PROPERTY DATA -------------------------
      do 10 i=1,nummat
      READ (9,1007) N,E(N),G(N),RO(N),WGHT(N)
 10   CONTINUE

 1007 FORMAT(I5,4F10.1)
C-----------------------------------------------------------------------
C----(5) READ AND PRINT GEOMETRIC PROPERTIES OF COMMON ELEMENTS.--------
      DO 30 I=1,NUMETP
      READ  (9,1002) N,(COPROP(N,J),J=1,6)
```

```
   30 CONTINUE
 1002 FORMAT(I5,6F10.8)
C-------------------------------------------------------------------------
C---- (6) ELEMENT LOAD MULTIPLIERS(gravity loads)-----------------------
      READ  (9,1006) ((EMUL(I,J),J=1,4),I=1,3)
 1006 FORMAT (4F10.0)
c-------------------------------------------------------------------------
c-----(7) READ AND PRINT ELEMENT DATA.GRNERATE MISSING INPUT------------
      do 155  n=1,numel
      READ (9,3000) I,INI(n),INJ(n),INK(n),IMAT,IMEL,ILC
     *,INELKI,INELKJ,INC
 155  CONTINUE
 3000 FORMAT (10I5,2I6,I8)
C-------------------------------------------------------------------------
C ----(8) DATA OF EXTERNAL LOAD ------------------------------------------
      do 161 j=1,10
      READ (9,1111)NNN(J),NNL(J),(R(j,I),I=1,6)
 161  CONTINUE
 1111 FORMAT (2I5,6F10.4)
      read (9,*)
C-------------------------------------------------------------------------
C-----(9) READ ELEMENT LOAD MULTIPLIERS ---------------------------------
      DO 50 L=1,LL
      READ  (9,1102)   (STR(I,L),I=1,4)
   50 CONTINUE
 1102 FORMAT (4F10.0)

      REWIND 9

C(1)*****-----------------------------------------------------------------
      WRITE (9,200)HED,NUMNP,NELTYP,LL,NF,NDYN,MODEX,NAD,KEQB,N10SV
  200 FORMAT (12A6/9I5)
C(2)*****-----------------------------------------------------------------
      DO 152 N=1,NUMNP
      WRITE (9,2005) N,(IID(N,I),I=1,6),X1(N),Y1(N),Z1(N)
 2005 format (7I5,3F10.5,I5,F10.0)
 152  CONTINUE
C(3)*****-----------------------------------------------------------------
      WRITE (9,2009) MTYPE,NUMEL,NUMETP,NUMFIX,NUMMAT
 2009 FORMAT (5I5)
C(4)*****-----------------------------------------------------------------
      do 101 N=1,nummat
 101  WRITE(9,2008) N,E(N),G(N),RO(N),WGHT(N)
C  101 G(N)=0.5*E(N)/(1.+G(N))
 2008 FORMAT(I5,4F10.1)
C(5)*****-----------------------------------------------------------------
      DO 102 N=1,NUMETP
      WRITE (9,2020) N,(COPROP(N,J),J=1,6)
  102 CONTINUE
 2020 FORMAT(I5,6F10.8)
C(6)*****-----------------------------------------------------------------
      do 31 i=1,3
      do 31 j=1,4
      emul(i,j)=0.0
 31   continue
      WRITE (9,2006) ((EMUL(I,J),J=1,4),I=1,3)
 2006 FORMAT (4F10.0)
C(7)*****-----------------------------------------------------------------
      do 156  n=1,numel
       write (9,4001) N  ,INI(n),INJ(n),INK(n),IMAT,IMEL,
     *ILC,INELKI,INELKJ,INC
 156  CONTINUE
 4001 FORMAT (10I5,2I6,I8)
C(8)*****-----------------------------------------------------------------
      do 166 j=1,10
      WRITE(9,2111) NNN(J),NNL(J),(R(j,I),I=1,6)
 166  CONTINUE
 2111 FORMAT (2I5,6F10.4)
      WRITE(9,*)
C(9)*****-----------------------------------------------------------------
      DO 59 L=1,LL
      WRITE (9,2102)  (STR(I,L),I=1,4)
   59 CONTINUE
 2102 FORMAT (4F10.0)
```

```
      RETURN
      END
cccccc*****************************************************************


      SUBROUTINE INI3(NCL,ID,NL,NR,A,Ix,Iy,Iz,NO,Mxo,Myo,Mzo,
     *SC,ist,RVV,RV,TYT,HED,XW,YW,ZW,YNM,VYM,p)
      implicit double precision(a-h,o-z)

      DOUBLE PRECISION Ix,Iy,Iz,NO,Mxo,Myo,Mzo
      CHARACTER*50 HED
      CHARACTER*30 TYT
      CHARACTER*1 IQ
      DIMENSION XW(1),YW(1),ZW(1),NL(1),NR(1),ID(1),NCL(1),
     * A(1),Ix(1),Iy(1),Iz(1),NO(1),Mxo(1),Myo(1),Mzo(1), SC(1),RVV(1),
     * RV(1),ist(1),YNM(1),VYM(1),p(1)

      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      WRITE(4,*) HED
      write(4,'(A30)')  TYT
      WRITE(4,*) NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      WRITE(4,*) (XW(I),YW(I),ZW(I),I=1,NND)
      WRITE(4,*) (NL(I),NR(I),I=1,NEL)
      WRITE(4,*) (ID(I),I=1,6*NND)
      WRITE(4,*) (NCL(I),I=1,NEL)
      WRITE(4,*) (A(I),Ix(I),Iy(I),Iz(I),NO(I),Mxo(I),Myo(I),Mzo(I)
     *,I=1,NCLA)
      WRITE(4,*) (SC(I),I=1,ISC*NN)
      WRITE(4,*) (RVV(I),RV(I),I=1,ISC)
      WRITE(4,*) (YNM(I),I=1,4)
      WRITE(4,*) (VYM(I),I=1,NV)
      WRITE(4,*) (IST(I),I=1,NEL)
      WRITE(4,*) EPSD,EPS,AL
      write(4,*) (p(i),i=1,nn)

      RETURN
      END
ccccc******************************************************************


      SUBROUTINE INDATA(TYT,XW,YW,ZW,A,IX,IY,IZ,NO,MXO,MYO,MZO,RVV,RV
     *,SC)
      DOUBLE PRECISION XW,YW,ZW,SC,A,IX,IY,IZ,NO,MXO,MYO,MZO,RVV,RV,
     *EPSD,EPS,AL,AA

      CHARACTER*30 TYT
      DIMENSION XW(1),YW(1),ZW(1),A(1),IX(1),IY(1),IZ(1),NO(1),MXO(1),
     *MYO(1),MZO(1),SC(1),RVV(1),RV(1),IST(1)

      COMMON /PAR/NEL,NND,ISC,MM,NN,NV,No,NZ,NSMS,NS,MS,KTTT,KTT,KT
     *,IUS,NCLA,LSP,IPRI,KWY,IPRINT,IT,NYC
      COMMON/RD/EPSD,EPS,AL,AA

      WRITE(2,*)TYT
      IF(IUS.EQ.0)WRITE(2,330)
      IF(IUS.GT.0)WRITE(2,334)
      IF(IUS.LT.0)WRITE(2,335)
c     IF(IHA.GT.0)WRITE(2,336)
      WRITE(2,333)(XW(I),YW(I),ZW(I),I=1,NND)
      WRITE(2,337)
      WRITE(2,338)(I,A(I),IX(I),IY(I),IZ(I),NO(I),MXO(I),MYO(I),MZO(I),
     *I=1,NCLA)
      IF(IUS.GT.0)WRITE(2,339)(RVV(I),RV(I),I=1,ISC)
      IF(IUS.LE.0)WRITE(2,340)(SC(I),I=1,NN)
      IF(IUS.GT.0)WRITE(2,340)(SC(I),I=1,NN*ISC)
      WRITE(2,341)EPSD,AL
330     FORMAT(1H+,35X,'ELASTIC ANALYSIS')
331     FORMAT(3H    ,I2.2,1H:,I2.2,1H:,I2.2)
332     FORMAT(2X,10A4)
333     FORMAT(1H ,'CO-ORDINATES',6F11.6)
334     FORMAT(1H+,35X,'****SHAKE DOWN****')
```

