POLISH ACADEMY OF SCIENCES
SYSTEMS RESEARCH INSTITUTE

THE INTERNATIONAL ECONOMIC COOPERATION

THEORETICAL FOUNDATIONS

STANISŁAW PIASECKI

POLISH ACADEMY OF SCIENCES
SYSTEMS RESEARCH INSTITUTE

THE INTERNATIONAL ECONOMIC COOPERATION

THEORETICAL FOUNDATIONS

STANISŁAW PIASECKI

Warszawa 1992

PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

* prices on the internal market do not correspond to social costs,

* there is lack of conviction as to correctness of exchange rates,

* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Planimg and Statistics, Warsaw, 1968).

If we distinquish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Łódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (Quarterly Journal of Economics, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

products to the advantage of the production intensity of the other, in such a way as to purchase as much of the non-produced semi-product for the money accruing from the sales of the surplus of the produced semi-product, as to ultimately increase the production of the final good. Since with increase of production of the final product production value as well as profit increase, too, then it is understandable that the growth of final production magnitude of an enterprise is almost always advantageous for an enterprise.

5. MODEL OF COOPERATION OF TWO ECONOMIC ORGANIZATIONS (WITH LIMITED EXCHANGE)

The model considered in the previous chapter assumed that exchange of goods is not anyhow limited (both at the sales end of the good we have in surplus and the purchase end of the good which we are in need of) and that this exchange takes place at the constant price ratio ω_2.

Such a situation does not exist when economic cooperation takes place between two economic organizations having limited capacities of absorbing exports. Exchange between them is then limited by the finite capacities of importing the surpluses of products turned out within the other organization, this limitation being a mutual one. Besides that, in bilateral agreements it is not necessary that existing international or national prices of products in question be valid. It may happen that, for instance, "contractual prices" are introduced, for purposes of keeping financial balances, these prices, in some cases, depending also upon the magnitude of exchange.

Consequently, we can approach the problem of determination of optimal cooperation strategies in a somewhat different manner.

Let us look at Figure 6 (and at Figure 7). In the first "quarter" of the coordinate system (α_1, α_2) the characteristics of the enterprise (or of national economy) number I is shown as defined by the parameters A_1^I, A_2^I, b_{21}^I, b_{12}^I.

The superscripts denote the enterprise (number I or II) to which a given quantity refers.

In the third "quarter" of the coordinate system the figure shows, in the same way as for the enterprise (economy) I - the characteristics of the enterprise (economy) II. It is defined via the values of parameters A_1^{II}, A_2^{II}, b_{21}^{II}, b_{12}^{II}.

The figure presents the "points of functioning" of both enterprises defined by the coordinate values (α_1^I,α_2^I) in the first "quarter", and $(\alpha_1^{II},\alpha_2^{II})$ in the third "quarter". These values indicate production levels of goods "1" and "2" in enterprises I and II.

In Figure 6 the segment $\overline{0,\alpha_1^I}$ is divided into segments $\overline{0,\mu_1^I}$, $\overline{\mu_1^I,\mu_1^I+\beta_1^I}$ (with $\mu_1^I+\beta_1^I = \alpha_1^I-\alpha_2^I b_{21}^I$), and $\overline{\mu_1^I+\beta_1^I,\alpha_1^I}$.

In a similar manner the segments $\overline{0,\alpha_2^I}$, $\overline{0,\alpha_1^{II}}$ and $\overline{0,\alpha_2^{II}}$ are also divided. Note also that the segment $\overline{0,\alpha_2^I}$ is composed of subsegments $\overline{0,\alpha_2^I-\alpha_1^I b_{12}^I}$ and $\overline{\alpha_2^I-\alpha_1^I b_{12}^I,\alpha_2^I}$, while the segment of the length β_1^I is composed of the subsegments $\overline{-\mu_2^I,0}$ and $\overline{0,\alpha_2^I-\alpha_1^I b_{12}^I}$, so that $\beta_1^I = \mu_1^I + (\alpha_2^I-\alpha_1^I b_{12}^I)$.

