New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

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IBS PAN iBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

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Modeling the influence of repeated marketing communication on the decision making process

Agnieszka Jastrzebska and Wladyslaw Homenda
Faculty of Mathematics and Information Science,
Warsaw University of Technology,
ul. Koszykowa 75, 00-662 Warsaw, Poland
{A.Jastrzebska, homenda}@mini.pw.edu.pl

Abstract

In this article we have described developed approach to consumers decision making modeling. On the background of our model lie the theory of psychophysical field and the concept of consumers bounded rationality. Developed methodology overcomes limitations of neoclassical theory of consumers choice. In our approach consumers are represented by premises vectors. Cognitive processes are replaced with triangular norms and their generalizations. In this article we took a closer look at the law of diminishing marginal utility. We applied it in an unconventional way: we look at the information as we look at goods we consume. The source of such information can be, for example, marketing communication. These data is translated into premises influencing the decision. In this article we discuss iteratively repeated information processing properties. According to our model's theoretical assumptions, a consumer with bounded rationality tends to simplify cognitive processes and make decisions with satisfactory enough confidence. Therefore, the rule of diminishing marginal utility of information should obey. We have investigated several t-conorms and their properties. We have shown operators, which allow to model consumer behavior, including the law of diminishing marginal utility of information and the concept of bounded rationality applied to multiple criteria decision making.

Keywords: decision making, marketing communications, consumer, diminishing marginal utility.

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1 Introduction

Complexity of human behavior requires appropriate modeling framework. Decision making, as any other cognitive process, is conditioned by various stimuli and difficult to catch cognitive phenomena. The amount of information, which humans receive and pass on is huge. Moreover, it is imprecise in its nature. We tend to express our attitudes in a fuzzy and sometimes even contradictory way. Most importantly, decision making, similarly as all cognitive processes, is bounded by available knowledge and subject's processing skills. Therefore, modeling human behavior requires appropriate theoretical research on the background of decision making, especially on psychology. There is plenty of good, thoroughly researched mathematical tools, which might be applied to model decision making based on imperfect information.

In this paper we focus on the properties of decision aggregation process for the purpose of decision making. We research various operators and their suitability to model real-life processes. The contextual background of this article lies in iteratively repeated marketing communications, which is influencing the decision. We discuss how recurring appearance of motivational stimuli is reflected in the decisions. We suggest operators, which allow to capture this process. Most importantly, we introduce the concept of marginal utility of information.

This article is structured as follows. In section 2 discussed is theoretical research on the background of human decision making processes. Presented are both psychological and economic points of view. Explained is the concept of Lewin's field theory, the concept of bounded rationality and the law of diminishing marginal utility. Section 3 contains information on developed approach to decision making modeling. Section 3.2 discusses mathematical models applied in our model. Article contains also an extensive case study, where our model is applied.

2 Theoretical background of consumers decision making

Decision making requires a multidimensional analysis. On its background lie psychophysical motivational tensions that determine our actions. It is obvious, that human beings even unstimulated by external factors would still be making plenty decisions. This natural phenomena results from inbuilt conscious and subconscious will. Decision making is a process, which is a consequence of our striving to satisfy arising needs. Needs come into being, whether we want them (or like them) or not. Therefore, decision making processes, which are natural implication of our endeavors to needs satisfaction should be analyzed also at the most basic levels. Moreover, understanding processes, which result and influence decision making is crucial, also from the point of view of economics. Needs satisfaction can be translated directly into purchasing decisions. Doubtless, the knowledge about psychology and patterns of it is priceless.

When we analyze a decision making process, we usually distinguish input data, processes and the outcome. Plenty theoretical models, including Lewin's field theory, have been based on this decomposition, [6] [p. 240]. The first element, which is input information, consists from all arguments that somehow may consciously or unconsciously influence the decision. These are motivational stimuli, which can be for example attitudes, beliefs, and so forth. Input data can be also called a set of premises. Processes performed on these arguments result in the decision. Of course, one's cognitive abilities determine the way of how we conduct decision making. The differences in premises processing depend, for example, on one's psychographical portrait. We can divide our personality between the part of emotional selves and reasonable selves. The combination of these two entities describes our reasoning possibilities and characteristics. This, and many other topics are core research problems of the psychology of purchasing.

Before we introduce the case study and describe our model, we would like to discuss the concept of bounded rationality, introduced by H. A. Simon, for which he was rewarded with a Nobel Prize in Economics in 1978. Simon is formally justifying the necessity of individual and multidimensional approach to decision making, which was also our goal, while we were building our model. Simon's bounded rationality concept proves that humans do not make decisions coherently with mainstream economics theory of rational choice, [8] [p. 4]. H. A. Simon assumes that subjects have bounded cognitive abilities. Various conditioning, including environmental, mental and so forth determine and bound one's ability to gather and evaluate information. It implies that decision making process is a subjective phenomena and decision maker acts not rationally. An objective and rational decision does not exist. H. A. Simon states that humans tend to simplify cognitive processes and make decisions with limited amount of premises. Moreover, the decision criterion is imprecise. Humans make decisions that satisfy them enough. What it implies –modeling decision making has to include imperfect information processing. In our approach we benefit from Simon's theory of bounded rationality.

If humans take irrational actions, the whole purpose of marketing communications starts to be really luxurious. Simon's proof that humans cannot be objective implies that it is thoroughly justified that skilful marketers may enforce (or induce) certain purchasing actions. Therefore, extremely important is the analysis of intentional influence on the decision making. It is a skill and an art, which have been practised in various shades of marketing communications. Marketing communications is founded on a vast set of principles of how to plan and manage all aspects of widely understood relations between the company, its products and the market. It is a dialog between company and its environment, [2] [p. 4]. It is a wider term than marketing, as its aspiration is to manage all aspects of relations that concern the company, including relations inside it. Marketing communications most active phase –transmission may exert influence through product's properties, packaging (its design and quality), product's and company's image, brand position and so forth [5] [p. 825]. Nevertheless, the most important factor, also from the traditional understanding of marketing, is promotion, [5] [p. 142]. It is intentional communication through explicit advertising, but also through subconscious messages, directed and constructed with specialist's knowledge of human behavior. Messages constructed and transmitted through various channels undergo a process of decoding. It is one of the most crucial aspects of marketing communication, which has been widely researched, and which is still being of constant interest [1] [p. 94]. Consumer attacked with promotional messages translates them into premises, which serve as input data for the decision making. In this article we would like to present our approach to modeling a process of decision making based on data received through marketing communications. We aim at capturing properties of uniform information processing. The details of our approach and methodology of constructed case study are given in sections 3 and 4.