```
 335      FORMAT(1H+,35X,'LIMIT ANALYSIS')
 336      FORMAT(1H+,55X,'HARDENING')
 337      FORMAT(1H ,34X,'***MATERIAL DATA***'/1X,'CLASS',8X,'AERA',11X,
     *    'Ix',13X,'Iy',13X,'Iz',14X,'No',12X,'Mxo',11X,'Myo',12X,'Mzo')
 338      FORMAT(1H ,I4,8D15.4)
 339      FORMAT(1H ,'RVV',6F8.3)
 340      FORMAT(1H ,'SC(I)',12F8.3)
 341      FORMAT(1H ,'ACCURACY',1D15.4,'     STEP',1D15.4)
          RETURN
      END
cccccc**************************************************************


      SUBROUTINE SIMPLEX(S,X,Y,M,ME,N,NF,IBL,EPS,IPRINT)
      DOUBLE PRECISION S,X,Y,EPS
      DIMENSION S(1), X(1), Y(1)
      DIMENSION NC(100), NR(900)
C
C   SOLUTION OF DUAL PROBLEM
C   OF LINEAR PROGRAMMING
C
      WRITE(*,*)'********SIMPLEX*********'
      IBL= 0
C   ARE THERE PROPER PARAMETERS?
      IF(M.GT.N.AND.N.GE.NF.AND.NF.GE.ME.AND.
     *ME.GE.0) GO TO 1
         IBL= 1
         WRITE(*,9000)
         GO TO 999
C
 1    MS=M+1
      NS=N+1
      IF(IPRINT.GT.1)CALL MDRUK(S,NS,MS,12,'ISIMPLSIMPL')
C
C   NUMBER OF COLUMNS AND ROWS AND IN MATRIX S
      DO 100 I= 1,NS
 100  NC(I)= I
      DO 200 I= 1,MS
 200  NR(I)= -I
C
C   ARE THERE ANY EQUATIONS?
      IF(ME.EQ.0) GO TO 2
         CALL EQSOUT(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
         IF(IBL.EQ.1) GO TO 999
C   ARE THERE FREE PARAMETERS?
 2    IF(NF.EQ.0) GO TO 3
         CALL FREEOUT(S,NC,NR,M,ME,NF,MS,NS,IBL,EPS,IPRINT)
         IF(IBL.EQ.1) GO TO 999
 3    CALL FEASIBLE(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
      IF(IBL.EQ.1) GO TO 999
      CALL OPTIMAL(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
      IF(IBL.EQ.1) GO TO 999
      CALL SOLUTION(S,X,Y,NC,NR,M,N,MS,NS,IPRINT)
 999  RETURN
 9000 FORMAT(//'*** WRONG PARAMRTERS ***')
      END

      SUBROUTINE JORDAN(A,NR,NC,M,N,K,L,IBL,EPS,IPRINT)
      DOUBLE PRECISION A,P,AA,DD,GG,EPS
      DIMENSION A(1), NR(1), NC(1)
C
C   JORDAN'S TRANSFORMATION.
C
C
      KL= (L-1)*M+K
      IF(DABS(A(KL)).GT.EPS) GO TO 1
         IBL= 1
         WRITE(*,9000)
         GO TO 999
 1    P= 1./A(KL)
      IF(K.EQ.1) GO TO 2
         I1= 1
         I2= K-1
         CALL ROW(A,M,N,K,L,I1,I2,P)
```

```
2     IF(K.EQ.M) GO TO 3
        I1= K+1
        I2= M
        CALL ROW(A,M,N,K,L,I1,I2,P)
3     DO 100 J= 1,N
        KJ= (J-1)*M+K
        A(KJ)= A(KJ)*P
100   CONTINUE
      A(KL)= P
      J= NR(K)
      NR(K)= NC(L)
      NC(L)= J
C
999   RETURN
9000  FORMAT(//5X,'*** ZERO OF DIRECTIONAL ELEMENT ***')
      END


      SUBROUTINE ROW(A,M,N,K,L,I1,I2,P)
      DOUBLE  PRECISION A,P,AIL
      DIMENSION A(1)
C
C   TRANSFORM ROW WITH RESPECT TO JORDAN TRANSFORMATION.
C
      DO 100 I= I1,I2
        IL= (L-1)*M+I
        AIL= A(IL)
        DO 200 J= 1,N
          J1= (J-1)*M
          IJ= J1+I
          KJ= J1+K
          A(IJ)= A(IJ)-A(KJ)*AIL*P
200     CONTINUE
        A(IL)= -AIL*P
100   CONTINUE
      RETURN
      END


      SUBROUTINE EQSOUT(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
      DOUBLE PRECISION S,P,R,EPS
      DIMENSION S(1), NC(1), NR(1)
C
      IF(IPRINT.EQ.0) GO TO 4
      WRITE(*,7000)
 4    CONTINUE
C
C   REDUCTION OF SOLUTION
C
C   MAIN LOOP
      DO 100 I= 1,ME
        P= -1.
        L= 0
C   SELECTION OF MAXIMUM ELEMENT AT i-th ROWS.
        DO 200 J= 1,NF
          IJ= (J-1)*MS+I
          R=DABS(S(IJ))
          IF((R.LE.P).OR.(NC(J).LT.0)) GO TO 200
            P= R
            L= J
200     CONTINUE
C
C   TEST ON LINEAR DEPENDENSE.
      IF(L.NE.0)GOTO11
      IBL=1
      WRITE(*,9000)
      GOTO 999
C
 11         CALL JORDAN(S,NR,NC,MS,NS,I,L,IBL,EPS,IPRINT)
100   CONTINUE
C   END OF MAIN LOOP.
C   ORDERING OF COLUMNS, ZERO,FREE,AND NEGATIVE VARIABLES.
      DO 300 I= 1,NF
        IF(NC(I).LT.0) GO TO 300
```

```
          DO 400 J= I,NF
             IF(NC(J).GT.0) GO TO 400
                CALL WEKSEC(I,J,S,NC,MS,NS,IPRINT)
      GOTO 300
400       CONTINUE
300   CONTINUE
999   RETURN
7000  FORMAT(//5X,'EQSOUT')
9000  FORMAT(/1X,'*** LINEAR DEPENDENCE EQUATIONS ***')
      END


      SUBROUTINE WEKSEC(K,L,S,NC,MS,NS,IPRINT)
      DOUBLE PRECISION S,Y
      DIMENSION S(1), NC(1)
C
C   CHANGES COLUMNS K,L BETWEEN ITSELVES.
C
      DO 100 I= 1,MS
         IK= (K-1)*MS+I
         IL= (L-1)*MS+I
         Y= S(IK)
         S(IK)= S(IL)
         S(IL)= Y
 100  CONTINUE
      N= NC(K)
      NC(K)= NC(L)
      NC(L)= N
C
      IF(IPRINT.GT.0)WRITE(*,7000) K,L
 7000 FORMAT(5X,'CHANGES OF THE COLUMNS',2I5)
C
      RETURN
      END

