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Book review:

STATISTICS AND ANALYSIS OF SHAPE

by

H. Krim, A. Yezzi, Jr., eds.

There are numerous definitions of a notion of shape of an object. These definitions usually highlight aspects relevant to a particular applications of interest. The development of digital imagery has triggered keen interest in further refining and unifying the notion of shape. Shape analysis and recognition is an essential element of many applications governing our daily lives. These applications include, among others: biomedical engineering (e.g. Magnetic Resonance Imaging), mechanical engineering (shape and topology optimization of structures or machine parts), user authentication, object retrieval from databases, or surveillance and security measures around sensitive areas. Shape analysis is emerging as an important area of image processing and computer vision.

The modern shape analysis is based on two main approaches:

1. the first one approximates object shapes by a finite-dimensional approximation (a set of characteristic points or landmarks), which is then subject to various transformations to derive shape models,
2. the second one interprets shapes as closed contours in infinite-dimensional space, which, when subject to transformations morph into other shapes, yielding a notion of similarity in the space of shapes.

In the first approach a shape of an object is described by its characteristic points. Mathematically, shape is understood here as all the geometrical information that remains when location, scale and rotation are filtered out from an object. A planar shape commonly coincides with a closed curve enclosed in a region of a plane in R^2, bearing landmarks given by a vector of points. These points, with additional constraints on their coordinates, represent a constrained subset of R^2, referred to as a preshape space. All rotations of the plane containing the preshape space create a set of orbits of a preshape space. Shape space, i.e., equivalence classes of shapes, is defined as a set of such orbits. This space forms a Riemannian manifold. A metric on this manifold, affording a comparison of different shapes, is induced by a metric on the preshape space or a sphere of preshapes. Graph based representations of two- or three-dimensional shapes and polygonal approximation of a shape as well as particle based shape models are also related to landmark based shapes.

The second approach uses either the parametric representation technique or the level set method to describe the contour of the object. Parametric representation, consisting in association to each point of a boundary of the object a value of a parameter, is an explicit description of the body contour. This representation is used in capturing object boundaries in snakes or active contour methods. This approach with its non unique parameterization presents difficulties in computation of averages or distances employed in shape analysis. The level set methods represent implicit description of the object boundary. They consist in defining a function, called level set function, which has zero value at points lying on the object boundary and which has value less (respectively, greater) than zero at points lying outside (respectively, inside) the object. In the level set method the problem of non unique parameterization appearing in parametric representation method, is replaced by the non unique choice of the level set function for points not belonging to the contour of the body. Geometric formulation of explicit\implicit representation of shape is employed to avoid infinite-dimensional ambiguities of both representations. Geometric formulation means adopting a signed distance function in the implicit representation or an arc-length parameterization in the explicit representation. Two fundamental problems in shape analysis are:

- to determine a shape space, which is usually non-linear,
- to derive a measure of distance between different shapes; the adoption of different norms in shape spaces yields different algorithmic techniques.

The goal of this contributed book is to expose these different approaches to shape analysis as well as unveil latest efforts in both schools of thought to overcome existing fundamental problems. These unsolved problems in both schools include, among others, either systematic, robust and automatic choice of landmarks for an unambiguous definition of shape or a definition of a natural norm in the Riemannian framework.

The book is divided into 15 chapters. Each chapter is written by leading experts in the field. The initial chapters 1 to 4 explore the statistical modeling of landmarks. These chapters are devoted to graph based representation of shapes. The statistical shape analysis has been initiated by Kendalls' works. Its goal is to construct a compact and stable description of mean and variability of a population of geometric objects. The subsequent chapters address the probabilistic modeling of entire shapes. Chapters 5 to 7 contain applications of level set methods to shape representation and analysis. Chapters 8 to 10 deal with extensions of graph based representation of shapes to three- or higher-dimensional shapes. These chapters concentrate on case studies as well as implementation-related and practical challenges in real systems. The discussed in Chapters 11-12 particle based shape models are motivated by pattern formation in statistical physics. These models are related to a class of landmark based shapes. Chapter 11 deals with a system of spin particles which may interact to yield a shape. The next chapter concerns the particles diffusing along the trajectory describing the shape of interest which may be also used to model a limiting case of a landmark based shape. Shape analysis in infinite-dimensional shape spaces is presented in Chapters 13 to 15, where different norms, measuring the distance between shapes are introduced and numerically computed. Each chapter provides a long list of references allowing for further studies.

Let us provide more details about the contents of the book. Chapter 1 deals with medial axis computation using average outward flux algorithm and its evolution governed by the motion of the bitangent boundary points. The examples of computed medial axes of three-dimensional human objects are given. Chapter 2 presents a general method called principal geodesic analysis for computing the variability of manifold valued data, i.e., shape variation of medial axis. This method is applied to describe the shape variability of medial representations and results are shown on a hippocampus data set. Chapter 3 is devoted to modeling of a topologically diverse class of shapes based on Morse theoretic skeleton graphs. Morse theory relates topology of a smooth manifold with a number of critical points of a Morse function defined on this manifold. This approach captures not only topology but also geometry of a shape. Applications include storage and shape classification. Chapter 4 is concerned with measuring similarity between shapes and exploiting it for object recognition. Two algorithms for rapid shape retrieval are presented. Applications for silhouettes and handwritten digits are presented.

