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TREATISE ON  
INTEGRAL  
CALCULUS

EDWARDS

Vol. I

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A TREATISE  
ON THE  
INTEGRAL CALCULUS

**A TREATISE ON THE INTEGRAL CALCULUS**  
**VOLUME I.**

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A TREATISE ON THE INTEGRAL CALCULUS

VOLUME I

A TREATISE  
ON THE  
INTEGRAL CALCULUS

WITH APPLICATIONS, EXAMPLES  
AND PROBLEMS

BY

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## PREFACE.

AN apology is due to readers of my *Treatise on the Differential Calculus* for the many years of delay between its publication and that of the present companion volumes. This delay has been due to several causes. In the first place it was due to the very severe pressure of other duties. In the second place, when several chapters of what now constitutes the first volume had been written, changes occurred in the regulations for the Mathematical Tripos and in the requirements of many of the class of students I have come into contact with, and I was not sure that such requirements were not already amply provided for by other existent text-books. I have been urged, however, by many from time to time to continue the work I had begun years ago, and to put upon record the experience I had gained in the teaching of the large number of advanced students it has been my lot to meet. And I must also confess that in acceding to this expressed desire, I have turned to this work with a sense of pleasure and of relief from the distracting circumstances of the great war through which we have recently passed.

In the preparation of the book for the press, I have endeavoured to collect together for the use of the reader all information necessary to give him a good working knowledge of the subject, both practically and theoretically, and to place before him this information as clearly as possible, with abundance of illustrative examples and instances of the application of the principles explained. To do this as fully as I desired, it has unfortunately been found necessary to enlarge the book beyond the ordinary bounds of a text-book, and to divide it into two volumes. Several of the matters treated of in the Second Volume are the subject of exhaustive treatises expressly devoted to the discussion

of those particular branches. So that such chapters as are there to be found treating of Conformal Representation, Contour Integration, Elliptic Integrals, Mean Values and Chances, Harmonic Analysis, etc., can only be regarded as an attempt to put together in a convenient form for the reader the most important theorems and processes used in dealing with the earlier parts of these subjects, and merely as introductory and in no way exhaustive. The mode and sequence of treatment is the same as that I have adopted in my advanced classes of students during the last five-and-thirty years.

Such a book is necessarily to a considerable extent a compilation, and though some of the results and proofs are, so far as I know, new, by far the greater part are to be found elsewhere. I have endeavoured to assign to their proper authorship as many of the results as possible, but it is very difficult to do this in many cases with certainty. A teacher learns from his pupils and from those he examines as well as from reading and research, and one meets in this way with many proofs of the same theorem; it may be, in some cases, that they are due to the ingenuity of the student, but in general it will be that such proofs, if not to be found in existing text-books, are due to one or other of the distinguished body of teachers engaged at the Universities of the Kingdom in teaching the subject. In such cases it is often impossible, however much one may desire it, to assign the authorship correctly.

A large number of works has been consulted, and I must acknowledge a great indebtedness to many authors. In particular, I am indebted for much information to the admirable and exhaustive works of Legendre, Laplace, Lacroix, Jacobi, Serret, Bertrand, Todhunter, Williamson, Boole, Cayley, Hobson, Forsyth, Greenhill, Airy, Chauvenet and others, as well as to articles by Glaisher, Culverwell and many more in various mathematical journals. I am also indebted to the mathematical editor of the *Educational Times* for permission to make use of some of the many very excellent examples on Chances and Mean Values, etc., to be found in that collection.

The early articles of Volume I. have been so written that a student already equipped with a knowledge of Graphical work and Elementary Applications of the Summation-definition of

Integration may begin at the second chapter at once, where the definition used is that of the inverse of the operation of differentiation, if he prefers to do so.

A considerable number of the examples are extracted from University and College Examination Papers, and the source of such examples is indicated when known. Many others are new. These examination papers define better than anything else the scope and extent of the knowledge expected of students by the distinguished mathematicians engaged from time to time in framing the regulations for such examinations and in conducting them.

My very grateful thanks are also due to the publishers, Messrs. Macmillan & Co., to whose encouragement the appearance of the book is in no small measure due. They are also due to the printers, Messrs. Robert MacLehose & Co., and to their Staff at the Glasgow University Press, who have with constant courtesy and unflinching care and patience carried through their part of a piece of work which must at times have been far from easy.

JOSEPH EDWARDS.

QUEEN'S COLLEGE, LONDON,  
*March, 1921.*

IN preparing a Second Edition of Vol. I. for the press I have to thank several kindly critics and correspondents for lists of errata and suggestions. The errors have been corrected so far as they have come to my knowledge and suggestions incorporated so far as possible. The reader will find further comments on the scope of the work in the Preface to Vol. II.

JOSEPH EDWARDS.

QUEEN'S COLLEGE, LONDON,  
*October, 1929.*

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## ABBREVIATIONS USED IN THE REFERENCES.

- Ox. I. P. or Ox. II. P., etc. =First or Second Public Examination, Oxford University.
- Math. Trip. I. or Math. Trip. II. =Mathematical Tripos Examination, Cambridge University, Parts I. or II.
- Ox. J. M. S. =Oxford, Junior Mathematical Scholarship.
- Colleges  $\alpha$ , etc.

To indicate the sources from which many of the Examples are derived in cases where a group of Cambridge Colleges have held an examination in common, the references are abbreviated as follows :

- ( $\alpha$ ) =St. Peter's, Pembroke, Corpus Christi, Queen's and St. Catharine's.
- ( $\beta$ ) =Clare, Caius, Trinity Hall and King's.
- ( $\gamma$ ) =Jesus, Christ's, Magdalen, Emmanuel and Sidney Sussex.
- ( $\delta$ ) =Jesus, Christ's, Emmanuel and Sidney Sussex.
- ( $\epsilon$ ) =Clare, Caius and King's.

I. C. S. =Examination for the Indian Civil Service and Home Office Clerkships, Grade I.

(R. P.) =Set in problem paper to his classes by the late Dr. Routh, possibly taken from examination papers or possibly original. Source unknown to the present author.

L. =London University Examinations.

E. F. = *Elliptic Functions*.

C. I. = *Calcul Intégral*.

References to *Diff. Calc.* are to the author's larger *Treatise on the Differential Calculus*.

ABBREVIATIONS USED IN THE REFERENCES

- 1. *Journal of the Royal Society of Medicine*
- 2. *British Medical Journal*
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- 4. *New England Journal of Medicine*
- 5. *Annals of the Royal College of Physicians*
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