      SUBROUTINE FREEOUT(S,NC,NR,M,ME,NF,MS,NS,IBL,EPS,IPRINT)
      DOUBLE PRECISION S,P,R,EPS
      DIMENSION S(1), NC(1), NR(1)
C
C   FREE PARAMETERS ELIMINATED.
C
      IF(IPRINT.EQ.0) GO TO 4
      WRITE(*,7000)
 4    CONTINUE
C
      IC= ME+1
C
C   MAIN LOOP
      DO 100 I= IC, NF
         P= -1.
         K= 0
C
C   SELECTION OF MAXIMUM ELEMENT AT i-th COLUMN.
         DO 200 J= IC, M
            JI= (I-1)*MS+J
            R= DABS(S(JI))
            IF(R.LE.P.OR.NR(J).GT.0) GO TO 200
               P= R
               K= J
 200     CONTINUE
         CALL JORDAN(S,NR,NC,MS,NS,K,I,IBL,EPS,IPRINT)
 100  CONTINUE
C   END OF MAIN LOOP.
C   ORDERING OF ROWS : FREE AND NEGATIVE VARIABLES.
      DO 300 I= IC,M
         IF(NR(I).GT.0) GO TO 300
            DO 400 J= I,M
               IF(NR(J).LT.0) GO TO 400
                  CALL WEKSER(I,J,S,NR,MS,NS,IPRINT)
                  GO TO 300
 400        CONTINUE
 300  CONTINUE
      RETURN
 7000 FORMAT(//5X,'FREEOUT')
```

```
      END

      SUBROUTINE WEKSER(K,L,S,NR,MS,NS,IPRINT)
      DOUBLE PRECISION S,X
      DIMENSION S(1), NR(1)
C
C   CHANGES OF ROWS K,L BETWEEN EACH OTHERS (ITSELVES).
C
      DO 100 I= 1,NS
         KI= (I-1)*MS+K
         LI= (I-1)*MS+L
         X= S(KI)
         S(KI)= S(LI)
         S(LI)= X
 100  CONTINUE
      N= NR(K)
      NR(K)= NR(L)
      NR(L)= N
C
      IF(IPRINT.GT.0)WRITE(*,7000) K,L
 7000 FORMAT(5X,'CHANGES OF THE ROWS',2I5)
C
      RETURN
      END


      SUBROUTINE FEASIBLE(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
      DOUBLE PRECISION S,P,R,EPS
      DIMENSION S(1), NC(1), NR(1)
C
C   ADMISSIBLE SOLUTION IS DETERMINED.
C
      IF(IPRINT.EQ.0) GO TO 8
      WRITE(*,7000)
 8    CONTINUE
C
      ITER= 0
      IC= ME+1
      JC= NF+1
      MSNS= MS*NS
      MSN= MS*N
C
C   MAIN LOOP.
 1    K= 0
      L= 0
      NEGLC= 0
      II= 0
      R= 1.E20
C
C   NEGLC= NUMBER OF NEGATIVE FREE VARIABLES.
      DO 100 I= JC,M
         INS= MSN+I
         IF(S(INS).GE.-EPS) GO TO 100
            IF(II.NE.0) GO TO 6
               K= I
 6          NEGLC= NEGLC+1
            II= 1
 100  CONTINUE
      IF(NEGLC.GT.0) GO TO 7
C
C   NEGLC=0  -ADMISSIBLE SOLUTION.
      WRITE(*,9002)
      GO TO 999
C
C   SEARCH OF NEGATIVE ELEMENT AT K-th ROWS.
7      CONTINUE
 754    IF(IPRINT.GT.0)WRITE(*,9000) ITER,NEGLC,S(MSNS)
      DO 300 I= IC,N
         KI= (I-1)*MS+K
         IF(S(KI).GE.-EPS) GO TO 300
            L= I
            GO TO 2
 300  CONTINUE
C
```

```
C   NEGATIVE RESLUTS - CONTRADICTION(RESTRICTION) LIMITATION.
      IBL= 1
      WRITE(*,9001)
      GO TO 999
C
C   SEARCH OF DIRECTIONAL ROW.
 2    K= 0
      DO 200 I= JC,M
         IL= (L-1)*MS+I
         INS= (NS-1)*MS+I
         IF(DABS(S(IL)).LE.EPS) GO TO 200
            P= S(INS)/S(IL)
            IF(P.LT.-EPS.OR.P.GE.R) GO TO 200
               IF(P.GT.EPS) GO TO 3
                  IF(S(IL).GT.EPS) GO TO 4
                     GO TO 200
 3                R= P
                  K= I
 200  CONTINUE
      GO TO 5
C
 4    K= I
 5    CALL JORDAN(S,NR,NC,MS,NS,K,L,IBL,EPS,IPRINT)
      ITER= ITER+1
      GO TO 1
C   END OF MAIN LOOP.
C
 999  RETURN
 7000 FORMAT(//5X,'FEASIBLE')
 9000 FORMAT(/5X,'*** ITER=',I5,5X,'NEGLC=',I5,
     *5X,'F=',E15.5,'***')
 9001 FORMAT(/5X,'**** CONTRADDICTION LIMITATION ***')
 9002 FORMAT(/5X,'**** SOLUTION  POSSIBLE ****')
      END

      SUBROUTINE OPTIMAL(S,NC,NR,M,ME,N,NF,MS,NS,IBL,EPS,IPRINT)
      DOUBLE PRECISION S,P,R,EPS
      DIMENSION S(1), NC(1), NR(1)
C
C   OPTIMAL SOLUTION IS DETERMINED.
C
      IF(IPRINT.EQ.0) GO TO 8
      WRITE(*,7000)
 8    CONTINUE
C
      ITER= 0
      IC= ME+1
      JC= NF+1
      MSNS= MS*NS
C
C   MAIN LOOP.
 1    K= 0
      L= 0
      NEGLR= 0
      II= 0
      R= 1.E20
C   NEGLR= NUMBER OF NEGATIVE FACTORS OF OBJECTIVE FUNCTION.
      DO 100 I= IC,N
         MSI= MS*I
         IF(S(MSI).GE.-EPS) GO TO 100
            IF(II.NE.0) GO TO 6
               L= I
 6          NEGLR=NEGLR+1
            II= 1
 100  CONTINUE
      IF(NEGLR.GT.0) GO TO 2
C
C   NEGLR= 0  - OPTIMAL SOLUTION.
         WRITE(*,9002)
         WRITE(*,9000) ITER,NEGLR,S(MSNS)
         GO TO 999
2        CONTINUE
      IF(IPRINT.GT.0)WRITE(*,9000)ITER,NEGLR,S(MSNS)
C
```

```
C
C   SEARCHING OF DIRECTIONAL ROW.
      DO 200 I= JC,M
         IL= (L-1)*MS+I
         INS= (NS-1)*MS+I
         IF(DABS(S(IL)).LE.EPS) GO TO 200
            P= S(INS)/S(IL)
      IF(P.LT.-EPS.OR.P.GT.R)GOTO 200
            IF(P.GT.EPS) GO TO 3
               IF(S(IL).GT.EPS) GO TO 4
                  GO TO 200
 3             R= P
               K= I
 200  CONTINUE
C
C   K=0  - MAX F  UNLIMITED.
      IF(K.GT.0) GO TO 5
         IBL= 1
         WRITE(*,9001)
         GO TO 999
C
 4    K= I
 5    CALL JORDAN(S,NR,NC,MS,NS,K,L,IBL,EPS,IPRINT)
      ITER= ITER+1
      GOTO1
C   END OF MAIN LOOP.
C
 999  RETURN
 7000 FORMAT(//5X,'OPTIMAL')
 9000 FORMAT(/5X,'*** ITER=',I5,5X,'NEGLR=',I5,
     *5X,'F=',E15.5,' ***')
 9001 FORMAT(/5X,'*** MAX F UNLIMITED ***')
 9002 FORMAT(/5X,'***** OPTIMUM SOLUTION *****')
      END