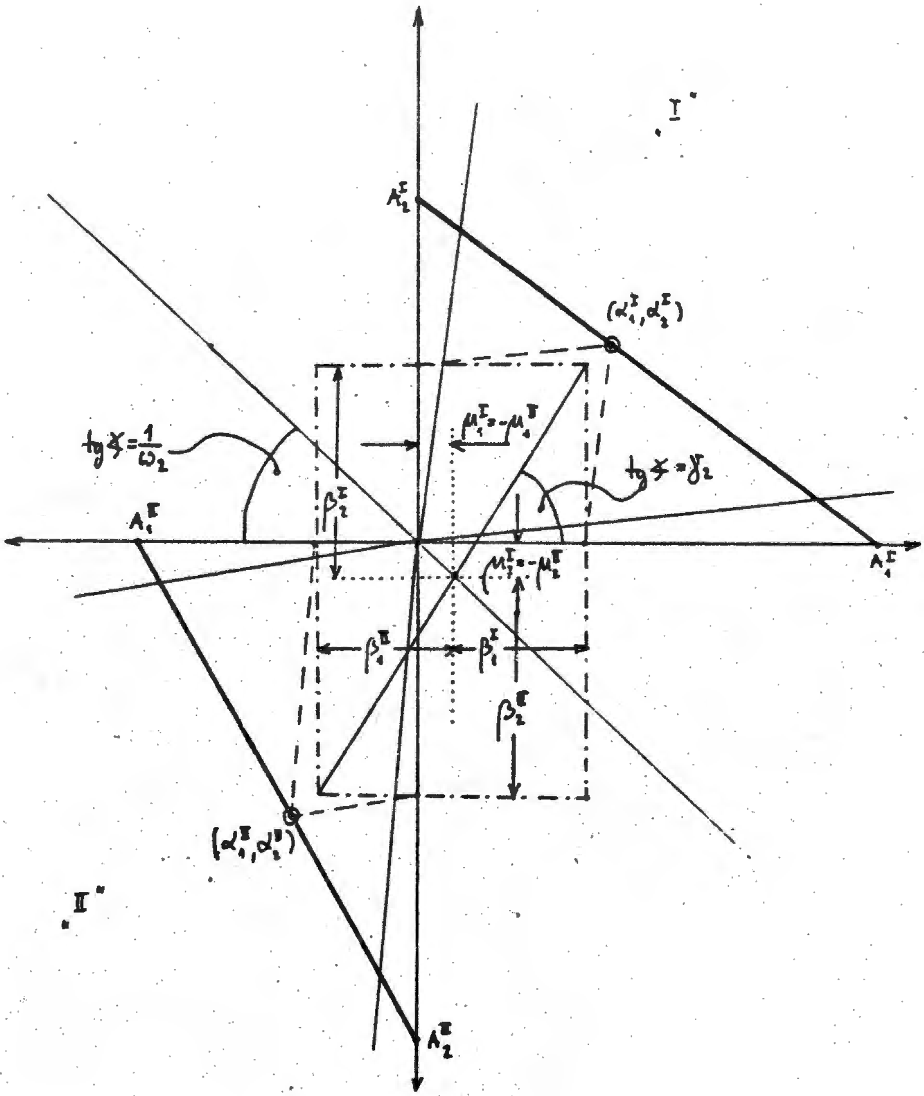
„I"
„II"
A_2^I
A_1^I
A_1^{II}
A_2^{II}
(α_1^I, α_2^I)
$(\alpha_1^{II}, \alpha_2^{II})$
$tg \sphericalangle = \frac{1}{\omega_2}$
$tg \sphericalangle = \gamma_2$
$\mu_1^I = -\mu_1^{II}$
$\mu_2^I = -\mu_2^{II}$
β_2^I
β_1^{II}
β_1^I
β_2^{II}

Figure 6

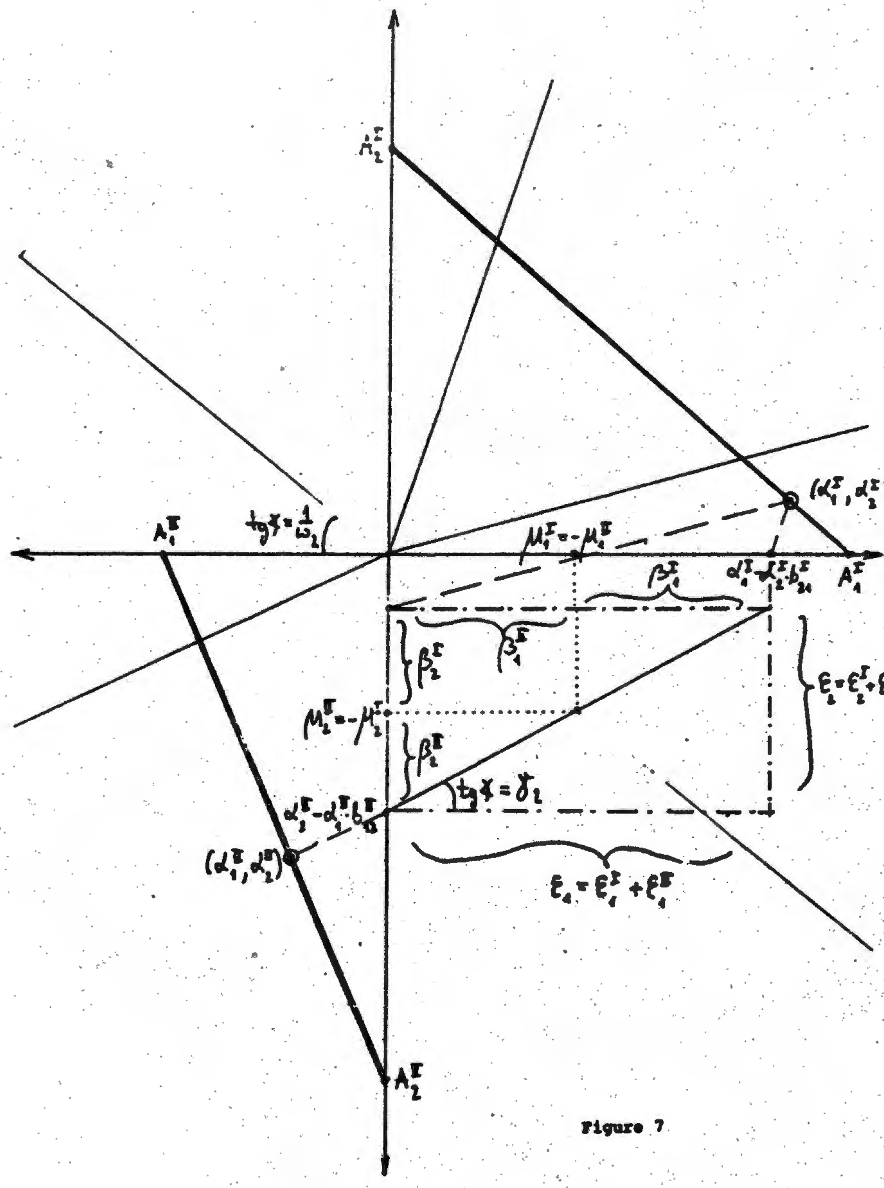
A_2^I
(α_1^I, α_2^I)
A_1^{II}
$tg\,\chi = \frac{1}{\omega_2}$
$\mu_1^I = -\mu_1^{II}$
β_1^I
$\alpha_1^I - \alpha_2^I \cdot b_{21}^I$
A_1^I
β_1^{II}
β_2^I
$\varepsilon_2 = \varepsilon_2^I + \varepsilon_2^{II}$
$\mu_2^{II} = -\mu_2^I$
β_2^{II}
$tg\,\chi = \gamma_2$
$\alpha_2^{II} - \alpha_1^{II} \cdot b_{12}^{II}$
$(\alpha_1^{II}, \alpha_2^{II})$
$\varepsilon_1 = \varepsilon_1^I + \varepsilon_1^{II}$
A_2^{II}