Another very important concept, known from mainstream economics is marginal utility. It is the change of increase of satisfaction associated with consumption. Buying an additional homogenous unit of one good influences the total satisfaction taken from this good or service. As it turns out, marginal utility of each (homogenous) unit decreases as the supply of units increases. The law of diminishing marginal utility is also called the first Gossen's law. It is in fact very intuitive property. Lets follow am example: we eat the first ice-cream. It gives us a lot of satisfaction. Second one as well, but satisfaction is slightly smaller, because we already have eaten the first ice-cream. And so forth. At some point, having another ice-cream would not give us any more satisfaction, it might even give us a stomachache. The law of diminishing utility is illustrated on figure 1.

Figure 1 on the vertical axis contains information about the marginal utility associated with the consumption of i-th unit of some good. We clearly see the diminishing tendency described before.

Reception of repetitive communication (for example: promotional messages) can be compared to consumption of information. Therefore, we propose to discuss the phenomena of diminishing utility of information. In following chapters we

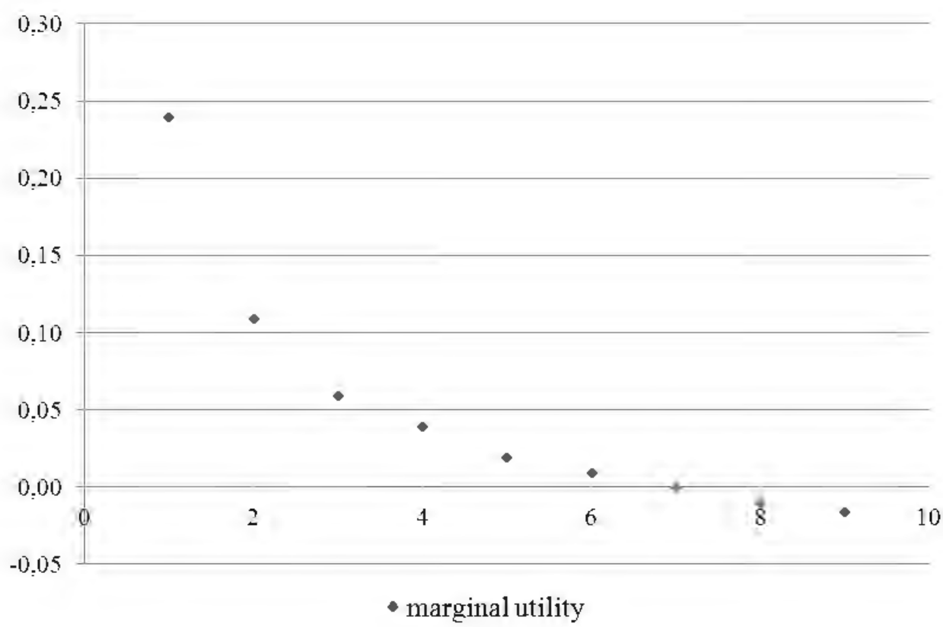

Figure 1: The law of diminishing utility

describe developed approach to decision making and introduce a case study.

3 Developed approach to decision making

In this section we would like to describe our approach to decision making. It is based on theoretical foundations described in section 2. While building our model we were inspired by Lewin's field theory and Simon's concept of bounded rationality.

We model a very basic level of decision making, which is a single decision. We analyze a nonempty set of premises, input data that influence the decision. In this article we want to analyze the most prime aspect of premises gathering, which is the characteristics of uniform information processing.

3.1 Consumers' decision making modeling

Consumers, for the purpose of our model, are described by premises vectors. These constitute all information received and decoded that have any impact on the decision. According to Simon's perspective of consumer's evaluation of reality, we do not apply crisp logic. Humans perceive positive information as an attitude, which range from very strong through mildly strong and weak strong to neutral. Analogically, negative information is also not perceived as a yes–no kind. It is a noncrisp evaluation of a particular information. Therefore, we apply fuzzy sets to our model. As a result, consumers premises vectors, constitute of a countable set of attitudes evaluated as numbers from the $[-1, 1]$ interval. The stronger is the absolute value of a premise, the more impact (negative or positive) it has on the decision. A premise with importance equals to 0 means that the argument is not influencing the decision. Premises, as was mentioned before, are all motivational stimuli, which elicit, control and sustain certain behaviors. In theory (for example in the mainstream economics) decision making can be based on an infinite amount of information. In our model we assume that the amount of analyzed premises is countable. It derives inter alia from the fact that humans tend to simplify cognitive processes. This fact was also indirectly described by H. A. Simon.

Premises vectors are aggregated to a decision. In the case study of this article, the aggregation operators, which we chose to investigate, are various t–conorms. Applied t–conorms are described to greater detail in section 3.2.

In this paper we focus on properties of uniform information processing that is a result of marketing communication. Consumer is bombarded with the same information. He decodes and processes it. Promotional message is translated into a premise. Each time consumer considers the same premise (the same promotional message) his evaluation of this premise remains unchanged. Hence, consumer's vectors are filled with homogeneous numbers. The goal is to observe the saturation of the decision, when we aggregate multiple premises with the same strengths using different operators. This corresponds to modeling the influence of repetitive marketing communications. The new and unconventional idea, which we try to investigate and research in this paper is that we look at the information as we look at goods we consume. In particular, we want to test how applied t–conorms reflect the law of diminishing marginal utility.

While we have been researching presented ideas, important questions have risen: What impact has repetitive marketing communications on the decision? Does the strength of the premise have any influence on the characteristics of processing? Which operators allow to model desired phenomena most accurately? We are giving answers to this questions in the case study in section 4.

3.2 Mathematical models for consumer decision making modeling

We recall basic notions of fuzzy sets and generalization of fuzzy connectives maximum and minimum to triangular norms. We will be expressing fuzzy sets in the form of membership functions. Namely, a fuzzy set A defined in the universe X is a mapping $\mu : X \to [0,1]$ or $\mu_{A,X} : X \to [0,1]$ if the names of the set and the universe should be explicitly stated.