      SUBROUTINE SOLUTION(S,X,Y,NC,NR,M,N,MS,NS,IPRINT)
      DOUBLE PRECISION S,X,Y
      DIMENSION S(1), X(1), Y(1), NC(1), NR(1)
C
C   X, Y IS DETERMINTED.
C
      IF(IPRINT.GT.0)WRITE(*,7000)
 7000 FORMAT(5X,'SOLUTION')
C
C   ZERO OF X, Y .
      DO 100 I= 1,M
 100     X(I)= 0.
      DO 200 I= 1,N
 200     Y(I)= 0.
C
C   ORDERING Y
      DO 300 I= 1,M
         IF(NR(I).LT.0) GO TO 300
            J= NR(I)
            INS= (NS-1)*MS+I
            Y(J)= S(INS)
 300  CONTINUE
C
C   ORDERING X
      DO 400 I= 1,N
         IF(NC(I).GT.0) GO TO 400
            J=-NC(I)
            MSI= (I-1)*MS+MS
            X(J)= S(MSI)
 400  CONTINUE
C
      RETURN
      END
```

****

A.4

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      Path: E:\SAPP
      File: CTANG     FOR     2,121  a    23-07-93 21:47:34          Page 1

      SUBROUTINE CTMAT INI,INJ,XIN,C,Q,DC,ND)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(6,12),Q(6,12),DC(3,3)
      COMMON/PLANXY/IAXIS
c
C     SUB DESCRIBES LOCAL MATRIX C FOR BEAM ELEMENT
C  INI- number of the first node of the element
c  INJ-              second
c  XLN    length of the element
c  C    local matrix c
c  ND   NUMBER OF FIXED DISK FOR WRITEN MATRIX C
c  IY1  COLUMN OF LOCATION OF THE FIRST  PART OF MATRIX C
c  IY2  COLUMN OF LOCATION OF THE SECOND PART OF MATRIX C
c  Q    TRANSFORMED MATRIX C TO THE GLOBAL COORDINATE
c  DC   DIRECTIONAL COSINUSES
c
      IY1=(INI-1)*6
      IY2=(INJ-1)*6
      DO 10 I=1,6
      DO 10 J=1,12
 10            C(I,J)=0.d0
      IF(IAXIS.EQ.0) THEN
               C(1,1) =-1.D0
               C(1,7) = 1.D0
               C(2,4) =-1.D0
               C(2,10)= 1.D0
               C(3,3) =-1.D0/XLN
               C(3,5) = 1.D0
               C(3,9) = 1.D0/XLN
               C(4,2) = 1.D0/XLN
               C(4,6) = 1.D0
               C(4,8) =-1.D0/XLN
               C(5,3) =-1.D0/XLN
               C(5,9) = 1.D0/XLN
               C(5,11)= 1.D0
               C(6,2) = 1.D0/XLN
               C(6,8) =-1.D0/XLN
               C(6,12)= 1.D0
c
c    TRANSFORM C TO GLOBAL COORDINATE
c
               DO 20 I=1,4,3
                  IS=I-1
               DO 20 J=1,4,3
                  JS=J-1
                       DO 30 IX=1,3
                          IXS=IS+IX
                       DO 30 JY=1,3
                          JYS=JS+JY
                          Q(IXS,JYS)  =0.D0
                          Q(IXS,6+JYS)=0.D0
                               DO 50 KN=1,3
                                JKN=JS+KN
                Q(IXS,JYS)=Q(IXS,JYS)+C(IXS,JKN)*DC(JY,KN)
               Q(IXS,6+JYS)=Q(IXS,6+JYS)+C(IXS,6+JKN)*DC(JY,KN)
 50                            CONTINUE
 30                    CONTINUE
 20            CONTINUE

               NC=6
      ELSE
               IY1=(INI-1)*3
               IY2=(INJ-1)*3
               C(2,2)=1.D0/XLN
               C(2,3)=1.D0
               C(2,5)=0.D0-C(2,2)
               C(1,1)=-1.D0
               C(1,4)=1.D0
               C(3,2)=C(2,2)
               C(3,5)=C(2,5)
               C(3,6)=1.D0
c
c      TRANSFORM TO GLOBAL COORDINATE
c
               DO 60 I=1,3
               DO 60 J=1,3
                       Q(I,J)=0.D0
                       Q(I,3+J)=0.D0
               DO 70 K=1,3
                       Q(I,J)=Q(I,J)+C(I,K)*DC(J,K)
                       Q(I,J+3)=Q(I,J+3)+c(I,3+K)*DC(J,K)
 70            CONTINUE
 60            CONTINUE
               NC=3
      ENDIF
      IF(IY1.LT.IY2) THEN
               ERITE(ND,*)IY1,NC,((Q(I,J),I=1,NC),J=1,NC),
     1               IY2,((Q(I,J),I=1,NC),J=NC+1,2*NC)
      ELSE
               WRITE(ND,*)IY2,NC,((Q(I,J),I=1,NC),J=NC+1,2*NC),
     1                IY1,((Q(I,J),I=1,NC),J=1,NC)
      ENDIF
      RETURN
      END
```

```
Path: E:\SAPP
File: ETANG    FOR      780  a   23-07-93 21:35:12          Page 1

      SUBROUTINE ETMATRIX (S,E,ND)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(12,12),E(6,6)
      COMMON/TOTEL/NTOTEL
      COMMON/PLANXY/ IAXIS

c   SUB DESCRIBES MATRIX OF ELASTICITY "E" FROM STIFNEX MATRIX S
c   FOR TANGENT ELEMENT
C   ND  NUMBER OF DISK FOR WRITEN E
C   NTOTEL  TOTAL NUMBER OF NATURAL DEGREE OF FREEDOM
      DO 10 I=1,6
      DO 10 J=1,6
 10           E(I,J)=0.
      IF(IAXIS.EQ.0) THEN
              E(1,1)=S(7,7)
              E(2,2)=S(10,10)
              E(3,3)=S(5,5)
              E(3,5)=S(5,11)
              E(4,4)=S(6,6)
              E(4,6)=S(6,12)
              E(5,3)=S(11,5)
              E(5,5)=S(11,11)
              E(6,4)=S(12,6)
              E(6,6)=S(12,12)
              N=6
              NTOTEL=NTOTEL+6
      ELSE
              E(1,1)=s(7,7)
              E(2,2)=s(6,6)
              E(2,3)=s(6,12)
              E(3,2)=s(12,6)
              E(3,3)=s(12,12)
              N=3
              NTOTEL=NTOTEL+3
      ENDIF
      WRITE(ND,*)N,((E(I,J),I=1,N),J=1,N)
      RETURN
      END
```

```
      Path: E:\SAPP
      File: STRVEC    FOR     2,274   a   23-07-93 22:11:44          Page 1

FOR GENRATING STRESS VECTOR


      SUBROUTINE BEAM
C
      IMPLICIT REAL*8(A-H,O-Z)
C
C     CALLS:  TEAM,STRSC
C     CALLED BY:  ELTYPE
C     FOR GENRATING STRESS VECTOR (STRVECT.DAT)
      COMMON /JUNK/ LT,LH,L,IPAD,SIG(20),N6,N7,N8,N9,N10,IFILL(381)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON /EXTRA/ MODEX,NT8,N10SV,NT10,IFILL2(12),iiextr(10)
      COMMON / /A(1)
      common /planxy/ iaxis
      common /totel/ ntotel
C
      IF(NPAR(1).EQ.0) GO TO 500
      N5A=N5+NUMNP
      N6=N5+NPAR(5) + NUMNP
      N7=N6+NPAR(5)
      N8=N7+NPAR(5)
      N9=N8+12*NPAR(4)
      N10=N9+6*NPAR(3)
      N11=N10+NPAR(5)
      IF(N11.GT.MTOT) CALL ERROR(N11-MTOT)
C
      CALL TEAM(NPAR(2),NPAR(3),NPAR(4),NPAR(5),A(N1),A(N2),A(N3),
     1          A(N4),A(N5A),A(N6),A(N7),A(N8),A(N9),A(N10),
     2          NUMNP,MBAND)
C
      RETURN
C
  500 WRITE (6,2002)
c
      itot=0
      neqpl=neq
c      if(iaxis.eq.1) neqpl=2*neq
      ibegtot=n3+lh*neq
c
      NUME=NPAR(2)
      DO 800 MM=1,NUME
      CALL STRSC (A(N1),A(N3),NEQ,0)
      WRITE (6,2001)
C
      DO 801 L=LT,LH
c
      itotal=ibegtot+(l-1)*ntotel+itot
C
      CALL STRSC (A(N1),A(N3),NEQ,1)
      WRITE(6,3002) MM,L,(SIG(I),I=1,12)
C***  STRESS PORTHOLE
      IF(N10SV.EQ.1)
     *WRITE (NT10) MM,L,(SIG(I),I=1,12)
      if(iaxis.eq.1) then
        a(itotal+1)=sig(7)
        a(itotal+2)=sig(6)
        a(itotal+3)=sig(12)
      else
        a(itotal+1)=sig(7)
        a(itotal+2)=sig(10)
        a(itotal+3)=sig(5)
        a(itotal+4)=sig(6)
        a(itotal+5)=sig(11)
        a(itotal+6)=sig(12)
      endif
  801 CONTINUE
      if(iaxis.eq.1) then
           itot=itot+3
      else
           itot=itot+6
      endif
  800 continue
      do 900 l=lt,lh
         itotal=ibegtot+(l-1)*ntotel
         write(18,*)(a(itotal+k),k=1,ntotel)
  900 continue
      RETURN
 2001 FORMAT (/)
 2002 FORMAT(/29H1.....BEAM FORCES AND MOMENTS//
     . 10H0BEAM LOAD,5X,5HAXIAL, 2(7X,5HSHEAR),5X,7HTORSION,
     .  2(5X,7HBENDING)/ 10H  NO.   NO. ,8X, 2HR1 ,10X, 2HR2 ,10X,
     .  2HR3,10X,2HM1,10X,2HM2,10X,2HM3)
 3002 FORMAT (I5,I4,1PE11.3,5E12.3/8X,6E12.3/)
      END
```