Figure 7

All the quantities mentioned here satisfy together the system of four fundamental relations:

$$\begin{aligned}
\alpha_1^{I} - \alpha_2^{I} b_{21}^{I} - \beta_1^{I} &= \mu_1^{I} \\
\alpha_2^{I} - \alpha_1^{I} b_{12}^{I} - \beta_2^{I} &= \mu_2^{I} \\
\alpha_1^{II} - \alpha_2^{II} b_{21}^{II} - \beta_1^{II} &= \mu_1^{II} \\
\alpha_2^{II} - \alpha_1^{II} b_{12}^{II} - \beta_2^{II} &= \mu_2^{II}
\end{aligned} \qquad (1)$$

Besides this, as can be seen in the figure, exchange is balanced in quantitative terms, so that

$$\begin{aligned}
\mu_1^{I} + \mu_1^{II} &= 0 \\
\mu_2^{I} + \mu_2^{II} &= 0
\end{aligned} \qquad (2)$$

If, as it is shown in Figure 6, the enterprise I exports commodity "1" (since $\mu_1^{I}>0$), then it buys for the proceeds resulting from these exports the commodity "2" so that

$$c_1\mu_1^{I} = c_2\mu_2^{I}$$

or

$$\mu_1^{I} + \mu_2^{I}\omega_2 = 0 \qquad (3)$$

and

$$\mu_1^{II} + \mu_2^{II}\omega_2 = 0$$

correspondingly, for the enterprise II. It can be verified that in the Figure 6 relation

$$-\frac{\mu_2^{I}}{\mu_1^{I}} = \omega_2$$

is in fact satisfied for enterprise I, similarly as an analogous one is satisfied for enterprise II.

Then, the condition of agreement of the structure of goods being at disposal with the structure of needs must be fulfilled, namely

$$\frac{\beta_2^I}{\beta_1^I} = \gamma_2, \qquad \frac{\beta_2^{II}}{\beta_1^{II}} = \gamma_2 \tag{4}$$

This relation is satisfied, for instance, for enterprise I, in Figure 6 (and 7), since the segment of the length

$$\beta_1^I = \alpha_1^I - \alpha_2^I b_{21}^I - \mu_1^I$$

forms, together with the segment of the length

$$\beta_2^I = \alpha_2^I - \alpha_1^I b_{12}^I - \mu_2^I \quad (\mu_2^I < 0)$$

the sides of the rectangle triangle defining the angle whose tangent equals γ_2.

Besides this, in view of the wish of maximizing the degree of satisfaction of needs, points (α_1^I, α_2^I) and $(\alpha_1^{II}, \alpha_2^{II})$ should be chosen in such a way as to lie on the constraints (see Fig.6.).