The Zadeh's model of fuzzy sets can be described as a system similar to set theory $(F(X), \cup, \cap, -)$, where $F(X)$ denotes fuzzy sets over the universe X and $\cup$, $\cap$, $-$ denote union, intersection and complement. This system is clearly inter–preted as $([0,1]^X, \max, \min, 1-)$, where $[0,1]^X$ denotes all mappings from the universe X into the unit interval $[0,1]$, i.e. the space of membership functions, and $\max$, $\min$ and $1-$ applied to membership functions implement union, inter–section and complement. We do not pay attention to the interpretation of fuzzy sets in terms of a lattice L^X.

Study of fuzzy sets system was enriched with triangular norms used in place of the $\max$ and $\min$ operators. Note: $\max$ and $\min$ are triangular norms as well.

Triangular norms, i.e. t–norms and t–conorms, are mappings from the unit square $[0,1] \times [0,1]$ onto the unit interval $[0,1]$ satisfying axioms of associativity, commutativity, monotonicity and boundary conditions (cf. [4, 7] for details), i.e.:

Definition 1 *Triangular norms are mappings $p : [0,1] \times [0,1] \to [0,1]$, where p stands for both t-norm and t-conorm, satisfying the following axioms:*

1.	$p(a, p(b,c)) = p(p(a,b), c)$	*associativity*
2.	$p(a,b) = p(b,a)$	*commutativity*
3.	$p(a,b) \le p(c,d)$ *if* $a \le c$ *and* $b \le d$	*monotonicity*
4.	$t(1,a) = a$ *for* $a \in [0,1]$	*boundary condition for t-norm*
	$s(0,a) = a$ *for* $a \in [0,1]$	*boundary condition for t-conorm*

In this paper we use only t–conorms as agregators of premisses and launching evaluation of a decision to be made.

The selected known t–conorms, which are used in this paper, are given in Table 1. t–conorms are bounded by maximum t–conorm, i.e. for any t–conorm s and any $x, y \in [0,1]$:

$$\max(x,y) \le s(x,y) \tag{1}$$

We focus our discussion on a special type of t–conorms called strict (contin–uous and strictly monotone) generated by *additive generators*. Let $f : [0,1] \to [0,+\infty]$ is a strictly increasing function such that $f(0) = 0$ and $f(1) = +\infty$, then the function $s : [0,1] \times [0,1] \to [0,1]$, $s(a,b) = f^{-1}(f(a) + f(b))$ is a t–conorm. The function f such that $f(x) = tangent(\pi/2 * x)$ and $f(1) = +\infty$ for $x \in [0,1)$ is an example of additive generator of a strong t–conorms. Its in–verse counterpart is defined by the formula $f(x) = 2/\pi * arctan(x)$ In case of

Table 1: Selected triangular conorms

t-conorm	
maximum	$\max(x, y)$
probabilistic sum	$x + y - x \cdot y$
nilpotent maximum	$\begin{cases} \max(x, y) & \text{if } x + y < 1 \\ 1 & \text{otherwise} \end{cases}$
strong/tan	$s(x, y) = f^{-1}(f(x) + f(y)), f(x) = arcsin(x)$
strong/arcsin	$s(x, y) = f^{-1}(f(x) + f(y)), f(x) = tan(\pi/2 * x)$
strong/arctanh	$s(x, y) = f^{-1}(f(x) + f(y)), f(x) = arctanh(x)$

generator's codomain being a finite interval $f : [0, 1] \to [0, r]$, the pseudo inverse is define as follows: $f^{-1}(\min(r, f(x) + f(y))$. An example of such the generator is the mapping $\arcsin$.

An extension of fuzzy sets called balanced fuzzy sets provides processing of negative data. We recall that triangular norms of classical fuzzy sets are generalized to balanced triangular norms, c.f. [3] for details. In our study we use balanced t-conorms. Focusing our discussion on strict (continuous and strictly monotone) balanced t-conorms generated by additive generators. Let $f : [-1, 1] \to [-\infty, +\infty]$ is a strictly increasing function such that $f(-1) = -\infty$, $f(0) = 0$, $f(1) = +\infty$ and $f(a) + f(b)$ is in the range of f or is equal to $-\infty$ or to $+\infty$, then the function $S : [-1, 1] \times [-1, 1] \to [-1, 1]$, $S(a, b) = f^{-1}(f(a) + f(b))$ is a balanced t-conorm. The function f such that $f(x) = tangent(\pi/2 * x)$ for $x \in (-1, 1)$, $f(-1) = -\infty$ and $f(1) = +\infty$ is an example of additive generator of a strong t-conorms. Its inverse counterpart is defined by the formula $f(x) = 2/\pi * arctan(x)$ in the interval $(-1, 1)$. In case of generator's codomain being a finite interval $f : [-1, 1] \to [-r, r]$, the pseudo inverse is define as follows: $f^{-1}(\min(r, f(x) + f(y))$ for $f(x) + f(y) \geq 0$ and $f^{-1}(\max(-r, f(x) + f(y))$ for $f(x) + f(y) \leq 0$. An example of such the generator is the mapping $\arcsin$.

4 Case study

In the case study we would like to discuss the decision making process based on uniform premises. Analyzed examples represent a situation, when a consumer makes a decision based on a single message, but repeated multiple times. The number of times, when the consumer records this message corresponds to the amount of premises (forces influencing the decision) in premises vector. We as–sume that his attitude towards received information does not change. Chosen operators should process gathered data in appropriate way. Decision is calculated using different t–conorms for six case study examples. Case studies A, B and C are associated with the positive–only premises scenarios. There we apply 5 differ–ent t–conorms. Case studies D, E and F concern consumers who recognized input messages as information negatively influencing decision. There we apply different 3 t–conorms. We investigate how many arguments should be present in premises vectors, in order to make decisions with a confidence satisfactory enough. We assume, that consumer is content with his decision, when he is sure to at least 0.9 degree. Of course this threshold can be lowered.

We research following 6 case study examples: consumers named A –F. We fill their premises vectors with desired numbers. The choice was performed ar–bitrarily, so that we can observe, if the law of diminishing utility of information is influenced by the strength of recognized input arguments. Premises vectors of consumers A–F are placed below.