A.5

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     File: OEL        DOC    14,062   a    23-07-93 20:31:18          Page 1

OUTPUT DATA FOR ELASTIC SOLUTION (SAP.IV)
(1)------------------------------------------------------------------------
SPACE.FRAME.3.cases.of.load.(9.Element)

 C O N T R O L    I N F O R M A T I O N
    NUMBER OF NODAL POINTS     =     9
    NUMBER OF ELEMENT TYPES    =     1
    NUMBER OF LOAD CASES       =     3
    NUMBER OF FREQUENCIES      =     0
    ANALYSIS CODE (NDYN)       =     0
      EQ.0,   STATIC
      EQ.1,   MODAL EXTRACTION
      EQ.2,   FORCED RESPONSE
      EQ.3,   RESPONSE SPECTRUM
      EQ.4,   DIRECT INTEGRATION
    SOLUTION MODE (MODEX)      =     0
      EQ.0,   EXECUTION
      EQ.1,   DATA CHECK
    NUMBER OF SUBSPACE
    ITERATION VECTORS (NAD)    =    42
    EQUATIONS PER BLOCK        =     0
    TAPE10 SAVE FLAG (N10SV)   =     0
(2)------------------------------------------------------------------------

1GENERATED NODAL DATA
0NODE   BOUNDARY CONDITION CODES                 NODAL POINT COORDINATES
 NUMBER  X    Y    Z    XX   YY   ZZ          X          Y          Z          T
    1    1    1    1    1    1    1          .000       .000       .000       .000
    2    1    1    1    1    1    1         6.000       .000       .000       .000
    3    0    0    0    0    0    0          .000       .000      6.000       .000
    4    0    0    0    0    0    0         3.000       .000      6.000       .000
    5    0    0    0    0    0    0         6.000       .000      6.000       .000
    6    0    0    0    0    0    0          .000      6.000      6.000       .000
    7    0    0    0    0    0    0         6.000      6.000      6.000       .000
    8    1    1    1    1    1    1          .000      6.000       .000       .000
    9    1    1    1    1    1    1         6.000      6.000       .000       .000
(3)------------------------------------------------------------------------

13 / D   B E A M   E L E M E N T S

  NUMBER OF BEAMS                    =     9
  NUMBER OF GEOMETRIC PROPERTY SETS=       1
  NUMBER OF FIXED END FORCE SETS     =     0
  NUMBER OF MATERIALS                =     1

(4)------------------------------------------------------------------------
 MATERIAL PROPERTIES

     MATERIAL        YOUNG*S       POISSON*S        MASS         WEIGHT
      NUMBER         MODULUS         RATIO        DENSITY       DENSITY

          1        .2100E+08        .3000       .0000E+00     .0000E+00
(5)------------------------------------------------------------------------

 BEAM GEOMETRIC PROPERTIES

     SECTION   AXIAL AREA   SHEAR AREA   SHEAR AREA     TORSION     INERTIA     INERTIA
      NUMBER      A(1)         A(2)         A(3)          J(1)        I(2)        I(3)

         1     .6875E-01    .0000E+00    .0000E+00    .5875E-04   .8893E-02   .1042E-02
(6)------------------------------------------------------------------------

 ELEMENT LOAD MULTIPLIERS
                 A                B                C                D
 X-DIR    .000000E+00      .000000E+00      .000000E+00      .000000E+00
 Y-DIR    .000000E+00      .000000E+00      .000000E+00      .000000E+00
 Z-DIR    .000000E+00      .000000E+00      .000000E+00      .000000E+00

(7)------------------------------------------------------------------------
13/D BEAM ELEMENT DATA

   BEAM    NODE    NODE    NODE   MATERIAL   SECTION   ELEMENT END LOADS    END CODES
  NUMBER    -I      -J      -K     NUMBER     NUMBER    A    B    C    D     -I    -J

     1       1       3       2        1          1      0    0    0    0      0     0
     2       2       5       1        1          1      0    0    0    0      0     0
     3       3       4       1        1          1      0    0    0    0      0     0
     4       4       5       1        1          1      0    0    0    0      0     0
     5       3       6       1        1          1      0    0    0    0      0     0
     6       5       7       2        1          1      0    0    0    0      0     0
     7       6       7       9        1          1      0    0    0    0      0     0
     8       6       8       3        1          1      0    0    0    0      0     0
     9       7       9       5        1          1      0    0    0    0      0     0
---------------------------------------------------------------------------
(8)------------------------------------------------------------------------

1N O D A L   L O A D S   (S T A T I C)   O R   M A S S E S   (D Y N A M I C)
   NODE    LOAD      X-AXIS        Y-AXIS        Z-AXIS        X-AXIS        Y-AXIS        Z-AXIS
  NUMBER   CASE      FORCE         FORCE         FORCE         MOMENT        MOMENT        MOMENT
     3      1     .10000E+01    .00000E+00    .00000E+00    .00000E+00    .00000E+00    .00000E+00
     3      2     .00000E+00    .10000E+01    .00000E+00    .00000E+00    .00000E+00    .00000E+00
     3      3     .00000E+00    .00000E+00   -.20000E+01    .00000E+00    .00000E+00    .00000E+00
     4      2     .00000E+00    .10000E+01    .00000E+00    .00000E+00    .00000E+00    .00000E+00
     4      3     .00000E+00    .00000E+00   -.20000E+01    .00000E+00    .00000E+00    .00000E+00
     5      2     .00000E+00    .10000E+01    .00000E+00    .00000E+00    .00000E+00    .00000E+00
     5      3     .00000E+00    .00000E+00   -.20000E+01    .00000E+00    .00000E+00    .00000E+00
     6      1     .10000E+01    .00000E+00    .00000E+00    .00000E+00    .00000E+00    .00000E+00
     6      3     .00000E+00    .00000E+00   -.20000E+01    .00000E+00    .00000E+00    .00000E+00
     7      3     .00000E+00    .00000E+00   -.20000E+01    .00000E+00    .00000E+00    .00000E+00