Thus, they should satisfy equations

$$\frac{\alpha_1^I}{A_1^I} + \frac{\alpha_2^I}{A_2^I} = 1 \tag{5a}$$

$$\frac{\alpha_1^{II}}{A_1^{II}} + \frac{\alpha_2^{II}}{A_2^{II}} = 1 \tag{5b}$$

Making use of the systems of equations (1), (2), (4) and (5) we obtain, by eliminating, for instance, the quantity μ, the following system of two equations:

$$\begin{aligned} \alpha_1^{I}(1+b_{21}\frac{A_2^{I}}{A_1^{I}}) + \alpha_1^{II}(1+b_{21}^{II}\frac{A_2^{II}}{A_1^{II}} &= A_2^{I}b_{21}^{I} + A_2^{II}b_{21}^{II} + \beta_1^{I} + \beta_1^{II} \\ \alpha_1^{I}(b_{12}^{I}+ \frac{A_2^{I}}{A_1^{I}}) + \alpha_1^{II}(b_{12}^{II}+ \frac{A_2^{II}}{A_1^{II}}) &= A_2^{I} + A_2^{II} - \gamma_2(\beta_1^{I} + \beta_1^{II}) \end{aligned} \qquad (6)$$

One should, of course, choose the "production points" of both enterprises (quantities α_1^{I} and α_1^{II}) in such a way as to utilize maximally their properties gain the greatest consumption of commodities "1" and "2" given the structure γ_2. This assumption reduces, formally, to maximization of the quantity $F=\beta_1^{II}+ \beta_1^{I}$, which is defined on the basis of the first of equations (6):

$$F = \beta_1^{I}+\beta_1^{II} = \alpha_1^{I}(1+b_{21}^{I}\cdot\frac{A_2^{I}}{A_1^{I}}) + \alpha_1^{II}(1+b_{21}^{II}\cdot\frac{A_2^{II}}{A_1^{II}}) - A_2^{I}b_{21}^{I} - A_2^{II}b_{21}^{II} \Rightarrow \max$$

or

$$F = \alpha_1^{I}B_{21}^{I} + \alpha_1^{II}B_{21}^{II} - G$$

where

$$B_{21}^{I} = 1 + \frac{A_2^{I}}{A_1^{I}}\cdot b_{21}^{I}, \qquad B_{21}^{II} = 1 + \frac{A_2^{II}}{A_1^{II}}\cdot b_{21}^{II}$$

and

$$G = A_2^{I}b_{21}^{I} + A_2^{II}b_{21}^{II}$$

Thus, we have the criterion function for choosing the values of α_1^{I} and α_1^{II} from the point of view of maximization of joint production. Magnitudes of α_1^I and α_2^{II} must also satisfy the following constraints:

$$0 \leq \alpha_1^{\ I} \leq A_1^{\ I}$$

$$0 \leq \alpha_1^{\ II} \leq A_1^{II}$$

Then, adding by sides the two equations mentioned, we obtain the relation between the values of α_1^I and α_1^{II} being an essential limitation upon the choice of their values from the pcint of view of maximization of the criterion *F*.

It will have the form of

$$\alpha_1^I B^I + \alpha_1^{II} B^{II} = D$$

where

$$B^I = \frac{1}{\gamma_2}(\frac{A_2^I}{A_1^I} + b_{12}^I) + (1 + \frac{A_2^I}{A_1^I} b_{21}^I)$$

$$B^{II} = \frac{1}{\gamma_2}(\frac{A_2^{II}}{A_1^{II}} + b_{12}^{II}) + (1 + \frac{A_2^{II}}{A_1^{II}} b_{21}^{II})$$

$$D = G + \frac{1}{\gamma_2}(A_2^I + A_2^{II})$$

If we determine from this equation

$$\alpha_1^{II} = \frac{D - B^I \alpha_1^I}{B^{II}} \geq 0$$

and substitute in the criterion function, then the latter shall take on the form of

$$F = \alpha_1^I E + B_{21}^{II} \frac{D}{B^{II}} - G \Rightarrow \max \qquad (8)$$

where

$$E = B_{21}^I - B_{21}^{II} \frac{B}{B^{II}}$$

and

$$B^{I}_{21} = 1 + \frac{A^{I}_{2}}{A^{I}_{1}} \cdot b^{I}_{21}$$

$$B^{II}_{21} = 1 + \frac{A^{II}_{2}}{A^{II}_{1}} \cdot b^{II}_{21}$$

$$G = A^{I}_{2} b^{I}_{21} + A^{II}_{2} b^{II}_{21}$$

as previously.

We can therefore conclude, that when condition

$$E = B^{I}_{21} \frac{B^{I}}{B^{II}} < 0$$

is satisfied, then we should choose possibly small α^{I}_{1}, such as e.g. $\alpha^{I}_{1}=0$. We can, in fact, take the value $\alpha^{I}_{1}=0$, but only in the case when

$$0 \le \alpha^{II}_{1} = \frac{D - \alpha^{I}_{1} B^{I}}{B^{II}} \le A^{II}_{1}$$

that is, when condition $\frac{D}{B^{II}} \le A^{II}_{1}$ holds, with $\frac{D}{B^{II}} > 0$.