Chapter 5 is concerned with the problem of visual recognition of two-dimensional planar shapes. Especially, an abstract setting is introduced where decision rules based on contrario methodology for pairing two shape elements are provided. The proposed matching methodology is illustrated through several experiments including logo recognition, comparing codes extracted from two views of old paintings. Chapter 6 introduces for closed planar contours a class of functionals obtained by performing integral operation, which are invariant with respect to the Euclidean groups. Based on these invariants a distance measure between shapes is proposed. The analysis of shapes at multiple scales is allowed. The examples of results of matching and retrieving noisy shapes from a database are shown. Chapter 7 deals with shape representation, registration and modeling using a level set method. Global and local registration of shapes through the alignment of the corresponding distance transforms is proposed. These transforms are performed by defining objective functions minimizing metrics between implicit representation of shapes. Validation of the proposed method through various applications is provided. Chapter 8 deals with the description of manifolds by point clouds. One of the most popular source of point clouds are three-dimensional shape acquisition devices such as laser range scanners. The computation of geodesic distances of objects represented by the point cloud is discussed. Using the theory of Gromov-Hausdorff distances, geometric framework for comparing manifolds given by point clouds is presented. Numerical experiments are provided. Chapter 9 focuses on simple estimators of the intrinsic dimension of the smooth Riemannian manifold embedded in R^d and of the entropy of the underlying sampling distribution on the manifold. Assuming compactness of the manifold and the boundedness of the Lebesgue sampling density supported on the manifold, proofs of strong consistency of these estimators are shown. Experimental results are provided. Chapter 10 deals with the construction of a metric for comparing configuration of object features to configuration of image features that is invariant to any affine transformation of the object or image. This chapter explores the generalized weak perspective camera model. An algorithm to compute the distances between shapes in image and object spaces is provided.

The particle based approach to shape analysis, motivated by statistical physics, is presented in Chapters 11 and 12. The first of these chapters deals with ill-posed problems of image processing using Gibbs field approach. The notions and examples of regular models are given. Chapter 12 concerns stochastic approximation of curve shortening flows based on the theory of interacting particle systems. The proposed approach is valid for arbitrary embedded planar curves. The curve evolution is interpreted as a semilinear diffusion equation. This leads to a coupled system of interacting particle systems, whose limiting behavior is the desired curve shortening flow.

Chapter 13 deals with characterizing shapes of continuous curves, both open and closed in R^2. Under appropriate constrains that remove shape preserving transformations these curves form infinite-dimensional non-linear shape spaces. Two Riemannian metrics are imposed on these spaces and properties of these structures are studied. Probability models on shape spaces are given. Examples of applications including mean shape computing, clustering of shapes, interpolation of shapes in echocardiographic image sequences and many others are enclosed. Chapter 14 focuses on the study of plane curve deformations as well as its evolution, comparison and matching. Diffeomorphic deformations in which a template curve is in one-to-one smooth correspondence with a target curve are the subject of interest. The deformation is driven by a data attachment term measuring the quality of the matching. Three kinds of attachment are reviewed. Momentum theorem relating the momentum of the Hamiltonian evolution to the differential of the data attachment term is proved. The existence of Hamiltonian flow is shown. Chapter 15 proposes a framework for dealing with two main problems of shape analysis: the definition of a set of shapes and that of defining the metric on it. Motivated by image analysis applications, a set of shapes consists from subsets of R^2 of positive reach in the sense of Federer with smooth boundaries of bounded curvature. For this particular shape space three considered topologies, i.e., L^2 norm of a difference of characteristic functions and L^∞, $\mathrm{W}^{1,2}$ norms of the difference between distance functions, are shown to be equivalent. A family of smooth approximations of the distance functions which are continuous with respect to the Hausdorff topology is proposed. The presented approach is illustrated on a variety of examples, in which means and variations of several fish or human hand curves are computed.

The book in clear, concise and interesting way describes the fascinating world of shapes as well as problems concerning shape recognition and retrieval. This self-contained book reports on the latest results and techniques in the field of statistical and probabilistic shape analysis. The book presents different approaches to shape analysis of objects using elements of geometry, topology, optimization, partial differential equations and probability theories. Therefore it requires from the reader the knowledge of basic statistics and probability theories, partial differential equation theory as well as the advanced calculus. The book is well written and organized at a difficulty level that precisely meets the target audience's needs. An explanatory introduction to each chapter, numerous illustrations and application examples help readers to overcome terse technical details of the probabilistic formalism and understand the text as well as makes the material more accessible to a wider cross-disciplinary audience. The wide variety of topics covered and the tutorial writing style by many of the contributors make this book suitable for specialists as well as young researchers who seek a more intuitive understanding of these relevant topics in the field.

Statistical and medical researchers, engineers, scientists as well as interested students will find this book to be an excellent resource to rapid introduction into the field of shape statistics and analysis. The book may also serve as a textbook for graduate level courses in statistics or image analysis as well as for an intensive course on shape analysis and modeling. The advanced techniques presented in the book may also be useful for experienced researchers and practitioners both from academia or industry.

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