(9)------------------------------------------------------------------------
 STRUCTURE          ELEMENT     LOAD     MULTIPLIERS
 LOAD CASE             A          B           C           D
     1               1.000       .000        .000        .000
     2               1.000       .000        .000        .000
     3               1.000       .000        .000        .000
---------------------------------------------------------------------------
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N O D E D I S P L A C E M E N T S / R O T A T I O N S

	NODE NUMBER	LOAD CASE	X-TRANSLATION	Y-TRANSLATION	Z-TRANSLATION	X-ROTATION	Y-ROTATION	Z-ROTATION
0	9	1	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		2	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		3	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
0	8	1	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		2	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		3	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
0	7	1	.18058E-03	.80302E-04	-.15074E-05	-.57030E-05	.38445E-04	.29630E-04
		2	.19147E-06	.33406E-03	-.14642E-05	-.23652E-04	.45170E-07	.14566E-05
		3	-.53626E-08	-.97977E-06	-.83195E-05	.56565E-06	-.42258E-07	.63586E-07
0	6	1	.18265E-03	-.80302E-04	.15074E-05	.57030E-05	.38933E-04	.29631E-04
		2	-.19147E-06	.33406E-03	-.14642E-05	-.23652E-04	-.45170E-07	.14566E-05
		3	.53626E-08	-.97977E-06	.83195E-05	.56565E-06	.42258E-07	.63586E-07
0	5	1	.37540E-03	.79881E-04	-.78917E-06	-.17574E-04	.37631E-04	.29524E-04
		2	-.19243E-06	.33539E-03	.14642E-05	-.73886E-04	-.31793E-07	.45636E-05
		3	-.51421E-06	.97615E-06	-.12460E-04	.29736E-06	-.68356E-04	-.63586E-07
0	4	1	.37644E-03	.96108E-09	-.25979E-06	-.45267E-19	-.18507E-04	.25178E-04
		2	.55819E-18	.34826E-03	.14165E-05	-.73886E-04	-.27271E-19	.55961E-19
		3	-.20061E-19	-.88077E-06	-.21777E-03	.29736E-06	-.34288E-21	.18165E-20
0	3	1	.37748E-03	-.79881E-04	.78917E-06	.17574E-04	.37977E-04	.29526E-04
		2	.19243E-06	.33539E-03	.14642E-05	-.73886E-04	.31793E-07	.45636E-05
		3	.51421E-06	-.97615E-06	.12460E-04	.29736E-06	-.68356E-04	.63586E-07
0	2	1	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		2	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		3	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
0	1	1	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		2	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
		3	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00

1.....BEAM FORCES AND MOMENTS

0BEAM NO.	LOAD NO.	AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	1	-1.899E-01	-3.205E-01	2.818E-01	-2.335E-03	-1.392E+00	-1.100E+00
		1.899E-01	3.205E-01	-2.818E-01	2.335E-03	-2.983E-01	-8.230E-01
1	2	-3.523E-01	-1.180E-04	-1.180E+00	-3.609E-04	5.840E+00	-4.701E-04
		3.523E-01	1.180E-04	1.180E+00	3.609E-04	1.240E+00	-2.381E-04
1	3	2.998E+00	2.488E-01	8.723E-04	-5.029E-06	-1.187E-02	4.969E-01
		-2.998E+00	-2.488E-01	-8.723E-04	5.029E-06	6.639E-03	9.958E-01
2	1	1.899E-01	3.193E-01	2.818E-01	-2.335E-03	-1.392E+00	1.095E+00
		-1.899E-01	-3.193E-01	-2.818E-01	2.335E-03	-2.983E-01	8.205E-01
2	2	-3.523E-01	-1.180E-04	1.180E+00	3.609E-04	-5.840E+00	-4.701E-04
		3.523E-01	1.180E-04	-1.180E+00	-3.609E-04	-1.240E+00	-2.381E-04
2	3	2.998E+00	2.488E-01	-8.723E-04	5.029E-06	1.187E-02	4.969E-01
		-2.998E+00	-2.488E-01	8.723E-04	-5.029E-06	-6.639E-03	9.958E-01
3	1	4.994E-01	2.740E-01	1.804E-01	2.780E-03	-5.413E-01	8.231E-01
		-4.994E-01	-2.740E-01	-1.804E-01	-2.780E-03	3.989E-05	-1.264E-03
3	2	9.261E-02	4.052E-16	-5.000E-01	1.041E-17	4.659E-01	2.320E-04
		-9.261E-02	-4.052E-16	5.000E-01	-1.041E-17	1.034E+00	-2.320E-04
3	3	2.475E-01	-1.000E+00	-1.799E-17	-2.033E-19	-3.958E-03	-1.001E+00
		-2.475E-01	1.000E+00	1.799E-17	2.033E-19	3.958E-03	-1.999E+00
4	1	4.994E-01	2.740E-01	1.804E-01	2.780E-03	-3.989E-05	1.264E-03
		-4.994E-01	-2.740E-01	-1.804E-01	-2.780E-03	-5.412E-01	8.206E-01
4	2	9.261E-02	4.054E-16	5.000E-01	6.939E-18	-1.034E+00	2.320E-04
		-9.261E-02	-4.054E-16	-5.000E-01	-6.939E-18	-4.659E-01	-2.320E-04
4	3	2.475E-01	1.000E+00	1.735E-18	-1.830E-19	-3.958E-03	1.999E+00
		-2.475E-01	-1.000E+00	-1.735E-18	1.830E-19	3.958E-03	1.001E+00
5	1	1.014E-01	-8.406E-02	-1.801E-01	-7.558E-05	5.436E-01	-2.955E-01
		-1.014E-01	8.406E-02	1.801E-01	7.558E-05	5.370E-01	-2.089E-01
5	2	3.200E-01	3.523E-01	9.272E-02	6.087E-06	-4.656E-01	1.240E+00
		-3.200E-01	-3.523E-01	-9.272E-02	-6.087E-06	-9.079E-02	8.737E-01
5	3	8.723E-04	1.887E-03	-1.321E-03	5.403E-03	3.963E-03	6.639E-03
		-8.723E-04	-1.887E-03	1.321E-03	-5.403E-03	3.963E-03	4.681E-03
6	1	-1.014E-01	8.406E-02	-1.801E-01	-6.441E-05	5.435E-01	2.955E-01
		1.014E-01	-8.406E-02	1.801E-01	6.441E-05	5.370E-01	2.089E-01
6	2	3.200E-01	3.523E-01	-9.272E-02	-6.087E-06	4.656E-01	1.240E+00
		-3.200E-01	-3.523E-01	9.272E-02	6.087E-06	9.079E-02	8.737E-01
6	3	8.723E-04	1.887E-03	1.321E-03	-5.403E-03	-3.963E-03	6.639E-03
		-8.723E-04	-1.887E-03	-1.321E-03	5.403E-03	-3.963E-03	4.681E-03
7	1	4.969E-01	2.787E-01	1.782E-01	9.021E-04	-5.347E-01	8.378E-01
		-4.969E-01	-2.787E-01	-1.782E-01	9.021E-04	-5.346E-01	8.342E-01
7	2	-9.214E-02	1.075E-16	-1.263E-15	1.735E-18	9.067E-02	-3.296E-04
		9.214E-02	-1.075E-16	1.263E-15	1.735E-18	-9.067E-02	3.296E-04
7	3	2.581E-03	-1.518E-17	-1.171E-17	4.743E-20	-3.958E-03	3.084E-04
		-2.581E-03	1.518E-17	1.171E-17	4.743E-20	3.958E-03	-3.084E-04
8	1	-3.627E-01	7.686E-02	-6.832E-01	-2.343E-03	8.378E-01	2.098E-01
		3.627E-01	-7.686E-02	6.832E-01	2.343E-03	3.261E+00	2.514E-01
8	2	3.523E-01	-3.200E-01	5.806E-04	1.152E-04	-3.357E-04	-8.737E-01
		-3.523E-01	3.200E-01	-5.806E-04	-1.152E-04	-3.148E-03	-1.046E+00
8	3	2.002E+00	-8.723E-04	1.260E-03	-5.029E-06	-5.094E-03	-4.681E-03
		-2.002E+00	8.723E-04	-1.260E-03	5.029E-06	-2.464E-03	-5.529E-04
9	1	3.627E-01	-7.686E-02	-6.770E-01	-2.343E-03	8.343E-01	-2.098E-01
		-3.627E-01	7.686E-02	6.770E-01	2.343E-03	3.228E+00	-2.514E-01
9	2	3.523E-01	-3.200E-01	-5.806E-04	1.152E-04	3.357E-04	-8.737E-01
		-3.523E-01	3.200E-01	5.806E-04	1.152E-04	3.148E-03	-1.046E+00
9	3	2.002E+00	-8.723E-04	-1.260E-03	5.029E-06	5.094E-03	-4.681E-03
		-2.002E+00	8.723E-04	1.260E-03	-5.029E-06	2.464E-03	-5.529E-04