It is only then that the optimal solution is given with the pair of values

$$\alpha^{I}_{1} = 0, \quad \alpha^{II}_{1} = \frac{D}{B^{II}}$$

If inequality

$$\frac{D}{B^{II}} \le A^{II}_{1}$$

does not hold, then we must take

$$\alpha^{II}_{1} = A^{II}_{1}$$

for it is the value that α_1^{II} may at most assume. In such a case, from equation

$$\frac{D - \alpha_1^I B^I}{B^{II}} = A_1^{II}$$

we obtain

$$\alpha_1^I = \frac{D - B^{II} A_1^{II}}{B^I}$$

If, to the contrary, the condition

$$B_{21}^I - B_{21}^{II} \cdot \frac{B^I}{B^{II}} > 0$$

holds, then we should choose a possibly high value of α_1^I. In particular, $\alpha_1^I = A_1^I$ when condition

$$0 \leq \alpha_1^{II} = \frac{D - \alpha_1^I B^I}{B^{II}} \leq A_1^{II}$$

holds, which means that

$$0 \leq D - A_1^I B^I$$

since the second condition,

$$\frac{D - A_1^I B^I}{B^{II}} \leq A_1^{II}$$

as can easily be verified, is always true. Then

$$\alpha_1^{II} = \frac{D - A_1^I B^I}{B^{II}}$$

Now, if condition

$$0 \leq D - A_1^I B^I$$

is not satisfied, then we take $\alpha_1^{II} = 0$ and, from equation

$$0 = \frac{D - \alpha_1^I B^I}{B^{II}}$$

we have

$$\alpha_1^I = \frac{D}{B^I}$$

Finally, in case of occurrence of equality

$$B_{21}^I - B_{21}^{II} \cdot \frac{B^I}{B^{II}} = 0$$

every pair of strategies $(\alpha_1^I, \alpha_1^{II})$ which satisfies the conditions

$$0 \le \alpha_1^{II} = \frac{D - \alpha_1^I B^I}{B^{II}} \le A_1^{II}$$

$$0 \le \alpha_1^I \le A_1^I$$

is equally good.

Let us now summarize our considerations. If we assume that it is advantageous for both cooperating sides to attain the maximum value of total consumption, i.e.

$$\beta_1 = \beta_1^I + \beta_1^{II}, \quad \beta_2 = \gamma_2 \beta_1$$

then the optimal production strategy for the party I will be

$$\overset{*}{\alpha}{}_1^I = \begin{cases} \min\{A_1^I, \frac{D}{B^I}\} & \text{if } E \ge 0 \\ \max\{0, \frac{D - B^{II} A_1^{II}}{B^I}\} & \text{in the opposite case} \end{cases} \tag{9}$$

with

$$\frac{D}{B^{I}} = \frac{(A_2^{I} + A_2^{II})(1 + \gamma_2 b_{21}^{I})}{\gamma_2\left(1 + b_{21}^{I}\cdot\frac{A_2^{I}}{A_1^{I}}\right) + b_{12}^{I} + \frac{A_2^{I}}{A_1^{I}}}$$

and

$$\frac{D - B^{II} - A_1^{II}}{B^{I}} = \frac{A_2^{I}(1 + \gamma_2 b_{21}^{I}) - A_1^{II}(\gamma_2 + b_{12}^{II})}{\gamma_2\left(1 + b_{21}^{I}\cdot\frac{A_2^{I}}{A_1^{I}}\right) + b_{12}^{I} + \frac{A_2^{I}}{A_1^{I}}}$$

On the other hand for the side II we obtain

$$\overset{*}{a}{}_1^{II} = \begin{cases} \max\{0, \dfrac{D - A_1^{I}B^{I}}{1}\} & \text{when } E \geq 0 \\ \min\{A_1^{II}, \dfrac{D}{B^{II}}\} & \text{in the opposite case} \end{cases} \tag{10}$$

with

$$\frac{D}{B^{II}} = \frac{(A_2^{I} + A_2^{II})(1 + \gamma_2 b_{21}^{I})}{\gamma_2\left(1 + b_{21}^{II}\cdot\frac{A_2^{II}}{A_1^{II}}\right) + b_{12}^{II} + \frac{A_2^{II}}{A_1^{II}}}$$

and

$$\frac{D - B^{I}A_1^{I}}{B^{II}} = \frac{A_2^{I}(1 + \gamma_2 b_{21}^{I}) - A_1^{II}(\gamma_2 + b_{12}^{II})}{\gamma_2\left(1 + b_{21}^{II}\cdot\frac{A_2^{II}}{A_1^{II}}\right) + b_{12}^{II} + \frac{A_2^{II}}{A_1^{II}}}$$

