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Chapter 1

Rough Sets in Medical Imaging: Foundations and Trends

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1.1 Introduction

Computational intelligence (CI) techniques and approaches encompass various paradigms dedicated to approximately solving real-world problems in decision making, pattern classification and learning. Prominent among these paradigms are fuzzy sets, neural networks, Genetic Algorithms, decision tree, rough sets, and a generalization of rough sets called near sets. Fuzzy sets [131] provide a natural framework for dealing with uncertainty. It offers a problem-

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solving tool between the precision of classical mathematics and the inherent imprecision of the real world. For example, imprecision in a segmented image can be represented and analyzed using fuzzy sets [61]. Neural networks [108] provide a robust approach to approximating real-valued, discrete-valued and vector-valued functions. The well-k nown back propagation algorithm that uses gradient descent to tune network parameters to best fit the training set with input-output pair, has been applied as a learning technique for the neural networks. Genetic algorithms [53, 47] are stochastic search techniques based on the principles of evolution. Extensive research has been performed exploiting the robust properties of genetic algorithms and demonstrating their capabilities across a broad range of problems. These evolutionary methods have gained recognition as general problem solving techniques in many applications, including function optimization, image processing, classification and machine learning, training of neural networks, and system control. Other approaches like case based reasoning and decision trees [25, 102] are also widely used to solve data analysis problems. Each one of these techniques has its own properties and features including their ability of finding important rules and information that could be useful for the medical field domain. Each of these techniques contributes a distinct methodology for addressing problems in its domain. This is done in a cooperative, rather than a competitive, manner. The result is a more intelligent and robust system providing a human-interpretable, low cost, exact enough solution, as compared to traditional techniques. Currently, development an efficient computer aided diagnosis system that assist the radiologist has thus become very interest, the aim being not to replace the radiologist but to over a second opinion [38, 8]. Consequently, many efforts have been made to incorporate techniques for image processing. Recently, various published algorithms have been applied to build a computer-aided analysis system in medical field [24, 23, 39, 38, 101]. The most commonly used algorithms are neural networks, Bayesian classifier, genetic algorithms, decision trees, and fuzzy theory [128, 113, 124, 105, 100, 118, 115]. Unfortunately, the techniques developed have not been sufficient to introduce an efficient computer-aided analysis in clinical use. A survey of the area can be found in [38].

Rough set theory introduced by Zdzisław Pawlak during the early 1980s [73, 75, 76] offers an approach to granular computing that is part of computational intelligence [87]. The basic idea underlying the rough set approach to information granulation is to discover to what extent a give set of objects (*e.g.*, pixel windows in an image) approximates another of set of objects of interest. Objects are compared by considering their descriptions. An object description is modelled as a vector function values represent object features [77]. It is possible for an object feature to be represented by one or more functions, *e.g.*, colour represented by functions that measure luminance (intensity), type of colour (hue), and purity of colour (saturation). This is a fairly new intelligent technique for managing uncertainty that has been applied to the medical domain and is used for the discovery of data dependencies, evaluates the importance of features, discovers the patterns of data, reduces all redundant objects and features, seeks the minimum subset of features, recognize and classify objects in medical imaging. Moreover, it is being used for the extraction of rules from databases. Rough sets have proven useful for representation of vague regions in spatial data. One advantage of the rough set is the creation of readable if-then rules. Such rules have a potential to reveal new patterns in the data material; furthermore, it also collectively functions as a classifier for unseen data sets. Unlike other computational intelligence techniques, rough set analysis requires no external parameters and uses only the information presented in the given data. One of the nice features of rough sets theory is that its can tell whether the data is complete or not based on the data itself. If the data is incomplete, it suggests more information about the objects needed to be collected in order to build a good classification model. On the other hand, if the data is complete, rough sets can determine whether there are more than enough or redundant information in the data and find the minimum data needed for classification model. This property of rough sets is very important for applications where domain knowledge is very limited or data collection is very expensive/laborious because it makes sure the data collected is just good enough to build a good classification model without sacrificing the accuracy of the classification model or wasting time and effort to gather extra information about the objects [63, 64, 66, 75].

The objective of this book chapter is to present to the rough sets and medical imaging research communities the state of the art in the rough applications to image processing and pattern recognition, and in particular in medical imaging, and motivate research in new trend-setting directions. Hence, we review and discuss in the following sections some representative methods to provide inspiring examples to illustrate how rough sets could be applied to resolve medical imaging problems and how medical images could be analyzed, processed, and characterized by rough sets. These representative examples include (i) Rough representation of a region of interest; (ii) Rough image entropy; (iii) Rough C-mean clustering; (iv) Rough Neural Intelligent Approach for Image Classification: A Case of Patients with Suspected Breast Cancer.