Stress Vector from Sap IV Program

Case Load No. "1"

No. Element	AXIAL(Nxr)	TORSION(Mxr)	BENDING(Myl)	BENDING(Mzl)	BENDING(Myr)	BENDING(Mzr)
1	+1.89893103900267E-001	+2.33508364080813E-003	-1.39230699280369	-1.10018554600969	-2.98276797867249E-001	-8.23048594350541E-001
2	-1.89893103900267E-001	+2.33498229666346E-003	-1.39230699280370	+1.09514122682048	-2.98276797867250E-001	+8.20532018073511E-001
3	-4.99356613400352E-001	-2.77981272878911E-003	-5.41254731551832E-001	+8.23124172114258E-001	+3.98852603318289E-005	-1.26387145608742E-003
4	-4.99356613400323E-001	-2.77981272878911E-003	-3.98852603296085E-005	+1.26387145608720E-003	-5.41174961031177E-001	+8.20596429202084E-001
5	-1.01359016347988E-001	+7.55777637167845E-005	+5.43589815192641E-001	-2.95496985138459E-001	+5.37036364045061E-001	-2.08864992776280E-001
6	+1.01359016347992E-001	+6.44111285729582E-005	+5.43509943327841E-001	+2.95496985138461E-001	+5.36956492180263E-001	+2.08864992776281E-001
7	-4.96889070322538E-001	-9.02067090981300E-004	-5.34692954597986E-001	+8.37773487692799E-001	-5.34613184077327E-001	+8.34215373221226E-001
8	+3.62725139804794E-001	+2.34340944708014E-003	+8.37849065456515E-001	+2.09767059867261E-001	+3.26144269184602	+2.51384980720109E-001
9	-3.62725139804795E-001	+2.34330810293547E-003	+8.34279784349799E-001	-2.09767059867262E-001	+3.22752107309355	-2.51384980720110E-001

Case Load No. "2"

No. Element	AXIAL(Nxr)	TORSION(Mxr)	BENDING(Myl)	BENDING(Mzl)	BENDING(Myr)	BENDING(Mzr)
1	+3.52329032618433E-001	+3.60917812550487E-004	+5.83972617000060	-4.70107688121184E-004	+1.24027620905811	-2.38097219489488E-004
2	+3.52329032618433E-001	-3.60917812550470E-004	-5.83972617000061	-4.70107688117923E-004	-1.24027620905811	-2.38097219487035E-004
3	-9.26055122812864E-002	-1.04083408558608E-017	+4.65912528400686E-001	+2.32010468632503E-004	+1.03408747159931	-2.32010468631286E-004
4	-9.26055122812867E-002	-6.93889390390723E-018	-1.03408747159930	+2.32010468631294E-004	-4.65912528400688E-001	-2.32010468630085E-004
5	-3.19999603490217E-001	-6.08675085698715E-006	-4.65551610588143E-001	+1.24027620905811	-9.07896680071859E-002	+8.73697986652491E-001
6	-3.19999603490231E-001	+6.08675085698078E-006	+4.65551610588141E-001	+1.24027620905811	+9.07896680071885E-002	+8.73697986652492E-001
7	+9.21429948550589E-002	-1.73472347597681E-018	+9.06744709816508E-002	-3.29628278529409E-004	-9.06744709816450E-002	+3.29628278530052E-004
8	-3.52329032618433E-001	-1.15197025538054E-004	-3.35715029386401E-004	-8.73697986652490E-001	-3.14759443558680E-003	-1.04629963428877
9	-3.52329032618433E-001	+1.15197025538064E-004	+3.35715029386999E-004	-8.73697986652492E-001	+3.14759443558928E-003	-1.04629963428877

Case Load No. "3"

