POLISH ACADEMY OF SCIENCES
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PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

* prices on the internal market do not correspond to social costs,

* there is lack of conviction as to correctness of exchange rates,

* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Planning and Statistics, Warsaw, 1968).

If we distinquish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Łódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (Quarterly Journal of Economics, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

If the goods considered were plants (e.g. wheat and corn) then for the given surfaces of cultivation of both these cereals the magnitudes A_1^I and A_2^I would be defined as the products of crop yields per surface unit and the surfaces S_1^I and S_2^I. The role of the bottleneck is played by the limited cultivation surface.

Having explained the assumptions adopted for further considerations, let us pass over now to a more detailed characterization of a single enterprise (or country) from the point of view of production and exchange of both distinguished products (raw materials, semi-products and final products). In considerations concerning single enterprises (countries) we shall be omitting indices I and II, assuming, that the reasonings presented would equally concern both partners.

2. THE MODEL OF ECONOMIC ACTIVITY

If we understand by α_j (the intensity of global output) the magnitude of production in a given year (or the so called scale of production), that is the quantity of the good "j" expressed in natural units proper for this good over a unit of time, one year, then net output of this product, ε_j, is defined by equation

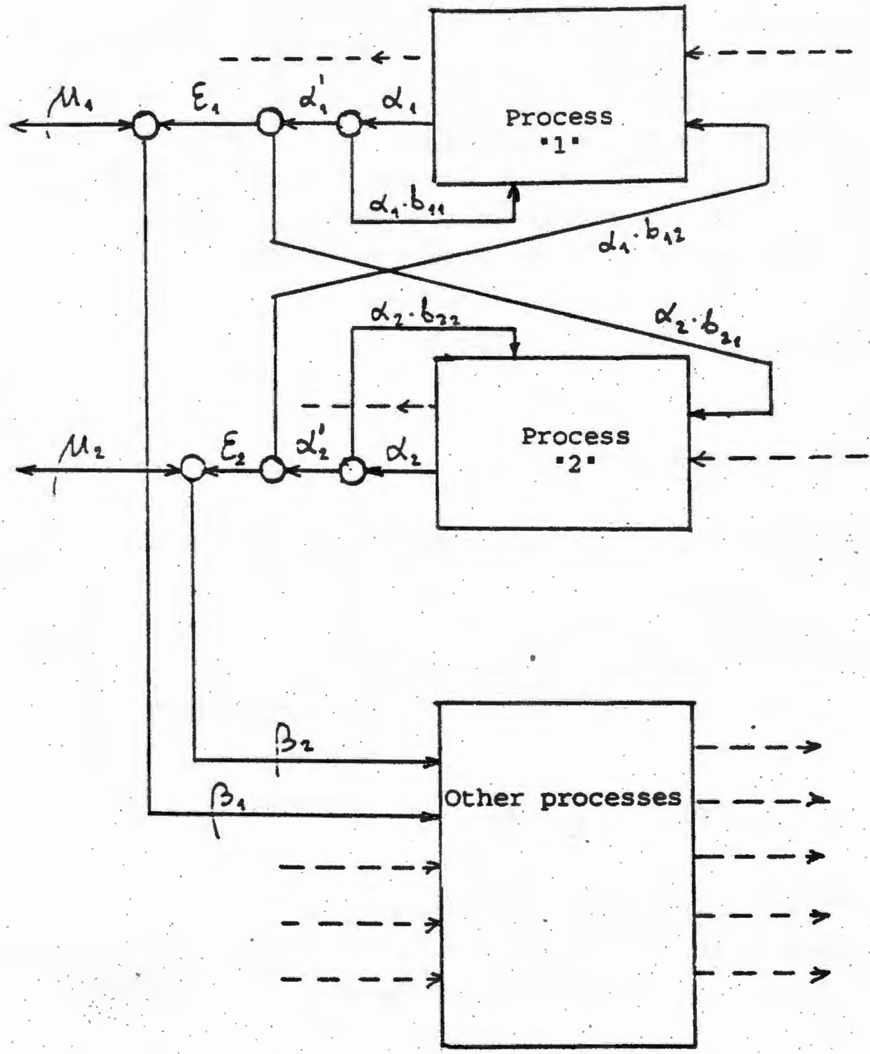
$$\varepsilon_j = \alpha_j - \sum_{i=1}^{J} \alpha_i b_{ij}, \qquad j=1,2$$

constituting a known model linear with respect to α_i. In our considerations this model shall be valid within the boundaries of variability of α_i determined by the inequality

$$\sum_{i=1}^{J} \frac{\alpha_i}{A_i} \le 1$$

These two relations define therefore together a nonlinear model (with the so called "constraint").

If we write down this model in extenso, then we get for $J=2$, corresponding to our case (see Fig. 1).



$$E_1 = \alpha_1 - \alpha_1 \cdot b_{11} - \alpha_2 \cdot b_{21}$$

$$E_2 = \alpha_2 - \alpha_1 \cdot b_{12} - \alpha_2 \cdot b_{22}$$

$$\frac{\alpha_1}{A_1} + \frac{\alpha_2}{A_2} \leq 1$$

Fig. 1.