Case of consumer A –aggregating multiple premises with strong positive evalu–ations, (vector P_A).

$$P_A = [\,0.80,\ 0.80,\ 0.80, \ldots,\ 0.80,\ 0.80,\ 0.80,\ 0.80\,]$$

Case of consumer B –aggregating multiple premises with medium positive eval–uations, (vector P_B).

$$P_B = [\,0.40,\ 0.40,\ 0.40, \ldots,\ 0.40,\ 0.40,\ 0.40,\ 0.40\,]$$

Case of consumer C –aggregating multiple premises with weak positive evalua–tions, (vector P_C).

$$P_C = [\,0.20,\ 0.20,\ 0.20, \ldots,\ 0.20,\ 0.20,\ 0.20,\ 0.20\,]$$

Case of consumer D –aggregating multiple premises with strong negative evalu–ations, (vector P_D).

$$P_D = [\,-0.80,\ -0.80,\ -0.80, \ldots,\ -0.80,\ -0.80,\ -0.80,\ -0.80\,]$$

Case of consumer E –aggregating multiple premises with medium negative eval–uations, (vector P_E).

$$P_E = [\,-0.40,\ -0.40,\ -0.40, \ldots,\ -0.40,\ -0.40,\ -0.40,\ -0.40\,]$$

Case of consumer F –aggregating multiple premises with weak negative evalua–tions, (vector P_F).

$$P_F = [\,-0.20,\ -0.20,\ -0.20, \ldots,\ -0.20,\ -0.20,\ -0.20,\ -0.20\,]$$

To the combinations of premises present above we apply t–conorms described in section 3.2. For the cases of consumers A–C, who have recognized positive premises only, we apply following operators:

- maximum,
- probabilistic sum,
- nilpotent t–conorm,
- t–conorm generated with arcsine generating function,
- t–conorm generated with tangent $(\pi/2)$ generating function.

In case study examples D–F we discuss t–conorms generated with both arcsine and tangent $(\pi/2)$ plus one additional t–conorm: generated with arc tangent hyperbolic. We do not consider t–conorm generated with arc tangent hyperbolic for cases A–C, because results are coherent with results obtained with other t–conorms. Adding one more t–conorm for cases A–C would make plots less readable and we would not gain much more insight in explaining consumer behavior.

On the plots (in Figures: 2 –7) placed in this section we are presenting paired plots:

- top plot: the decision obtained after i–th iteration of promotional message,
- bottom plot: marginal utility of passed promotional message for given consumer.

Decision for consumer A (placed in Figure 2), whose all premises were strong positive, saturate very fast to a strong positive, even certain positive one. For t–conorm generated with *tangent* function the decision will never reach 1, but it rapidly saturates to a value close to 1. For *arcsine* it reaches 1. In both cases we need less than 4 iterations (in the most pessimistic case of *tangent*) to reach the value of the decision greater than 0.9. For probabilistic sum the decision reaches 1 after two repetitions of promotional message. The initial steepness of the bottom plot for consumer A indicates that marginal utility of information decreases quickly for probabilistic sum and t–conorms generated with functions *arcsine* and *tangent*. It almost immediately reaches 0 (or value very very close to 0) and does not change, no matter how many times we repeat the promotional message.

Two operators behave differently. These are: nilpotent t–conorm and maximum. Decisions obtained for consumer A with these two functions do not change at all, independently of how many times we repeat promotional message. In these two cases the law of diminishing utility is not functioning. Therefore these two operators prove to be not suitable to model desired properties for the case of a consumer with strong positive premises.

In Figure 3 placed are plots describing decisions and marginal utility of information consumed by B.

Decisions obtained for B, whose all premises were medium positive also saturate to 1. The tempo of this process is slower, than in the case of consumer A. Good properties, compliant with the theory of bounded rationality and the law of diminishing marginal utility, are associated with t–conorms generated by *arcsine* and *tangent*. In these two cases the marginal utility always declines. The decision rapidly saturates to 1 or values very close to it. Similarly as before, *tangent* saturates slower than *arcsine*. Decisions obtained with these two t–conorms are satisfactory enough for consumer B to make the decision quickly. It is accordant with H. A. Simon's understanding of human cognitive processes. Also satisfactory are results obtained with probabilistic sum. In this case, the decision after 8 iterations reaches 1 and does not change.

Substantially different is the behavior of maximum and nilpotent maximum t–conorms. In the case of maximum the decision never changes from the initial 0.4, which is the value of all premises in P_B. Therefore, the law of diminishing marginal utility is not incorporated in this operator. For nilpotent maximum t–conorm, the decision after 2 repetitions of promotional message reaches the value of 0.4 and then never changes, no matter how many times we will repeat that message. It corresponds to a situation, when we can convince somebody to some extent (to some threshold level) that he needs a particular commodity or a service. Consumers cognitive abilities block further reception of promotional messages and we cannot influence him any more.

Figure 4 contains graphical description of decision making process based on uniform premises for consumer C. His all premises are weak positive.

As we have expected, the saturation of decision happens significantly slower than in the case of consumer A and B. When we aggregate the decision with t–conorm generated with *tangent* the saturation is the slowest. After 18 iterations we receive the decision equal to 0.89. This characteristics of information aggregation is highly desired. Consumer C is weakly positive about some feature, about which he is informed through an advertising channel. Constant and repeated campaigns slowly convince him more and more about the product. In order to make his decision relatively strong positive, it is required to repeat the advertisement multiple times. Similar behavior, but with faster saturation applies to t–conorm generated with *arcsine* and to probabilistic sum. According to Simon's theory of bounded rationality, it is unlikely that C will buy advertised product. Humans tend to simplify cognitive processes and make decisions on as limited amount of premises