No. Element	AXIAL(Nxr)	TORSION(Mxr)	BENDING(Myl)	BENDING(Mzl)	BENDING(Myr)	BENDING(Mzr)
1	-2.99811343885691	+5.02881941785197E-006	-1.18722635897108E-002	+4.96948286277748E-001	+6.63862033161553E-003	+9.95772812778876E-001
2	-2.99811343885691	-5.02881941785230E-006	+1.18722635897110E-002	+4.96948286277748E-001	-6.63862033161548E-003	+9.95772812778876E-001
3	-2.47465736896546E-001	+2.03287907341032E-019	-3.95831001925761E-003	-1.00117547349887	+3.95831001925765E-003	-1.99882452650113
4	-2.47465736896545E-001	+1.82959116606929E-019	-3.95831001925766E-003	+1.99882452650113	+3.95831001925767E-003	+1.00117547349887
5	-8.72273876349250E-004	-5.40266071999682E-003	+3.96333883867546E-003	+6.63862033161554E-003	+3.96333883867548E-003	+4.68074652690442E-003
6	-8.72273876349278E-004	+5.40266071999682E-003	-3.96333883867542E-003	+6.63862033161550E-003	-3.96333883867543E-003	+4.68074652690440E-003
7	-2.58077419985699E-003	+4.74338450462408E-020	-3.95831001925759E-003	+3.08377341931653E-004	+3.95831001925767E-003	-3.08377341931751E-004
8	-2.00188656114309	+5.02881941785194E-006	-5.09428337806516E-003	-4.68074652690442E-003	-2.46368414372605E-003	-5.52896731191093E-004
9	2.00188656114309	-5.02881941785228E-006	+5.09428337806507E-003	-4.68074652690439E-003	+2.46368414372576E-003	-5.52896731191055E-004

```
OUTPUT RESULTS FILE (SSS.DAT)
SHAKEDOWN ANALYSIS

Y.C= 1 (lower bound)
+                                  ****SHAKEDOWN****

CO-ORDINATES    .000000    .000000    .000000   6.000000    .000000    .000000
CO-ORDINATES    .000000    .000000   6.000000   3.000000    .000000   6.000000
CO-ORDINATES   6.000000    .000000   6.000000    .000000   6.000000   6.000000
CO-ORDINATES   6.000000   6.000000   6.000000    .000000   6.000000    .000000
CO-ORDINATES   6.000000   6.000000    .000000

                                  ***MATERIAL DATA***
CLASS   AERA        Ix         Iy         Iz         No         Mxo        Myo        Mzo
  1   .6875D-01  .5875D-04  .8893D-02  .1042D-02  .2918D+01  .3700D-01  .1000D+01  .2710D+00

RVV    .000   1.000    .000   1.000    .000   1.000

SC(I)   1.000    .000    .000    .000    .000    .000    .000    .000    .000    .000    .000    .000
SC(I)    .000    .000    .000    .000    .000    .000   1.000    .000    .000    .000    .000    .000
SC(I)    .000    .000    .000    .000    .000    .000    .000   1.000    .000    .000    .000    .000
SC(I)    .000   1.000    .000    .000    .000    .000    .000   1.000    .000    .000    .000    .000
SC(I)    .000    .000    .000    .000    .000    .000    .000    .000    .000    .000    .000    .000
SC(I)    .000    .000  -2.000    .000    .000    .000    .000    .000  -2.000    .000    .000    .000
SC(I)    .000    .000  -2.000    .000    .000    .000    .000    .000  -2.000    .000    .000    .000
SC(I)    .000    .000  -2.000    .000    .000    .000

ACCURACY      .1000D-06     STEP      .2000D-01


***MECHANISM INFORMATION***

   2   6   8  40  43  47  59  92  94 107 143 157 169 173 189
 195 199 249 250 254 266 268 270 282 288


 **PLASTIC MULIPLIER**

 .5865D-02 .1009D-02 .3665D-02 .1119D-01 .7031D-02
 .1439D-01 .1952D-01 .7126D-02 .1010D-01 .2607D-02
 .5935D-03 .5754D-03 .3105D-02 .6436D-03 .1173D-02
 .9097D-03 .9257D-03 .3225D-02 .9995D-03 .3077D-02
 .2757D-03 .4566D-03 .7058D-02 .8082D-02 .6768D-03


RESIDUAL STRESS VALUE=MM

 FORCE R   .0175749  -.0003081  -.1810031  -.0491563   .0430655  -.0134044
 FORCE R   .0333466  -.0002250   .1479127  -.1213957  -.0657958  -.0292133
 FORCE R   .0228724   .0003177   .1429354   .0106663  -.0614046  -.0201202
 FORCE R   .0228724   .0003177   .0614046   .0201202   .0201262  -.0295741
 FORCE R  -.0501665  -.0027381  -.1432435   .0427478   .2179173   .0816096
 FORCE R   .0134908  -.0003608  -.0203512   .0661135   .0337257   .1150583
 FORCE R   .0584789   .0001031  -.2181725   .0992915  -.0340060  -.0955975
 FORCE R  -.0201106  -.0002552   .0965534  -.0817127   .3289938   .0328919
 FORCE R  -.0308110  -.0002804  -.0959583  -.1149552  -.2415406  -.0562786


 DISPACEMENT VALUE=NN

 DISPL   .23334809E+00   .19253482E-01   .36119106E-02  -.32089136E-02   .38891348E-01   .32184001E-01
 DISPL   .22744529E+00   .11580549E+00  -.11306213E+00  -.83545007E-01  -.24667200E-01   .29211566E-01
 DISPL   .22655178E+00   .19561839E+00  -.10197825E-01  -.13078401E-01  -.34288102E-01   .26604301E-01
 DISPL   .43804903E-01   .18852917E-01  -.25019936E-02  -.31421528E-02   .73008172E-02   .31015152E-01
 DISPL   .44433882E-01   .19393166E+00  -.56716026E-02  -.35747493E-02   .52826817E-03   .29179791E-01

 SHAKEDOWN MULTPLIER+1.14275551180104E-001
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Path: E:\SAPP
File: S11 DAT 3,508 a 23-07-93 19:56:18 Page 1

FILE "PLOT.DAT" (FOR DEFORMED CONFIGURATION) Y.C=1

NUNBER OF STEP = 50 (SPACE FRAME)

No.	X(I)	Y(I)	Z(I)	SD(MULT.)
1	0.000000E+000	0.000000E+000	6.00000	1.240677E-001
2	1.795367E-002	4.151886E-003	6.00000	1.231204E-001
3	3.765606E-002	6.782428E-003	5.99994	1.221286E-001
4	5.726407E-002	9.518770E-003	5.99981	1.211681E-001
5	7.677997E-002	1.235691E-002	5.99962	1.202450E-001
6	9.620595E-002	1.529304E-002	5.99937	1.193395E-001
7	1.156330E-001	1.814810E-002	5.99905	1.184744E-001
8	1.349721E-001	2.110060E-002	5.99867	1.176335E-001
9	1.542254E-001	2.414664E-002	5.99822	1.168164E-001
10	1.733964E-001	2.727993E-002	5.99772	1.160126E-001
11	1.926160E-001	3.025676E-002	5.99715	1.151870E-001
12	2.117775E-001	3.327996E-002	5.99652	1.143845E-001
13	2.308818E-001	3.634930E-002	5.99583	1.136056E-001
14	2.499298E-001	3.946459E-002	5.99507	1.128389E-001
15	2.699298E-001	4.109767E-002	5.99423	1.120665E-001
16	2.898599E-001	4.305761E-002	5.99332	1.113195E-001
17	3.095555E-001	4.532101E-002	5.99235	1.106094E-001
18	3.290489E-001	4.787431E-002	5.99132	1.099253E-001
19	3.483255E-001	5.070650E-002	5.99024	1.092559E-001
20	3.681685E-001	5.286789E-002	5.98907	1.085711E-001
21	3.881685E-001	5.417162E-002	5.98783	1.078711E-001
22	4.081685E-001	5.564989E-002	5.98652	1.071928E-001
23	4.281685E-001	5.728978E-002	5.98514	1.065340E-001
24	4.481685E-001	5.906701E-002	5.98369	1.058826E-001
25	4.681685E-001	6.048230E-002	5.98218	1.052425E-001
26	4.881685E-001	6.191083E-002	5.98060	1.046178E-001
27	5.081685E-001	6.340252E-002	5.97895	1.040079E-001
28	5.281685E-001	6.495742E-002	5.97724	1.034077E-001
29	5.481685E-001	6.634085E-002	5.97545	1.028120E-001
30	5.681685E-001	6.777793E-002	5.97360	1.022112E-001
31	5.881684E-001	6.926868E-002	5.97168	1.016041E-001
32	6.081685E-001	7.081312E-002	5.96970	1.010099E-001
33	6.281685E-001	7.241122E-002	5.96764	1.004276E-001
34	6.481684E-001	7.406300E-002	5.96551	9.985610E-002
35	6.681685E-001	7.576854E-002	5.96332	9.940500E-002
36	6.833054E-001	5.707228E-002	5.96186	9.895391E-002
37	7.033054E-001	5.886780E-002	5.95955	9.840342E-002
38	7.233054E-001	6.071724E-002	5.95717	9.786288E-002
39	7.433054E-001	6.262064E-002	5.95473	9.733202E-002
40	7.633054E-001	6.457803E-002	5.95221	9.681059E-002
41	7.833054E-001	6.658950E-002	5.94962	9.629633E-002
42	8.029583E-001	6.861743E-002	5.94701	9.577772E-002
43	8.223810E-001	7.067243E-002	5.94437	9.524906E-002
44	8.423810E-001	7.072623E-002	5.94160	9.473425E-002
45	8.623810E-001	7.077912E-002	5.93876	9.422922E-002
46	8.823810E-001	7.083280E-002	5.93586	9.373381E-002
47	9.023810E-001	7.088725E-002	5.93288	9.324831E-002
48	9.223810E-001	7.094090E-002	5.92984	9.277040E-002
49	9.423810E-001	7.099687E-002	5.92673	9.227971E-002
50	9.623810E-001	7.105356E-002	5.92355	9.178872E-002