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Book review:

Empirical Process Techniques for Dependent Data

by

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The empirical process techniques for independent data constitute a very well–developed and fundamental branch of statistics and probability theory. The classical example in this field is the widely known Glivenko–Cantelli theorem for convergence of empirical distribution functions $F_n(x)$ to a distribution function $F(x)$ in case of independent, identically distributed (i.e. *iid*) random variables. This theorem is a foundation for modern mathematical statistics and plays the central and important role in many tests, e.g. in the Kolmogorov–Smirnov test.

However, practical problems are much more complicated than the standard *iid* case and real–life data often reveal various kinds of *dependence*. This type of data may be found in different fields – such as geology, finance, insurance, meteorology and many others. Also in theory it is very tempting to weaken the assumptions of the theorems and lemmas. We can see this tendency in many variants of the Central Limit Theorem with the gradually reduced independence assumption.

To model the dependence structure in complicated data sets and to extend the limits of the theory one needs new results in the area of empirical process techniques for dependent data. The book reviewed is a very important and essential answer to this demand.

"Empirical Process Techniques for Dependent Data", edited by Herold Dehling, Thomas Mikosch and Michael Sørensen, is a proceeding of the MaPhySto Instructional Workshop organized by Thomas Mikosch, Søren Feodor Nielsen and Michael Sørensen in November 2000 at the University of Copenhagen. The book contains fourteen skilfully assembled papers, classified into six parts.

In the first part we find one article, written by Herold Dehling and Walter Philipp. This paper provides a tutorial on empirical processes techniques for dependent data. The authors present the fundamental knowledge in this field, from the papers of Hermann Weyl up to very recent developments, such as empirical process of U-statistics structure. Also other subjects, like various mixing concepts and essential tools of empirical process theory, are discussed. This part allows the reader to extend his knowledge about the principles of theories surveyed further in this book and their historical development.

The following part is devoted to the techniques for the empirical process of stationary sequences. It contains papers written by Patrick Ango Nze, Paul Doukhan, Jérôme Dedecker, Sana Louhichi, Sara A. van de Geer, Florence Merlevède, Magda Peligrad, István Berkes and Lajos Horváth. These authors present their research on the classical empirical process in the case of dependent data. We may find in this part a variety of interesting subjects: a new concept of weak dependence based on the covariances of functions of the stationary sequences, examination of maximal inequalities important for proving tightness of the empirical process for mixing sequences, generalization of Hoeffding's inequality to the case of dependent variables, application of coupling to obtaining uniform strong laws of large numbers and central limit theorems in case of dependent sequences, review of behaviour of empirical sequences of residuals for *iid* case and time–series models ARMA, ARCH and GARCH.

The third part, entitled "The Empirical Process of Long Range Dependent Processes" consists of articles by Hira L. Koul, Donatas Surgailis, Liudas Giraitis and Miguel A. Arcones. They focus on the asymptotic behaviour of empirical processes. In the first paper the authors discuss the results on the asymptotic distribution of long memory moving averages of empirical processes. They also present applications to goodness-of-fit testing for the marginal stationary error distribution and and M-estimation in the one sample location model. In the second article, the extension of the reduction principle by Dehling and Taqqu is provided. The last paper in this part is an interesting review of results concerning limit theorems for sums of nonlinear function of a stationary sequence in case of Gaussian random variables.

The next part concerns empirical spectral process techniques. The papers contained in this part are written by Rainer Dahlhaus, Wolfgang Polonik and Philippe Soulier. In the first paper the analogy between classical techniques of empirical process and spectral analysis for time-series is precisely presented. This idea is complemented by application of goodness-of-fit tests, nonparametric Whittle likelihood estimation and other statistical methodologies to the time-series instance. This topic is continued in the next paper, where parametric and adaptive estimation is considered. The discussion is extended to application of different empirical functions to strongly dependent processes and presentation of functional limit theorem for the empirical process of the periodogram process at the Fourier frequencies.

The last two parts seem "not to follow the standard patterns" of literature about empirical process techniques, as indicated by the editors in the preface, and present the developments from some other fields. The fifth part entitled "The Tail Empirical Process in Extreme Value Theory", is constituted by the paper of Holger Drees. The author reviews the limit theorems for tail processes of absolutely regular time series, from Rootzén result for the uniform tail empirical process, through its generalizations up to the developments for the uniform tail quantile process.

The last part concerns bootstrap techniques for dependent data and we find there the papers by Dragan Radulović and Efstathios Paparoditis. In the first article, the author not only introduces the basic ideas and techniques in this field, but also reviews recent developments, such as bootstrap for Markov chains. The second paper is devoted to nonparametric resampling methods of the periodogram and their applications.

The book reviewed provides a very good survey of empirical process techniques for dependent data. It presents an introduction to classical theory, applications to practical problems and also very recent developments. The mathematical rigor of notation is worth stressing. This book gives the reader a rare opportunity of noticing similar techniques used in quite different research areas. The presented approach is rather unique in the literature. However, some readers may be confused by the diversity of the subjects discussed and may prefer a stricter sticking to the main topic. The book should attract the attention of specialists in various fields, both theoreticians and practitioners. Further, the abundance of references after each of the papers may serve as additional help for the readers in deepening their knowledge of the interesting concepts used in this book.

The volume was very carefully prepared to meet the high standards of Birkhäuser Boston.

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