Note yet that formulae (9) and (10) may be written down in a simpler form. This requires noticing that if the inequality given below holds:

$$A_1^{I} > \frac{D}{B^{I}}$$

then also the following inequality holds:

$$\frac{D - A_1^{I}B^{I}}{B^{II}} < 0$$

Consequently, when $E<0$, then

$$(\overset{*}{\alpha}{}_1^{I}, \overset{*}{\alpha}{}_1^{II}) = \begin{cases} (0, \dfrac{D}{B^{II}}) & \text{when } A_1^{II} \geq \dfrac{D}{B^{I}} \\ (\dfrac{D - B^{II}A_1^{II}}{B^{I}}, A_1^{II}) & \text{in the opposite cas} \end{cases} \tag{9'}$$

On the other hand, when

$$(\overset{*}{\alpha}{}_1^{I}, \overset{*}{\alpha}{}_1^{II}) = \begin{cases} (\dfrac{D}{B^{I}}, 0) & \text{when } A_1^{I} \geq \dfrac{D}{B^{I}} \\ (A_1^{I}, \dfrac{D - A_1^{I}B^{I}}{B^{II}}) & \text{in the opposite case} \end{cases} \tag{10'}$$

Knowing values of $\overset{*}{\alpha}{}_1^{I}$ and $\overset{*}{\alpha}{}_1^{II}$ we have, of course, immediately, from equation (5):

$$\begin{aligned} \overset{*}{\alpha}{}_2^{I} &= A_2^{I} - \frac{A_2^{I}}{A_1^{I}} \cdot \overset{*}{\alpha}{}_1^{I} \\ \overset{*}{\alpha}{}_2^{II} &= A_2^{II} - \frac{A_2^{II}}{A_1^{II}} \cdot \overset{*}{\alpha}{}_1^{II} \end{aligned} \tag{11'}$$

We should turn our attention to the fact that solution to the problem of optimum cooperation is defined uniquely excepting the case when we have $E=0$, that is

$$B_{21}^{I} - B_{21}^{II} \cdot \frac{B^{I}}{B^{II}} = 0$$

which, after substitution of appropriate expressions for B_{21}^{I}, B_{21}^{II},

B^{I} and B^{II}, shall take on the form

$$\frac{1 + b_{21}^{\mathrm{I}} \cdot \dfrac{A_2^{\mathrm{I}}}{A_1^{\mathrm{I}}}}{1 + b_{21}^{\mathrm{II}} \cdot \dfrac{A_2^{\mathrm{II}}}{A_1^{\mathrm{II}}}} = \frac{b_{12}^{\mathrm{I}} + \dfrac{A_2^{\mathrm{I}}}{A_1^{\mathrm{I}}}}{b_{12}^{\mathrm{II}} + \dfrac{A_2^{\mathrm{II}}}{A_1^{\mathrm{II}}}} \tag{12}$$

In such a case every pair of strategies

$$\alpha_1^{\mathrm{I}}, \quad \alpha_1^{\mathrm{II}} = \frac{D - \alpha_1^{\mathrm{I}} B^{\mathrm{I}}}{B^{\mathrm{II}}}$$

which satisfies constraints

$$0 \le \alpha_1^{\mathrm{I}} \le A_1^{\mathrm{I}}$$
$$0 \le \alpha_1^{\mathrm{II}} \le A_1^{\mathrm{II}}$$

is equally optimal.

Conclusions drawn from our considerations allow formulation of the following statement:

"Optimum cooperation of two economic organizations characterized by their production potentials A_i^k and matrices of technological coefficients $[b_{ij}^k]$ is defined by the production strategy α_i^{*k} of the 'bang-bang' type, involving full specialization, consisting in maximum production of one commodity by each cooperating side, with simultaneous stopping of production of the other good".

This conclusion is analogous to the optimum control principle for the linear systems, known from control theory.

Our considerations, though, do not only lead to the above, intuitively comprehensible conclusion, but also form the basis for the following

Theorem 2.

Optimum cooperation - from the point of view of maximization of the overall consumption of commodities, given a definite consumption structure, is determined by the formulae (9) and (10) for the optimum production values $\overset{*}{\alpha}{}_1^{I}$, $\overset{*}{\alpha}{}_1^{II}$ and formula (11) for $\overset{*}{\alpha}{}_2^{I}$, $\overset{*}{\alpha}{}_2^{II}$, while the maximum level of consumption, $\beta_1 = \beta_1^{I} + \beta_1^{II}$, is defined by the formula (8) after $\alpha_1^{I} = \overset{*}{\alpha}{}_1^{I}$ is substituted in it. In case equation (12) is satisfied cooperation does not bring advantages - it does not increase the value of β_1 - and every feasible strategy is equally optimal.

In this manner we have solved the problem of optimum cooperation of two organizations, with the notion "optimal" understood as the one which ensures maximum consumption of goods with a given consumption structure γ_2 within both organizations.

Still, this is not the end of troubles, with cooperation, for there remains the question of dividing the quantity β_1 resulting from cooperation into two parts: β_1^{I} and β_1^{II}.

It is understandable that if sharing is "unfair" then there may be no cooperation at all or, if it occurs, it might be broken. In connection with the above, ensuring of an adequate sharing of β_1 could be considered the necessary condition of existence of cooperation.