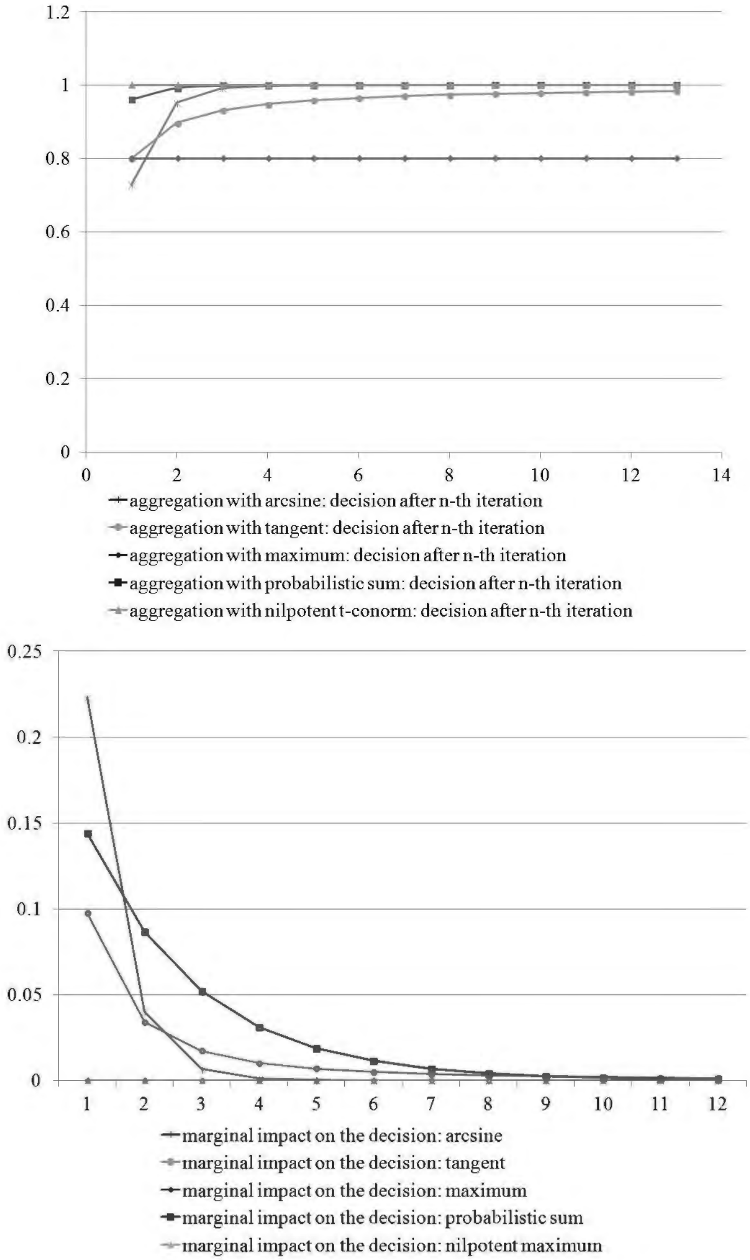

Figure 2: Case of P_A −graphical description

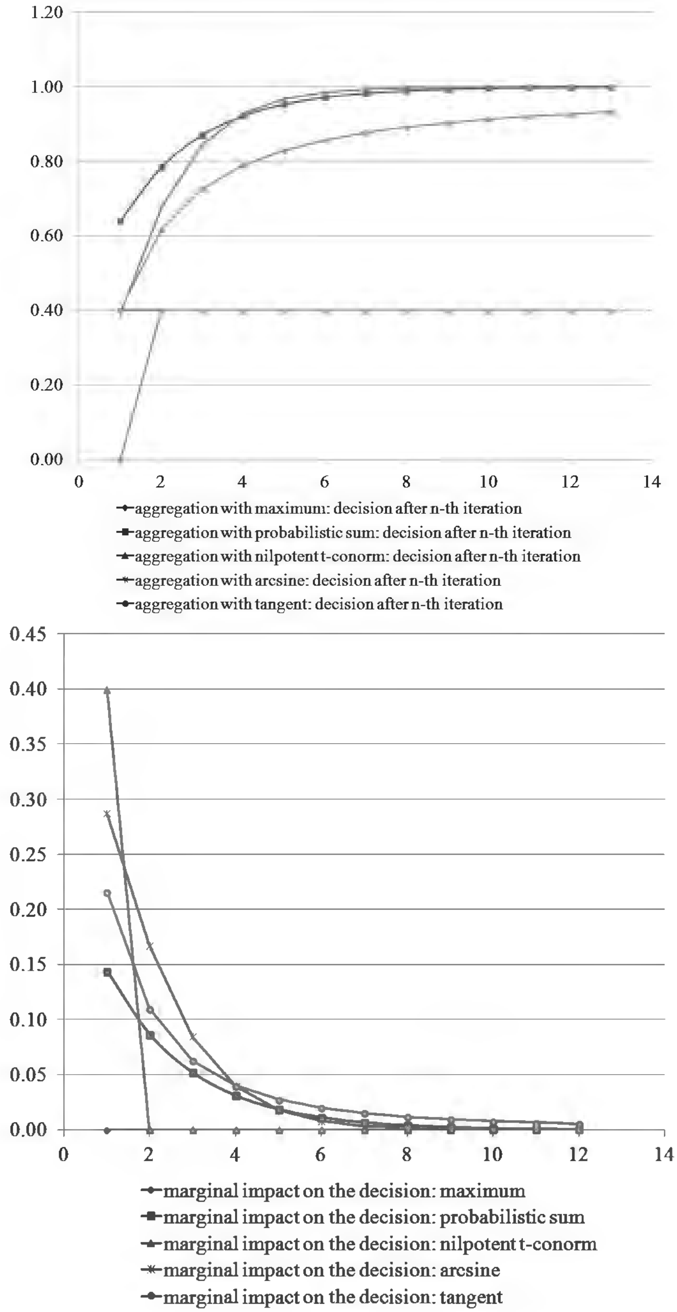

Figure 3: Case of P_B −graphical description

as possible. Moreover, the criterion for the decision has to be strong enough. It means, that we do not require to be certain about something for example to buy it. If we join these two observations and apply it to consumer C, we have to admit that the decision, whether he will buy the product depends on the threshold that is required for him to make up his mind.

Maximum and nilpotent maximum behaviors are similar as they were in the case of consumer A and B. Maximum is not suitable to model consumer behavior, as for aggregation of uniform data it computes always a constant number – the value of aggregated premise. In the case of consumer C it is 0.2. Nilpotent maximum, on the other hand, reflects the law of diminishing utility. The saturation (in the case analyzed in the P_C vector) happens only to the value of premises in the vector.

We see that for uniform mild and weak evaluations consumer have to make decision with greater amount of premises. Marketing communications involves careful campaign planning, including its finances. Therefore, modeling these aspects is highly justified. We see, that for some consumers, which can be described by nilpotent maximum t-conorm, putting greater financial efforts and repeating advertisements does not bring desired effects. Some consumers, after reaching a threshold may not be susceptible to more stimuli.

Next, we discuss decisions obtained for consumers D-F, whose premises vectors contain negative information. In these cases we apply different operators, who are fitted also to the negative domain. These are:

- t-conorm generated with $arcsine(x)$ generating function,
- t-conorm generated with $tan(\pi/2 * x)$ generating function,
- t-conorm generated with arctangent hyperbolic $(arctanh(x))$ generating function.

In Figure 5 we place plots of the decision and marginal utility of information obtained for D after i-th iteration of promotional message.