To provide useful insights for rough sets applications in medical imaging. We structure the rest of this chapter in six further sections, where Section 2 provides an explanation of the basic framework of rough set theory, along with some of the key definitions. Section 3 provides an introduction to the rough image processing including the rough image, rough representation of a region of interest, rough image entropy, and rough-based medical image applications including object extraction and medical image segmentation and clustering. Section 4 provides a brief review of the rough sets in feature reduction and image classification.Section 5 provides a brief review of the joint rough sets with other intelligent approaches including rough neural network, rough fuzzy, rough genetic algorithms, etc. It also provide detailed descriptions of using rough neural network in image mammogram breast cancer analysis. Section 6 provides other applications of rough sets in medical domain such as Rough sets in medical image retrieval and in medical data mining and decision systems.

Challenges and future trends are addressed and presented in Section 7.

1.2 Rough sets: Foundations

Rough sets theory is a new intelligent mathematical tool proposed by Pawlak [63, 64, 73, 75, 76]. It is based on the concept of approximation spaces and models of the sets and concepts. In rough sets theory, the data is collected in a table, called a decision table. Rows of a decision table correspond to objects, and columns correspond to features. In the data set, we assume that the a set of examples with a class label to indicate the class to which each example belongs are given. We call the class label a decision feature, the rest of the features are conditional. Let $\mathcal{O}, \mathcal{F}$ denote a set of sample objects and a set of functions representing object features, respectively. Assume that $B \subseteq \mathcal{F}, x \in \mathcal{O}$. Further, let $[x]_B$ denote

$$[x]_B = \{y : \ x \sim_B \ y)\}.$$

Rough sets theory defines three regions based on the equivalent classes induced by the feature values: lower approximation $\underline{B}X$, upper approximation $\overline{B}X$ and boundary $BND_B(X)$. A lower approximation of a set X contains all equivalence classes $[x]_B$ that are subsets of X, and upper approximation $\overline{B}X$ contains all equivalence classes $[x]_B$ that have objects in common with X, while the boundary $BND_B(X)$ is the set $\overline{B}X \setminus \underline{B}X$, *i.e.*, the set of all objects in $\overline{B}X$ that are not contained in $\underline{B}X$. So, we can define a rough set as any set with a non-empty boundary.

The indiscernibility relation $\sim_B$ (or by Ind_B) is a mainstay of rough set theory. Informally, $\sim_B$ is a set of all objects that have matching descriptions. Based on the seletion of B, $\sim_B$ is an equivalence relation partitions a set of objects $\mathcal{O}$ into equivalence classes (also called elementary sets [73]). The set of all classes in a partition is denoted by $\mathcal{O}/\sim_B$ (also by $\mathcal{O}/Ind_B$). The set $\mathcal{O}/Ind_B$ is called the quotient set. Affinities between objects of interest in the set $X \subseteq \mathcal{O}$ and classes in a partition can be discovered by identifying those classes that have objects in common with X. Approximation of the set X begins by determining which elementary sets $[x]_B \in \mathcal{O}/\sim_B$ are subsets of X.

Here we provide a brief explanation of the basic framework of rough set theory, along with some of the key definitions. A review of this basic material can be found in sources such as [63, 64, 66, 75, 62, 132, 76].

Information System and Approximation

DEFINITION 1.1 *(Information System) Information system is a tuple (U, A), where U consists of objects and A consists of features. Every $a \in A$ corresponds to the function $a : U \to V_a$ where V_a is a's value set. In applications, we often distinguish between conditional features C and decision features D, where $C \cap D = \emptyset$. In such cases, we define decision systems (U, C, D).*

DEFINITION 1.2 *(Indiscernibility Relation) Every subset of features $B \subseteq A$ induces indiscernibility relation*

$$Ind_B = \{(x, y) \in U \times U : \forall_{a \in B}\, a(x) = a(y)\}$$

For every $x \in U$, there is an equivalence class $[x]_B$ in the partition of U defined by Ind_B.

Due to imprecision which exists in real world data, there are sometimes conflicting classification of objects contained in a decision table. Here conflicting classification occurs whenever two objects have matching descriptions, but are deemed to belong to different decision classes. In that case, a decision table contains an inconsistency.