Before passing directly to this question let us explain in a bit more detail these situations in which cooperation gives no advantages from the point of view of increase of the value of β_1.

BIBLIGRAPHY

1. Aganbegyan, A., Bagrinovski, K., Granberg, A.: Modele matematyczne w planowaniu gospodarczym (mathematical models in economic planning, in Polish). PWE, Warszawa 1974.

2. Ameljańczyk, A.: Niefinansowy model sterowania współpracą międzynarodową (A non-financial model of controlling the international cooperation, in Polish). *Organizacja i Kierowanie*, No. 3/1976.

3. Ameljańczyk, A., Hołubiec, J., Piasecki, S.: Optimisation of international economic cooperation. *Ricerche Economiche*, no. 1, 1978.

4. Ameljańczyk, A., Hołubiec, J.: Optimal international economic cooperation and solution of a multi-person cooperative game. *Ricerche Economiche*, no. 1/2, 1981.

5. Ameljańczyk, A.: Multicriterial optimisation of international economic cooperation. Vth International Conference on Systems Science, Wrocław (Poland), 1978. *Prace Naukowe ICT Politechniki Wrocławskiej*, no. 39, 1978.

6. Ameljańczyk, A., Hołubiec, J.: Modelling of international cooperation in energy by multiperson cooperative game. *Proc. of the First International Conference on Applied Modelling and Simulation*, AMSE, Lyon, 1981 (vol. V).

7. Ameljańczyk, A., Hołubiec, J.: Compromise solution of multiperson cooperative game and its application to international energy consumption. *Large Scale systems, Theory and Applications*. Pergamon Press, London, 1983.

8. Ameljańczyk, A., Hołubiec, J.: The influence of political conditions on the international economic cooperation. *Proc. of SWIIS Workshop: Supplemental Ways for Improving International Stability*. Pergamon Press, Oxford, 1984.

9. Ameljańczyk, A., Hołubiec, J.: Modelling of economic cooperation among blocks of countries with different politico-economic objectives. *9th World Congress of IFAC*. Pergamon Press (IFAC Proc. Series, no. 6), Oxford 1985.

10. Aumann, R.J.: Existence of competitive equilibria in markets with a continuum of traders. *Econometrica*, 34/1966.

11. Bondareva, O.N.: O teoretiko-igrovykh model'akh v ekonomik'e (On game theoretical models in economics, in Russian). *Izd. Leningradskogo Universiteta*, Leningrad 1974.

12. Czerwiński, Z.: Problematyka planowania cen w ujęciu matematycznym (The problem of price planning in the mathematical perspective, in Polish). PTPN PAN, Poznań 1963.

13. Czerwiński, Z.: Podstawowe problemy konstrukcji racjonalnego systemu cen. *Poznańskie Roczniki Ekonomiczne*. Poznań 1969.

14. Debreu, G.: Economies with a finite set of equilibria. *Econometrica*, 38, 1970.

15. Debreu, G., Scarf, H.: A limit theorem on the core of an economy. *Internat. Econ. Rev.*, 4, no.3, 1963.

16. Gale, D.: On optimal development in a multi-sector economy. *Review of Economic Studies*, 34/1, no.97, 1976.

17. Gale, D.: The Thory of Linear Economic Models. McGraw-Hill, New York 1960.

18. Gale, D.: On optimal development in a multi-sector economy. *Rev.Econ.Studies*, 34, no.1, 1967.

19. Georgesen-Roegen, N.: Some properties of a generalized Leontief model. *Activity Analysis of Production and Allocation*, T.C.Koopmans, ed., New York 1951.

20. Gambarelli, G., Hołubiec, J.: Modelling and optimization of international economic cooperation via fuzzy mathematical programming and cooperative games. *Control and Cybernetics*, no.4, 1988

21. Graham, D.: The theory of international values examined. *Qarterly Journal of Economics*, 38, no.1.

22. Hahn, F., Matthews, R.C.O.: The theory of economic growth. A survey. *Economic Journal*, 74, 1964.

23. Hołubiec, J., Piasecki, S.: La collaborazione economica internazionale e la uniformita dei prezzi internazionali (International economic cooperation and the uniformity of

international prices, in Italian). *Rivista di Politica Economica*, no.12, 1978.

24. Holubiec, J.: Modelling of the International Economic Cooperation. *System Theory and Mathematical Economics*. Pitagora Ed., Bologna, 1986.

25. Intriligator, M.: Mathematical Optimization and Economic Theory. Prentice Hall, New York 1971.

26. Kemeny, J.G., Morgenstern, O., Thompson, G.L.: A generalization of the von Neumann model of an expanding economy. *Econometrica*, 24, no.2, 1956.

27. Kotyński, J.: Teoria wymiany międzynarodowej a programowanie liniowe. PWE, Warszawa 1970.

28. Kulikowski, R., ed.: Modelowanie systemowe społeczno gospodarczego rozwoju kraju (Systems modelling of the socioeconomic development of a country, in Polish). PWN, Warszawa1979.

