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TREATISE ON  
INTEGRAL  
CALCULUS

EDWARDS

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A TREATISE  
ON THE  
INTEGRAL CALCULUS

A TREATISE ON THE INTEGRAL CALCULUS

VOLUME II. PROBLEMS

BY  
JOSEPH EDWARDS, M.A.

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VOLUME II

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WITH APPLICATIONS, EXAMPLES  
AND PROBLEMS

BY

JOSEPH EDWARDS, M.A.

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## PREFACE.

THE remarkable progress made in recent years in the Theory of General Functions has revolutionised the method of treatment of many of the higher branches of Pure Mathematics; and the brilliant work of Riemann, Weierstrass, and their followers has opened out new paths for research. The discovery by Stokes and Seidel of the fundamental principles underlying the convergence of an infinite series has been far-reaching, and the question of uniformity or non-uniformity of approach to a limit which arises in dealing with such series and of continuity in the limiting values of functions dependent upon more than one variable when those variables are made to approach definitely assigned values, are matters which necessitate close attention. Professor Chrystal, in his *Algebra*, vol. ii., discusses such questions at considerable length in a most useful chapter on "The Convergence of Infinite Series and Products."

A general discussion of Abel's Theorem regarding the general integration of Algebraic Functions and of its development by Liouville and others is given by Bertrand (*Calc. Intég.*, ii., ch. v.), and an account of the general problem of integration of a function of a single variable, its possibilities and its barriers, is to be found in No. 2 of the *Cambridge Mathematical Tracts* (2nd ed.) by Mr. G. H. Hardy. A clear and careful exposition of the modern theory of Integration from Riemann's point of view, and of the question of Convergence of Infinite Integrals, is given in Professor Carslaw's work on the *Theory of Fourier's Series*.

It was my original intention to incorporate into this book some account of the more recent developments of the subject, and a long chapter was written for Volume I. with that view. But the further I progressed the stronger was my conviction, gained from many years of experience of work with post-graduate students, that there is in these days far too great a tendency on the part of teachers to push on their pupils so fast to the Higher Branches of Analysis or to Physical Mathematics that many have neither

time nor opportunity for the cultivation of real personal proficiency, or for the acquirement of that individual manipulative skill which is essential to any real confidence of the student in his own power to conduct unaided investigation, and without the possession of which any temporary interest he may have gained as a student must speedily die a natural death. I therefore felt that I should best serve the interests of the majority of readers by endeavouring to help them to cultivate and consolidate their knowledge, and to acquire an adequate mastery over the common processes of the Calculus rather than by pointing out the direction of the more modern trends of thought and by indicating further vistas for research. To do this, it has been necessary to exhibit a large number of worked-out illustrative examples, in addition to furnishing an adequate selection for personal practice. A great part of what I had prepared with regard to modern work was regretfully withdrawn, and other projected and partially completed portions either abandoned or drastically abridged, as they dealt with matters which would rather be of interest to specialists than helpful to the average reader.

The functions considered are for the most part combinations of the Elementary Functions of Ordinary Analysis, continuous and in general bounded, and for such the definition of integration as used by Cauchy and generally adopted in text-books will suffice, and form an adequate instrument for the treatment of the particular classes discussed. The more elaborate definition by Riemann, which furnishes a more powerful and delicate, but at the same time somewhat complex instrument for the discussion of generalised functions, introduces certain difficulties of conception likely to be an unnecessary source of trouble to the ordinary student in his earlier studies. It is therefore postponed until it is to be expected that he has arrived at a thorough mastery of the common processes to be used in the various applications of the Calculus, and has gained a riper experience for its consideration. And it does not appear that any danger is to be apprehended in such delay, seeing that Riemann's definition is specially devised to meet generalities which will only have to be dealt with in a later stage of specialisation.

JOSEPH EDWARDS.

QUEEN'S COLLEGE, LONDON,

*July, 1922.*

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