DEFINITION 1.3 *(Lower and Upper Approximation) In the Rough Sets Theory, the approximations of sets are introduced to deal with inconsistency. A rough set approximates traditional sets using a pair of sets named the lower and upper approximation of the set. Given a set $B \subseteq A$, the lower and upper approximations of a set $Y \subseteq U$, are defined by equations (1) and (2), respectively.*

$$\underline{B}Y = \bigcup_{x:[x]_B \subseteq X} [x]_B. \tag{1.1}$$

$$\overline{B}Y = \bigcup_{x:[x]_B \cap X \neq \emptyset} [x]_B. \tag{1.2}$$

DEFINITION 1.4 *(Lower Approximation and positive region) The positive region $POS_C(D)$ is defined by*

$$POS_C(D) = \bigcup_{X:X \in U/Ind_D} \underline{C}X.$$

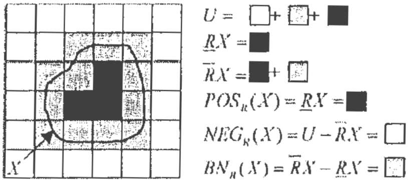

FIGURE 1.1: Illustrated example of approximation [29]

$POS_C(D)$ is called the positive region of the partition U/Ind_D with respect to $C \subseteq A$, i.e., *the set of all objects in U that can be uniquely classified by elementary sets in the partition U/Ind_D by means of C [76].*

DEFINITION 1.5 *(Upper Approximation and Negative Region) The negative region $NEG_C(D)$ is defined by*

$$NEG_C(D) = U - \bigcup_{X:X \in U/Ind_D} \overline{C}X,$$

i.e., *the set of all all objects that can be definitely ruled out as members of X.*

DEFINITION 1.6 *(Boundary region) The boundary region is the difference between upper and lower approximation of a set X consists of equivalence classes have one or more elements in common with X. It given by the following formula:*

$$BND_B(X) = \underline{B}X - \bar{B}X \tag{1.3}$$

Fig. (1.1) illustrated an example of approximation.

Reduct and Core

An interesting question is whether there are features in the information system (feature-value table) which are more important to the knowledge represented in the equivalence class structure than other features. Often we wonder whether there is a subset of features which by itself can fully characterize the knowledge in the database. Such an feature set is called a reduct. Calculation of reducts of an information system is a key problem in RS theory

[63, 64, 66, 75]. We need to get reducts of an information system in order to extract rule-like knowledge from an information system.

DEFINITION 1.7 *(Reduct) Given a classification task related to the mapping $C \to D$, a reduct is a subset $R \subseteq C$ such that*

$$\gamma(C, D) = \gamma(R, D)$$

and none of proper subsets of R satisfies analogous equality.

DEFINITION 1.8 *(Reduct Set) Given a classification task mapping a set of variables C to a set of labeling D, a reduct set is defined with respect to the power set $P(C)$ as the set $R \subseteq P(C)$ such that $Red = \{A \in P(C) : \gamma(A, D) = \gamma(C, D)\}$. That is, the reduct set is the set of all possible reducts of the equivalence relation denoted by C and D.*

DEFINITION 1.9 *(Minimal Reduct) Given a classification task mapping a set of variables C to a set of labeling D, and R, is the reduct set for this problem space. A minimal reduct R is the reduct such that $\|R\| \leq \|A\|, \forall A \in R$. That is, the minimal reduct is the reduct of least cardinality for the equivalence relation denoted by C and D..*

DEFINITION 1.10 *(Core) Attribute $c \in C$ is a core feature with respect to D, if and only if it belongs to all the reducts. We denote the set of all core features by $Core(C)$. If we denote by $R(C)$ the set of all reducts, we can put:*

$$Core(C) = \bigcap_{R \in R(C)} R \tag{1.4}$$

The computation of the reducts and the core of the condition features from a decision table is a way of selecting relevant features. It is a global method in the sense that the resultant reduct represents the minimal set of features which are necessary to maintain the same classification power given by the original and complete set of features. A straighter manner for selecting relevant features is to assign a measure of relevance to each feature and choose the features with higher values. Based on the generated reduct system we will generate list of rules that will be used for building the classifier model of the new object with each object in the reduced decision table (i.e. reduct system) and classify the object to the corresponding decision class. The calculation of all the reducts is fairly complex (see [102, 15, 19, 23, 103]).

Nevertheless, in many practical applications including medical imaging problems, it is not necessary to calculate all the reducts, but only some of them. For example, in Slowinski et al. [110] the following heuristic procedure has been used to obtain the most satisfactory reduct. Starting from single features, the one with the greatest quality of classification is chosen; then to the chosen feature, another feature is appended that gives the greatest increase to the quality of classification for the pair of features; then yet another feature is appended to the pair giving the greatest increase to the quality of classification for the triple, and so on, until the maximal quality is reached by a subset of features. At the end of this procedure, it should be verified if the obtained subset is minimal, i.e. if elimination of any feature from this subset keeps the quality unchanged. Then, for further analysis, it is often efficient to take into consideration a reduced data table, where the set Q of features is confined to the most satisfactory reduct.