29. Leontief, W.: Factor proportion and the structure of American trade. Further theoretical and empirical analysis. *Review of Economics and Statistics*, 38, no.4.

30. Leontief, W.: Structure of American economy 1919-1929 Cambridge-Masschusetts, Harvard Univ. Press 1941.

31. Leontief, W., Carter A.P., Petri P.A.: The Future of the World Economy. New York 1977, Oxford Univ. Press.

32. Luce R.D., Reiffa H.: Games and Decisions. Willey and Sons, New York 1957.

33. Maciejewski, W.: Algorytmy optymalizacji struktury handlu zagranicznego i kursów kierunkowych (Algorithms of optimization of the structure of foreign trade and the directional rates, in Polish). *Zastosowanie metod matematycznych do analizy ekonomicznej*. ZBKS HZ, Warszawa, 1968.

34. Maciejewski, W.: Ekonometryczne modele wymiany międzynarodowej (Econometric models of international trade, in Polish). PWN, Warszawa 1981.

35. Mycielski, J., Trzeciakowski, W.: Critères du choix des investissements rapidement rentables (Criteria of choice of quick

repayment investments, in French). *Economies and Societies*, no.1, 1970.

36. Mycielski, J., Rey, K., Trzeciakowski, W.: Decomposition and Optimization of Short-Run Planning in a Planned Economy (T.Barna, ed.). London 1963.

37. Nash, J.F.: Equlibrium points in *n*-person games. *Proc.Nat.Acad. Sciences*, USA, 36, 1950.

38. von Neumann, J., Morgenstern, O.: Theory of Games and Economic Behaviour. Princeton 1974.

39. Nikaido, H.: Convex Structures and Economic Theory. Academic Press, New York - London 1968.

40. Pawłowski, Z.: Ekonometria (Econometrics, in Polish) PWN, Warszawa 1978 (fifth edition).

41. Piasecki, St., Ameljańczyk, A.: A control of international economic cooperation. IVth International Conference on Systems Science, Wrocław, Poland, 1977. *Prace Naukowe ICT Politechniki Wrocławskiej*, no.47, 1977.

42. Piasecki, St., Hołubiec, J.: Sterowanie wymianą gospodarczą (Controlling the economic exchange, in Polish) *Prace IBS PAN*, no.32, Warszawa 1979.

43. Piasecki, St., Hołubiec, J.: Wieloletnia koordynacja międzynarodowego rozwoju gospodarczego (Multiannual coordination of the international economic development, in Polish). *Prace IBS PAN*, no.52, Warszawa 1980.

44. Piasecki, St., Hołubiec J.: International economic cooperation and the uniform international prices (in polish). Prace IBS PAN Nr 52, Warszawa 1980.

45. Piasecki, St., Hołubiec, J.: Pewne zagadnienie międzynarodowej współpracy gospodarczej (On a certain problem in international economic cooperation, in Polish). *Prace IBS PAN*, no.77, 1981.

46. Piasecki, St., Hołubiec, J., Ameljańczyk, A.: Międzynarodowa kooperacja gospodarcza (The International Economic Cooperation, in Polish). PWN, Łódź - Warszawa 1982.

47. Piasecki S.: Model of Socio-Economic National Development

(in polish). Prace IBS PAN, No 164 Warszawa 1988.

48. Plaszczyński, W.: Matematyczne modele teorii handlu międzynarodowego (Mathematical Models of the Theory of International Trade, in Polish). PWE, Warszawa 1974.

49. Rosati, D.: Polityka eksportowa. PWE, Warszawa 1990.

50. Rosenmüller, J.: Kooperative Spiele und Market (Cooperative Games and Market, in German). Springer Verlag, Berlin - Heidelberg - New York 1971.

51 .Roth, A.E.: Subsolutions and supercore of cooperative games. *Mathematics of Operations Research*, vol.1, no.1, 1976.

52. Shapley, L.S., Shubik, M.: On market games. *J.Economic Theory 1*, no.1, 1969.

53. Shubik, M.: Edgeworth market games. *Annals of Math. Studies*, vol.IV, no.40, 1959.

54 .Solow, R.M.: A contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70, 1956.

55. Shagalov, G.L.: Problemy optimalnogo planirovaniya vneshne-ekonomicheskikh sviazey (Problems of optimum planning of external economic relations, in Russian). Izd. Moskva 1973

56. Trzeciakowski, W.: Modele pośredniego kierowania gospodarką planową w sterowaniu handlem zagranicznym (Models of indirect management of a planned economy in the control of foreign trade, in Polish). PWE, Warszawa 1975.

57. Tsukui, J.: Turnpike theorem in a generalized dynamic input-output system. *Econometrica*, 34, no.2, 1966.

58. Uzawa, H.: Optimal growth in a two-sector model of capital accumulation. *Review of Economic Studies*, 31, 1964.

For order and other information, please write to:

Systems Research Institute, Polish Academy of Sciences
ul. Newelska 6, 01-447 Warsaw, Poland
Fax: +4822/37 27 72

ISBN 83-90-00412-5-1