We shall be using further on two magnitudes: net product α_i' and surplus ε_i. Net product is simultaneously gross product if $b_{11}=b_{22}=0$. In case the latter two coefficients differ from zero, we can define net product with the formulae ($i=1$ and $i=2$):

$$\alpha_1' = \alpha_1 - \alpha_1 b_{11} = \alpha_1(1-b_{11})$$
$$\alpha_2' = \alpha_2 - \alpha_2 b_{22} = \alpha_2(1-b_{22})$$

and surplus product with formulae:

$$\varepsilon_1 = \alpha_1' - \alpha_2' \cdot \frac{b_{21}}{1 - b_{22}}$$
$$\varepsilon_2 = \alpha_2' - \alpha_1' \cdot \frac{b_{12}}{1 - b_{11}}$$

subject to the constraint

$$\frac{\alpha_1'}{A_1(1 - b_{11})} + \frac{\alpha_2'}{A_2(1 - b_{22})} \le 1$$

If we introduce the following notation:

$$b_{21}' = b_{21} \cdot \frac{1}{1 - b_{22}}, \qquad b_{12}' = b_{12} \cdot \frac{1}{1 - b_{11}},$$
$$A_1' = A_1(1 - b_{11}), \qquad A_2' = A_2(1 - b_{22})$$

then previous formulae can be written down in a simplified form, namely:

$$\varepsilon_1 = \alpha_1' - \alpha_2' b_{21}'$$
$$\varepsilon_2 = \alpha_2' - \alpha_1' b_{12}'$$
$$\frac{\alpha_1'}{A_1'} + \frac{\alpha_2'}{A_2'} \le 1$$

equivalent formally to expressions valid for $b_{11}=b_{22}=0$.

Magnitudes $1-b_{11}$, $1-b_{22}$ appearing in the denominators of the formulae listed above must be different from zero. In fact, it would be economically impossible to conduct an activity in which "consumption" of a given product in the process of its production would be greater than (or equal to) the quantity thereby produced. Since we have demonstrated that the case of b_{11} and b_{22} different from zero can be transformed to expressions equivalent to the ones valid for b_{11} and b_{22} equal zero by adequate modification of the quantities A_1, A_2 and b_{12}, b_{21}, then we shall be using in further considerations just such a simplified model.

Thus, if in reality quantities b_{11} and b_{22} are different from zero then in all adequate formulae one should replace quantities b_{12}, b_{21} with, respectively, $b_{12}(1-b_{11})$, $b_{21}(1-b_{22})$, quantities A_1, A_2 - with, respectively, $A_1(1-b_{11})$, $A_2(1-b_{22})$, and quantities α_1, α_2 - with, respectively, $\alpha_1(1-b_{11})$, $\alpha_2(1-b_{22})$.

After such a substitution quantities α_1, α_2 can be interpreted as gross production . In order to explain the manner of determining the values of b_{ij} assume that we are considering cooperation of two industries: the machine tools industry ("1"), and the car industry ("2"). Products "1" of the machine tools industry are being used up by the very machine tools industry which turns them out for, at least, replacement of the worn out own machines, so that $b_{11}\neq 0$. They are also used by car industry, and therefore $b_{21}\neq 0$ as well. Analogously, products "2" - of the car industry - are used up by the industry which turns them out, for, at least, replacement of own transport means, so that $b_{22}\neq 0$, and by the machine tools industry, and so $b_{12}\neq 0$, too. Thus, we are dealing with two industries, constituting a distinct part of economy, which turn out two kinds of final product: machine tools and cars, and whose productive potential is characterized by the quantities A_1 and A_2, corresponding to maximum production capacities, of machine tools or cars, resulting from the limited capacities of casting shops. Technological normatives b_{ij}, defining the quantity of units of commodity "j" used for production of one unit of commodity "i", determine thereby the quality of the technological processes and have, in our case, the following interpretation:

Quantity b_{11} corresponds to the number of machine tools used up in production of one machine tool. This quantity is determined in the following manner: if z_{11} is the number of hours of work of the machine tool shop of the enterprise and z_1 is the durability of a machine tool expressed in hours it can work from the moment it was produced until being scrapped, then

$$b_{11} = \frac{z_{11}}{z_1}$$

Quantity b_{21} corresponds to the number of machine tools used up in producing one car and it is determined in an analogous way:

$$b_{21} = \frac{z_{21}}{z_1}$$

where z_{21} is the number of work hours of the machine tool shop necessary for producing one car.

Now, quantity b_{22} corresponds to the number of cars used up in producing one car, and it can be determined in the following manner: we take the ratio of the number of kilometers z_{22} covered during a year (in connection with car production) by cars owned by the enterprise, and the product of the annual car production α_2 and car durability z_2:

$$b_{22} = \frac{z_{22}}{\alpha_2 z_2}$$

where z_2, the durability of cars, is expressed in kilometers.

We can analogously determine b_{12} - "consumption" of cars per one produced machine tool, for a given output of machine tools, α_1, according to the formula

$$b_{12} = \frac{z_{12}}{\alpha_1 z_2}$$

where z_2 is the number of kilometers covered by the cars owned by the enterprise in connection with production of machine tools.

We may proceed in a similar way in other cases, for other commodities and other kinds of industries.

The technological structure of the portion of enterprise which is of interest for us from the point of view of production of goods "1" and "2" is shown in Fig. 1.

3. AUTARCHIC ECONOMY

Let us try now to determine the quantities α_1 and α_2, which maximize satisfaction of demand for commodities "1" and "2", given a predetermined structure γ_2, when we have no possibility of exchange. The productive potential we dispose of is characterized by the quantities A_1 and A_2 and by the matrix of technological normatives b_{ij} $(i,j=1,2)$.

Since we assume no exchange, the structure of surplus product must correspond to the structure given, γ_2:

$$\frac{\varepsilon_2}{\varepsilon_1} : \frac{\alpha_2 - \alpha_1 b_{12}}{\alpha_1 - \alpha_2 b_{21}} = \gamma_2$$

On the other hand, we would be able to maximally satisfy the

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