The degree of dependency

DEFINITION 1.11 *(The degree of dependency) The degree of dependency $\gamma(P, Q)$ of a set P of features with respect to a set Q of class labeling is defined as:*

$$\gamma(P, Q) = \frac{| POS_P(Q) |}{| U |} \tag{1.5}$$

where $| S |$ denotes the cardinality of a set S.

The degree of dependency provides a measure of how important P is in mapping the data sets examples into Q. If $\gamma(P, Q) = 0$, then classification Q is independent of the features in P, hence the decision features are of no use to this classification. If $\gamma(P, Q) = 1$, then Q is completely dependent on P, hence the features are indispensable. Values $0 < \gamma(P, Q) < 1$ denote partial dependency, which shows that only some of the features in P may be useful, or that the data set was flawed to begin with. In addition, the complement of $\gamma(P, Q)$ gives a measure of the contradictions in the selected subset of the data set.

It is now possible to define the significance of an feature. This is done by calculating the change of dependency when removing the feature from the set of considered conditional features.

Significant of the features

Significance of features enable us to evaluate of features by assigning a real number from the closed interval [0,1], expressing how important an feature in an information table is. Significance of an feature a in a decision table DT can be evaluated by measuring the effect of removing of an feature a in C from feature set C on a positive region defined by the table DT. As shown in definition (9), the number $\gamma(C, D)$ express the degree of dependency between feature C and D or accuracy of approximation of U/D by C.. The formal definition of the significant is given as follows:

DEFINITION 1.12 *(Significance) For any feature $a \in C$, we define its significance ζ with respect to D as follows:*

$$\zeta(a, C, D) = \frac{|POS_{C \setminus \{a\}}(D)|}{|POS_C(D)|} \tag{1.6}$$

1.7-1.12 are used to express importance of particular features in building the classification model. For a comprehensive study we refer to [104]. As one of importance measures, one can use frequency of occurrence of features in reducts. Then, one can also consider various modifications of Definition 1.7, for example approximate reducts, which preserve information about decisions only to some degree [102]. Further, positive region in Definition 1.4 can be modified by allowing for approximate satisfaction of inclusion $[x]_C \subseteq [x]_D$, as proposed, e.g., in VPRS model [132]. Finally, in Definition 1.2, the meaning of $IND(B)$ and $[x]_B$ can be changed by replacing equivalence relation with similarity relation, especially useful while considering numeric features. For further reading, we refer to, e.g., [63, 66].

Decision Rules

In the context of supervised learning,an important task is the discovery of classification rules from the data provided in the decision tables. The decision rules not only capture patterns hidden in the data as they can also be used to classify new unseen objects. Rules represent dependencies in the dataset, and represent extracted knowledge which can be used when classifying new objects not in the original information system. When the reducts were found, the job of creating definite rules for the value of the decision feature of the information system was practically done. To transform a reduct into a rule, one only has to bind the condition feature values of the object class from which the reduct originated to the corresponding features of the reduct. Then, to complete the rule, a decision part comprising the resulting part of the rule is added. This is done in the same way as for the condition features. To classify objects, which has never been seen before, rules generated from a training set will be used. These rules represent the actual classifier. This classifier is used to predict to which classes new objects are attached. The nearest matching rule is determined as the one whose condition part differs from the feature vector of re-image by the minimum number of features. When there is more than one matching rule, we use a voting mechanism to choose the decision value. Every matched rule contributes votes to its decision value, which are equal to the t times number of objects matched by the rule. The votes are added and the decision with the largest number of votes is chosen as the correct class. Quality measures associated with decision rules can be used to eliminate some of the decision rules. We list below some of these quality measures such as support, strength, accuracy and coverage and other (see [65]).

1.3 Rough Image Processing

Rough image processing is the collection of all approaches and techniques that understand, represent and process the images, their segments and features as rough sets [120]. In gray scale images boundaries between object regions are often ill defined because of grayness and/or spatial ambiguities [105, 106]. This uncertainty can be handled by describing the different objects as rough sets with upper (or outer) and lower (or inner) approximations. Here the concepts of upper and lower approximation can be viewed, respectively, as outer and inner approximations of an image region in terms of granules. Pal et. al [106] defined the rough as an image as follows:

DEFINITION 1.13 *(Rough image) Let the universe U be an image consisting of a collection of pixels. Then if we partition U into a collection of non-overlapping windows of size mn, each window can be considered as a granule G. Given this granulation, object regions in the image can be approximated by rough sets.*

Rough image is a collection of pixels and the equivalence relation induced partition as pixels lying within each non-overlapping window over the image. With this definition the roughness of various transforms (or partitions) of the image can be computed using image granules for windows of different sizes.