Consumer D has evaluated premises influence on the decision about the purchase of a particular service or commodity as a strong negative one (equal to -0.8). Decisions for consumer D quickly saturate to values very close to -1. In this case, what came as no surprise, consumer is very quickly able to make a decision. He is certain enough that he is not buying this product or service. In the cases of all three operators, which we have applied, consumer D after only 2 iterations achieves a decision with its absolute value greater than 0.9. Applied operators allow almost immediately to make a decision with a confidence level that satisfies the consumer.

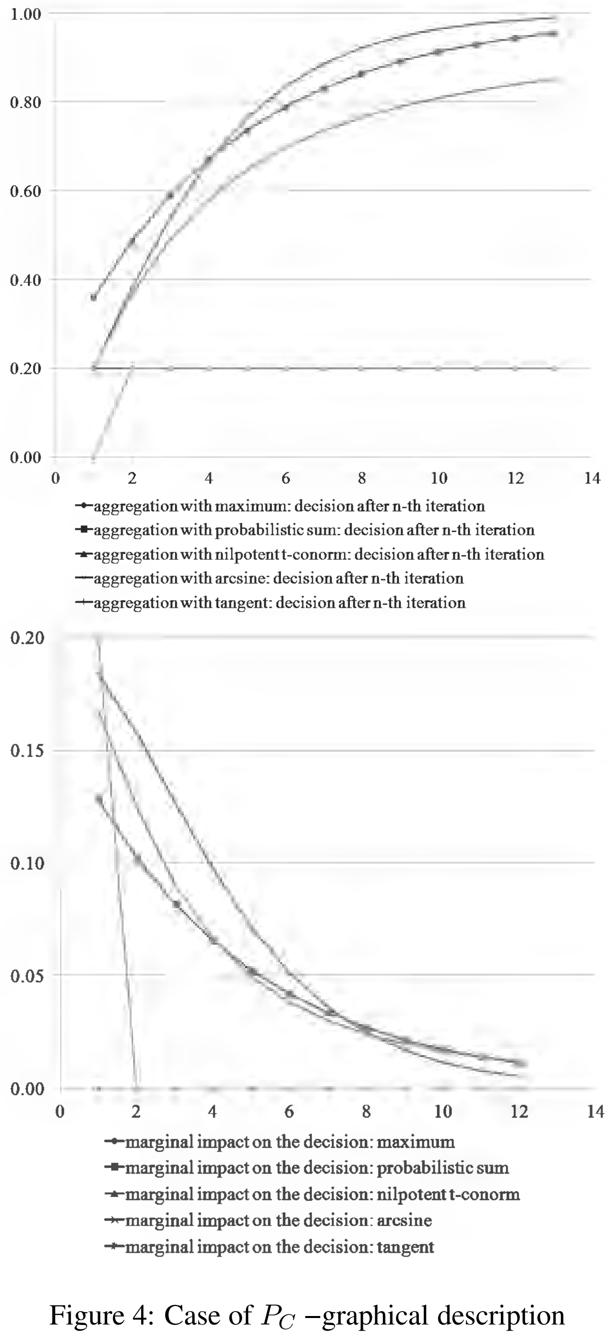

Figure 4: Case of P_C −graphical description

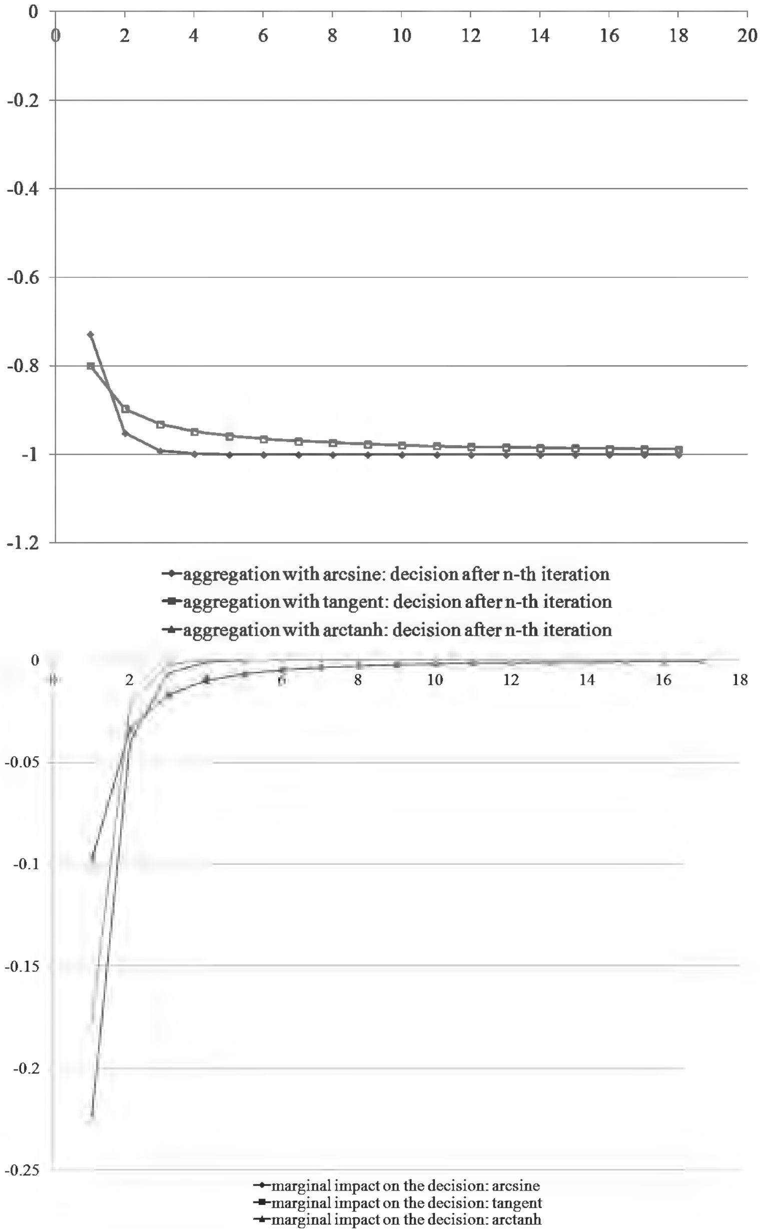

Figure 5: Case of P_D −graphical description

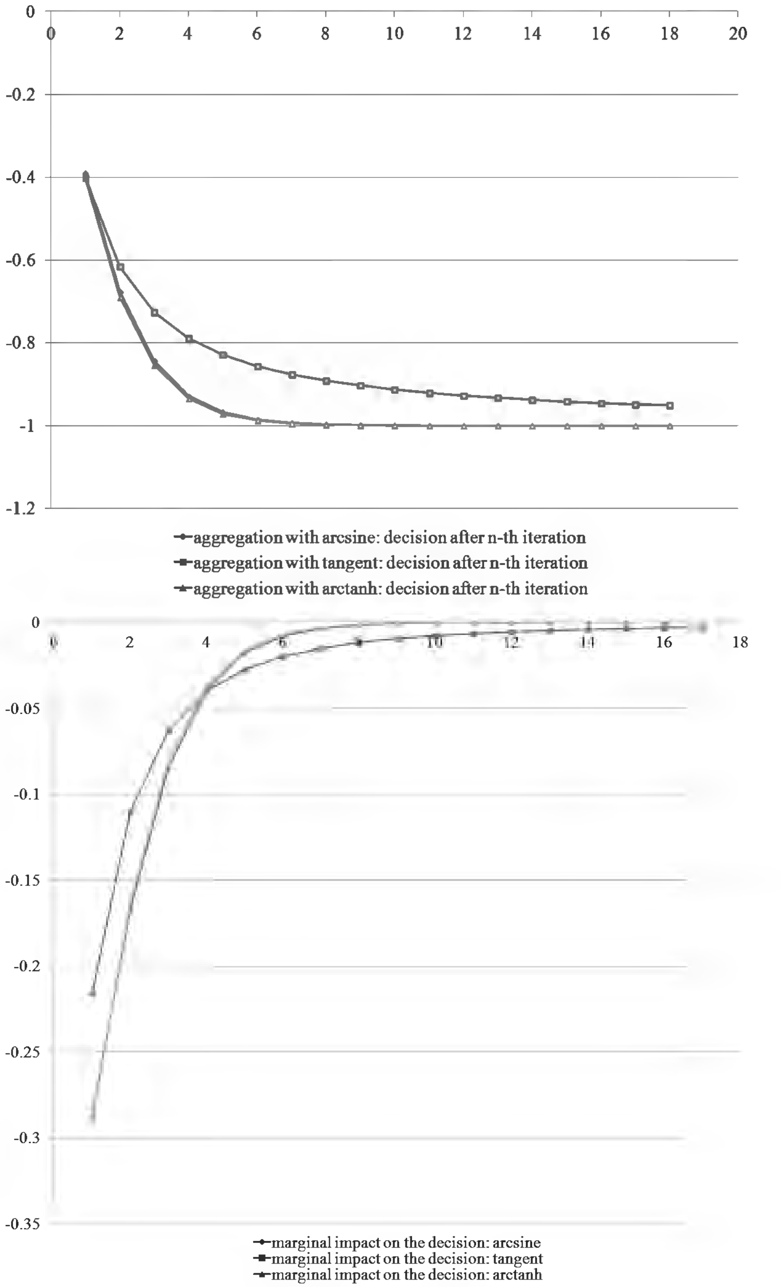

Figure 6: Case of P_E −graphical description

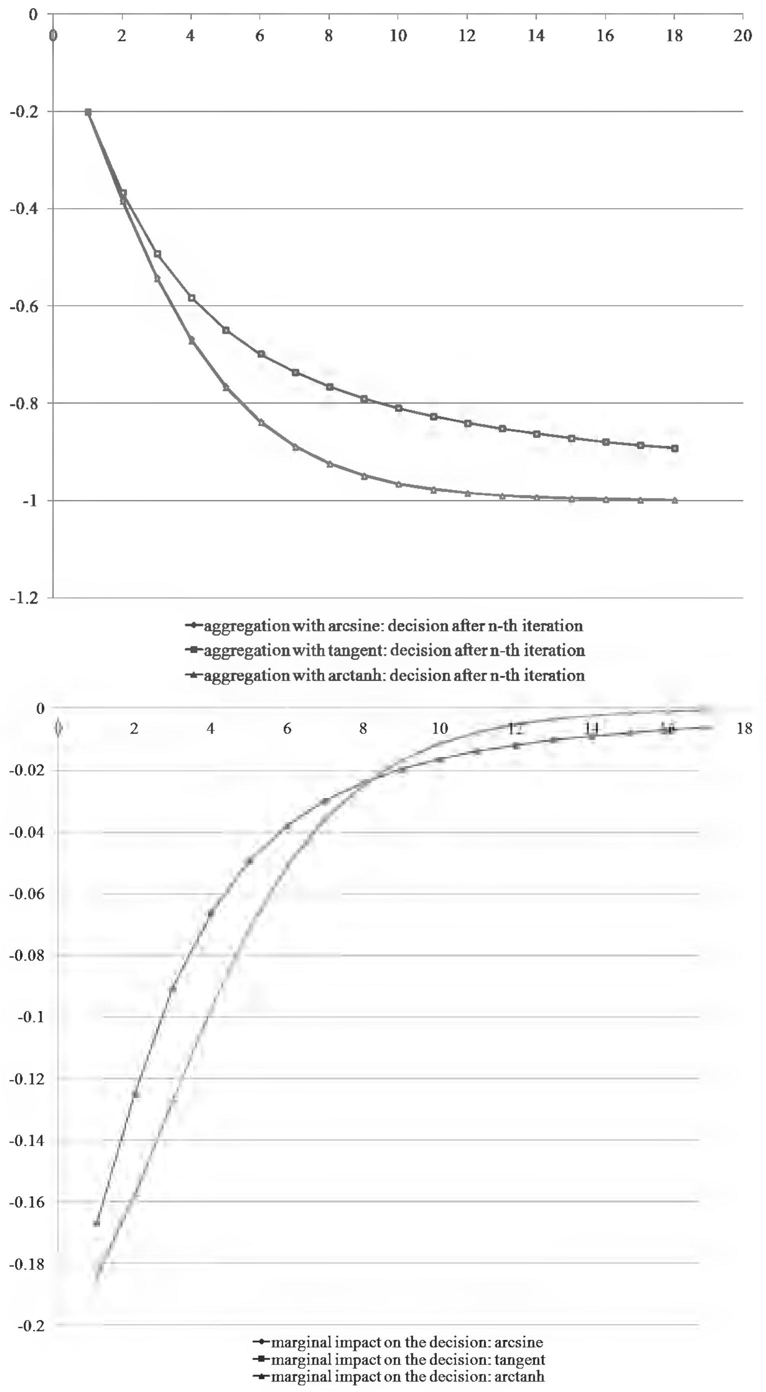

Figure 7: Case of P_F −graphical description

On the bottom plot in Figure 5 we observe that absolute changes of marginal utility of information for the case of consumer D are diminishing. Analogically as in the cases of A–C, the law described in section 2 obliges.