Rough representation of a region of interest

A Region of Interest, often abbreviated *ROI*, is a selected subset of samples within an image identified for a particular purpose. For example: the boundaries of an object in 2D images and the contours or surfaces outlining an object in a volume dataset. The concept of an *ROI* is commonly used in medical imaging. For example, the boundaries of a tumor may be defined on an image or in a volume, for the purpose of measuring its size. The endocardial border may be defined on an image, perhaps during different phases of the cardiac cycle, say end-systole and end-diastole, for the purpose of assessing cardiac function.

Hirano and Tsumoto [29] introduced the rough direct representation of *ROIs* in medical images. The main advantage of this method is its ability to represent inconsistency between the knowledge-driven shape and image-driven shape of a *ROI* using rough approximations. The method consists of three steps including preprocessing. (1) derive discretized feature values that describe the characteristics of a *ROI*, (2) build up the basic regions in the image using all features, so that each region includes voxels that are indiscernible on all features, (3) according to the given knowledge about the *ROI*, they construct an ideal shape of the *ROI* and approximate it by the basic categories according to the given knowledge about the *ROI*.

They discussed how to approximate a region of interest (*ROI*) when we are given multiple types of expert knowledge. The method contains three steps including preprocessing. First, they derive discretized feature values that describe the characteristics of a ROI. Secondly, using all features, they build up the basic regions (namely categories) in the image so that each region contains voxels that are indiscernible on all features. Finally, according to the given knowledge about the *ROI*, they construct an ideal shape of the *ROI* and approximate it by the basic categories. Then the image is split into three regions: a set of voxels that are:

- (1) certainly included in the *ROI* (Positive region),
- (2) certainly excluded from the *ROI* (Negative region),
- (3) possibly included in the *ROI* (Boundary region).

The *ROI* is consequently represented by the positive region associated with some boundary regions. In the experiments we show the result of implementing a rough image segmentation system.

Shoji and Shusaku [112, 29] described the procedures for rough representation of *ROIs* under single and multiple types of classification knowledge. Usually, the constant variables defined in the prior knowledge, for example some threshold values, do not meet the exact boundary of images due to inter-

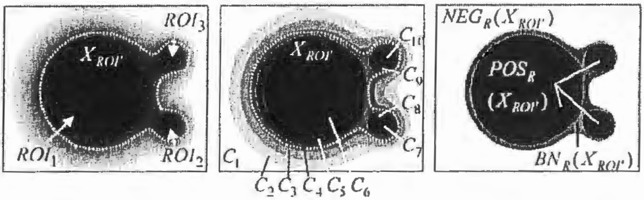

FIGURE 1.2: Rough ROI representation. Left: an original image. Middle: elementary categories C1C9. Right: roughly segmented ROI [29]

image variances of the intensity. The approach tries to roughly represent the shape of the ROI by approximating the given shapes of the ROI by the primitive regions derived from feature of the image itself. It is reported that the simplest case where we have only information about intensity range of the ROI. In this case intensity thresholding is a conventional approach to obtain the voxels that fall into the given range. Let us denote the lower and upper thresholds by ThL and ThH, respectively. Then the ROI can be represented by:

$$ROI = \{x(p) \mid Th_L \leq I(x)P \leq Th_P\} \tag{1.7}$$

where $x(p)$ denotes a voxel at location p and $I(x(p))$ denotes intensity of voxel $x(p)$.

Fig. (1.2) illustrates the concept of rough ROI representation. The left image is an original grayscale image. Suppose that the $ROIs$ are three black circular regions: ROI_1, ROI_2, and ROI_3. Also suppose that we are given a prior knowledge about the $ROIs$, that is, the lower threshold value Th_L of the $ROIs$, derived from some knowledge base. With this knowledge we can segment an ideal ROI $X_{R\hat{O}I}$ as follows:

$$X_{R\hat{O}I} = \{x(p)|Th_L \leq I(p)\} \tag{1.8}$$

However, $X_{R\hat{O}I}$ does not correctly match the expected ROIs. This is because Th_L was too small to separate the ROIs. Th_L is a global threshold determined on the other sets, therefore, it should not be directly applied to this image. Then, represent the possible boundary of the ROIs according to the low-level feature of this image, for more details see [29].

Rough Image Entropy

Entropy-based information theoretic approach has received considerable interest in image analysis recently such as image registration [34]. Previous work on entropic thresholding is based on Shannon's entropy. The idea is to calculate Shannon's entropy based on a co-occurrence matrix and use it as a criterion for selecting an appropriate threshold value. The approach using relative entropy for image thresholding has been shown very competitive compared to Pal and Pal's methods where the relative entropy is chosen to be a thresholding criterion of measuring mismatch between an image and a thresholded image. Currently there are various published approaches using relative entropy and applying to medical images, multispectral imagery, temporal image sequences, multistage thresholding and segmentation.

Pal et al. [106] presented a new definition of image entropy in a rough set theoretic framework, and its application to the problem of object extraction from images by minimizing both object and background roughness. Granules carry local information and reflect the inherent spatial relation of the image by treating pixels of a window as indiscernible or homogeneous. Maximization of homogeneity in both object and background regions during their partitioning is achieved through maximization of rough entropy; thereby providing optimum results for object background classification.

DEFINITION 1.14 *(Rough Image Entropy)[106] Rough image entropy ($R_I E$) is defined by:*

$$R_I E = -\frac{e}{2}[R_{O_T} log_e(R_{O_T}) + R_{B_T} log_e(R_{B_T})] \tag{1.9}$$

Sankar noted that the value of $R_I E$ lies between 0 and 1 and it has has a maximum value of unity when $R_{O_T} = R_{B_T} = \frac{1}{e}$, and minimum value of zero when $R_{O_T}, R_{B_T} \in \{0, 1\}$.

Fig. (1.3) shows plot of rough entropy for various values of roughness of the object and background [106].

Pal et al. [106] reported that a maximization of homogeneity in both object and background regions during their partitioning is achieved through maximization of rough entropy; thereby providing optimum results for object-background classification. Also, maximization of the rough entropy measure minimizes the uncertainty arising from vagueness of the boundary region of the object. Therefore, for a given granule size, the threshold for object-background classification can be obtained through its maximization with respect to different image partitions. The rough entropy concepts may be applicable for many application in image processing, in particulars in medical

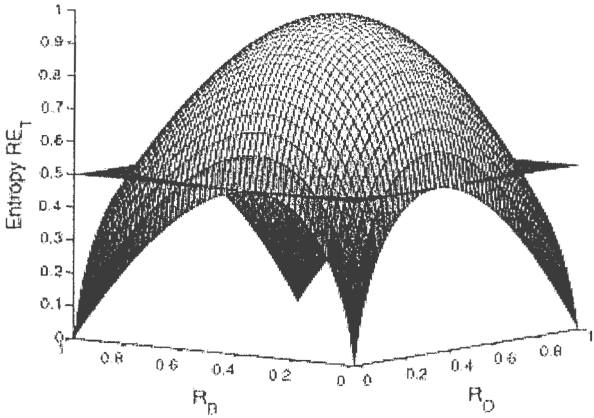

FIGURE 1.3: Rough entropy for various values [106]

imaging problems such as feature extraction and medical image segmentation problems for example; feature extraction in mammogram images and identification of lymphomas by finding follicles in microscopy images.

Rough Sets for Object Extraction

Identification of anatomical features is a necessary step for medical image analysis. Automatic methods for feature identification using conventional pattern recognition techniques typically classify an object as a member of a predefined class of objects, but do not attempt to recover the exact or approximate shape of that object. For this reason, such techniques are usually not sufficient to identify the borders of organs when individual geometry varies in local detail, even though the general geometrical shape is similar.

Pal et al.[106] demonstrated a new application of rough sets for object extraction from gray scale image. In gray scale images boundaries between object regions are often ill-defined. This uncertainty can be handled by describing the different objects as rough sets with upper (outer) and lower (inner) approximations. The set approximation capability of rough sets is exploited in the present investigation to formulate an entropy measure, called rough entropy, quantifying the uncertainty in an object-background image. They viewed the object and background as two sets with their rough representation by computing the inner approximation of the object ($\underline{Q}_T$), outer approximation of the object ($\overline{Q}_T$),inner approximation of the background ($\underline{B}_T$) and outer approximation of the background ($\overline{B}_T$) as follows:

$$\underline{Q}_T = \bigcup G_i | p_j > T, \forall j = 1, ..., mn \tag{1.10}$$

$$\overline{Q}_T = \bigcup G_i, \exists j, p_j > T, j = 1, .., mn \tag{1.11}$$

$$\underline{B}_T = \bigcup G_i | P_j > T, \forall j = 1, ..., mn \tag{1.12}$$

$$\overline{B}_T = \bigcup G_i, \exists j, p_j \leq T, j = 1, .., mn \tag{1.13}$$

Where p_j is a pixel in G_i. The rough set representation of the image for a given I_{mn} depends on the value of T.