Figure 6 contains graphical description of the case of consumer E.

On the top plot of Figure 6 we clearly see that all negative decisions after less than 10 iterations are stronger than -0.9 (decision's absolute value is greater than 0.9). *Tangent* t–conorm generating function's slope steepness is most mild. Therefore, the decision is stronger than -0.9 (decision's absolute value is greater than 0.9) after 9 iterations. Generating functions with different properties (*arcsine* and *arc tangent hyperbolic*) achieve the same result after only 4 repetitions of premise aggregation. Choosing the right function to model desired behaviors requires the knowledge about subject's psychographical description (susceptibility to external stimuli). More steep generating functions correspond to the cases, when consumer behavior changes more rapidly. Eagerness and even volatility of preferences can be easily exploited by marketers.

As in the previous cases, marginal utility (deltas of utility) are decreasing as the consumption of information progresses. For the first time, when a consumer hears advertisement it negatively influences the decision to the greatest extent. Later we may say that he becomes more and more immune to the promotional message, with which he is being attacked. Even though received stimuli is evaluated with the same strength, its influence on the decision declines with each iteration of aggregation. Deltas of the marginal utility tend to 0. For *tan*$(\pi/2 * x)$ and *arctanh*(x) they will never reach 0. It is because these two generating functions are asymptotic to -1 (and 1).

Figure 7 represents the last case study example – consumer F. His premises are weak negative (all are equal to -0.2).

Analogically to previously discussed cases, weak negative arguments after appropriate amount of iterations saturate and achieve values close to -1. Comparing consumer F with D, the tempo of this process is significantly slower. t–conorms generated with *arcsine* and *arc tangent hyperbolic* behave almost identically. After 8 iterations of aggregations procedure the decision reaches -0.9. It corresponds to a strong enough confidence level assuring us that F will not buy advertised product. If we apply *tangent* t–conorm generating function, we will have to repeat the procedure of aggregation 11 times. We see that someone, whose attitude towards analyzed premise is weakly negative, would not be that rapid to make strong negative decision. Nevertheless, the behavior is nondecreasing, so the decision based on analyzed vector P_F will never become positive. In order to apply the rule of bounded rationality, we would have to take a closer look at the threshold required by F to make a confident decision. Moreover, we would have to take into account the fact, that consumers do not want to analyze too many arguments to make a decision.

By analogy to previous examples, the weakness of premises is not interfering with the law of diminishing marginal utility. We observe that the change of impact of each next premise declines constantly.

5 Conclusions

There is a variety of well known triangular norms, which are suitable to model decision making. In the article we have investigated the application of triangular norms and their generalizations to modeling the influence of uniform premises on the decision. Such situation may happen, for example, when a consumer is influenced by marketing communications. Chosen operators allow to describe various psychographical types of consumers. We have shown, that initial attitude determines the tempo of decision saturation. It means, that the stronger is the content of premises vector, the less arguments we need to make a decision with a confidence level that satisfies the consumer. We can apply suggested modeling approach to marketing communications research. We can investigate the relation between the number of promotional messages targeted to consumers and their efficiency. What it implies, we can use this approach to determine how costly it is to convince people with different susceptibility to external influence to buy certain product.

Most important concept introduced in this article is the diminishing marginal utility of information. This is a property, conventionally analyzed in the context of commodities consumption. Here, we apply it to marketing communications messages. The new and unconventional concept, which we have investigated in this paper is that we look at the information as we look at goods we consume. We treat information (for example advertisement) sent to consumer as consumed service. Humans act according to the rule of bounded rationality. This implies, that our cognitive processing of input arguments undergoes this law. The marginal utility of information decreases, while we listen to next iterations of promotional message. It is a property hidden in cognitive processes, which can be reflected with applied operators. In the article analyzed were 6 case study examples, where we have illustrated concepts briefly concluded here.

We have analyzed several triangular norms and balanced norms. As it turns out, maximum is not suitable to model consumers behavior. It does not allow to include the phenomena of diminishing marginal utility of information. Very interesting behavior is observed for nilpotent maximum t–conorm. It reflects the law of diminishing utility. Unfortunately, this operator is also not well suited to model decision making. In the case of uniform premises aggregation, nilpotent maximum does not allow the decision to saturate to values higher than the value of premises. Without the property of decision strengthening with each next itera– tion of aggregation, the output is not saturating to values high enough to allow to make a decision. It reflects the phenomena of information blockage. Aggregating with nilpotent maximum describes an effect of immunity to marketing communi– cations, which is very rarely observed in real life. Probabilistic sum and t–conorms generated with various generating functions (in this paper we analyzed *tangent*, *arc sinus* and *arc tangent hyperbolic*) are satisfactory. These operators both in– clude the law of diminishing marginal utility and allow us to model behavior of a consumer with bounded rationality.

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References

[1] Baack D., Clow K. E., *Integrated Advertising, Promotion, and Marketing Communications. 2nd ed.*, Pearson Education Inc., Upper Saddle River, 2004.

[2] Broderick A., Pickton D., *Integrated Marketing Communications 2nd ed.*, Pearson Education Inc., Upper Saddle River, 2005.

[3] Homenda W., Balanced Fuzzy Sets, Information Sciences 176 (2006) 2467–2506.

[4] Klement E. P.,Mesiar R.,Pap E., Triangular norms, Kluwer Academic Pub–lishers, Dordrecht, 2000

[5] Kotler Ph., Armstrong G., Saunders J., Wong V., *Marketing. Podrcznik Europejski*, PWE, 2002.

[6] Lewin K., *Field theory in social science; selected theoretical papers.* D. Cartwright (Ed.)., USA, New York: Harper & Row, 1951.

[7] Schweizer B., Sklar A., Probabilistic Metric Spaces, North Holland, NY, 1983

[8] Simon H. A., Egidi M., Viale R., Marris R. L., *Economics, Bounded Rationality and the Cognitive Revolution*, Edward Elgar Publishing, 2008.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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