Pal et.al [106] define the roughness (R) of the object O_T and the backgroung B_T as follows:

$$R_{O_T} = 1 - \frac{|\underline{Q}_T|}{|\overline{Q}_T|} \tag{1.14}$$

$$R_{B_T} = 1 - \frac{\underline{B}_T}{\overline{B}_T} \tag{1.15}$$

Where $|S|$ is the cardinality of the set.

The presented method may be applicable for many application in image processing, in particulars in medical imaging problems such as automatically identify the myocardial contours of the heart,segmentation of knee tissues in CT image and segmentation of brain tissues in MR image, etc.

Rough Sets in Medical Image Segmentation and Clustering

Image segmentation is one of the most critical steps toward image analysis and understanding and therefore it has been the subject of considerable research activity over the last four decades. During all this time we have witnessed a tremendous development of new, powerful instruments for detecting, storing, transmitting, and displaying images but automatic segmentation still remained a challenging problem. This fact is easy to notice in medical applications, where image segmentation is particularly difficult due to restrictions imposed by image acquisition, pathology and biological variation. Biomedical image segmentation is a sufficiently complex problem that no single strategy has proven to be completely effective. Due to a complex nature of biomedical images, it is practically impossible to select or develop automatic segmentation methods of generic nature, that could be applied for any type of these images, e.g. for either micro- and macroscopic images, cytological and histological ones, MRI and X-ray, and so on. Medical image segmentation is an indispensable process in the visualization of human tissues. However, medical images always contain a large amount of noise caused by operator performance, equipment and environment. This leads to inaccuracy with segmentation. A robust segmentation technique is required.

The basic idea behind segmentation-based rough sets is that while some cases may be clearly labelled as being in a set X called positive region in rough sets theory), and some cases may be clearly labelled as not being in set X called negative region in rough sets theory, limited information prevents us from labelling all possible cases clearly. The remaining cases cannot be distinguished and lie in what is known as the boundary region. A little bit effort has been done in uses the rough sets in image segmentation and in particulars in medical segmentation problems.

Peters et al. [86] presented a new form of indiscernibility relation based on K-means clustering of pixel values. The end result is a partitioning of a set of pixel values into bins that represent equivalence classes. The proposed approach makes it possible to introduce a form of upper and lower approximation specialized relative to sets of pixel values. This approach is particularly relevant to a special class of digital images for power line ceramic insulators. Until now the problem of determining when a ceramic insulator needs to be replaced has relied on visual inspection. With the K-means indiscernibility relation, it is now possible to automate the detection of faulty ceramic insulators. The contribution of this article is the introduction of an approach to classifying power line insulators based on a rough set methods and K-means clustering in analyzing digital images. The introduced form of indiscernibility relation based on K-means clustering of pixel values is very interest in many medical application.

Among many difficulties in segmenting MRI data, the partial volume effect (PVE) arises in volumetric images when more than one tissue type occurs in a voxel. In such cases, the voxel intensity depends not only on the imaging sequence and tissue properties, but also on the proportions of each tissue type present in the voxel. Widz Sebastian et. al. [122, 123] discussed the partial volume effect problem in the segmentation of magnetic resonance imaging data that entails assigning tissue class labels to voxels. They employ the rough sets to identify automatically the partial volume effect, which occurs most often with low resolution imaging-with large voxels.

Kobashi et al. [40, 50] introduced rough sets treat nominal data based on concepts of categorization and approximation, into medical image segmentation. The proposed clustering method extracts features of each pixel by using thresholding and labeling algorithms. Thus, the features are given by nominal data. The ability of the proposed method was evaluated by applying it to human brain MRI images.

An interesting strategy for color image segmentation using rough set theory has been presented by Mohabey et.al. [58]. A new concept of encrustation of the histogram, called histon, has been proposed for the visualization of multi-dimensional color information in on integrated fashion and its applicability in boundary region analysis has been shown. The histon correlates with the upper approximation of a set such that all elements belonging to this set are clarified as possibly belonging to the same segment or segments showing similar color value. The proposed encrustation provides a direct means of segregating pool of inhomogeneous regions into its components. Experimental results for various images have been presented in their work. They extended their work in [59] and introduced a hybrid rough set theoretic approximations and fuzzy C-means algorithm for color image segmentation. They segmented natural images with regions having gradual variations in color value. The technique extracts color information regarding the number of segments and the segments center values from the image itself through rough set theoretic approximations and presented it as input to Fuzzy C-mean block for the soft evaluation of the segments. The performance of the algorithm has been evaluated on various natural and simulated images.

Lots of clustering algorithms [44] have been developed and applied in medical imaging problems, while most of them cannot process objects in hybrid numerical/nominal feature space or with missing values. In most of them, the number of clusters should be manually determined and the clustering results are sensitive to the input order of the objects to be clustered. These limit applicability of the clustering and reduce the quality of clustering. To solve this problem, an improved clustering algorithm based on rough set (RS) and entropy theory was presented by Chun-Bao et. al. [10] it aims at avoiding the need to prespecify the number of clusters, and clustering in both numerical and nominal feature space with the similarity introduced to replace the distance index. At the same time, the RS theory endows the algorithm with the function to deal with vagueness and uncertainty in data analysis. Shannon's entropy was used to refine the clustering results by assigning relative weights to the set of features according to the mutual entropy values. A novel measure of clustering quality was also presented to evaluate the clusters. The experimental results confirm that performances of efficiency and clustering quality of this algorithm are improved.

Widz et al. [122, 123] introduced an automated multi-spectral MRI segmentation technique based on approximate reducts derived from the theory of rough sets. They utilized the $T1$, $T2$ and PD MRI images from the simulated Brain Database as a gold standard to train and testing their segmentation algorithm. The results suggest that approximate reducts, used alone or in combination with other classification methods, may provide a novel and efficient approach to the segmentation of volumetric MRI data sets. Segmentation accuracy reaches 96for the highest resolution images and 89% for the noisiest image volume. They tested the resultant classifier on real clinical data, which yielded an accuracy of approximately 84%.

Adaptation of C-means to Rough Set Theory

C-means clustering is an iterative technique that is used to partition an image into $C-$ clusters. Fuzzy C-Means (FCM) is one of the most commonly used fuzzy clustering techniques for different degree estimation problems, especially in medical image processing [4, 2]. Lingras [47] described modifications of clustering based on Genetic Algorithms, K-means algorithm, and Kohonen Self-Organizing Maps (SOM). These modifications make it possible to represent clusters as rough sets. In their work, Lingers established a rough k-means framework and extended the concept of c-means by viewing each cluster as an interval or rough set [45]. Here is a brief summary of his pioneer clustering work.

K-means clustering is one of the most popular statistical clustering techniques used in segmentation of medical images [127, 121, 21, 60, 86, 9]. The name K-means originates from the means of the k clusters that are created from n objects. Let us assume that the objects are represented by m-dimensional vectors. The objective is to assign these n objects to k clusters. Each of the clusters is also represented by an m-dimensional vector, which is the centroid or mean vector for that cluster. The process begins by randomly choosing k objects as the centroids of the k clusters. The objects are assigned to one of the k clusters based on the minimum value of the distance $d(v, x)$ between the object vector $v = (v_1, ..., v_j, ..., v_m)$ and the cluster vector $x = (x_1, ..., x_j, ..., x_m)$. After the assignment of all the objects to various clusters, the new centroid vectors of the clusters are calculated as:

$$x_j = \frac{\sum_{v \in x} v_j}{SOC}, where 1 \leq j \leq m. \tag{1.16}$$

Where SOC is the size of cluster x.

Lingers [47] mentioned that incorporate rough sets into K-means clustering requires the addition of the concept of lower and upper bounds. Calculation of the centroids of clusters from conventional K-Means needs to be modified to include the effects of lower as well as upper bounds. The modified centroid calculations for rough sets are then given by:

$$cen_j = W_{low} \times \frac{\sum_{v \in \underline{R}(x)}}{|\underline{R(x)}|} + w_{up} \times \frac{\sum_{v \in (BN_R(x))}}{|\underline{BN_R(x)}|} \tag{1.17}$$

Where $1 \leq j \leq m$. The parameters w_{lower} and $w_(upper)$ correspond to the relative importance of lower and upper bounds, and $w_{low} + w_{up} = 1$. If the upper bound of each cluster were equal to its lower bound, the clusters would be conventional clusters. Therefore, the boundary region $BN_R(x)$ will be empty, and the second term in the equation will be ignored. Thus, the above equation will reduce to conventional centroid calculations. The next step in the modification of the K means algorithms for rough sets is to design criteria to determine whether an object belongs to the upper or lower bound of a cluster, for more details refer to.

Mitra [53] proposed an evolutionary rough c-means clustering algorithm. Genetic algorithms are employed to tune the threshold, and relative importance of upper and lower approximations of the rough sets modeling the clusters. The DaviesBouldin clustering validity index is used as the fitness function, that is minimized while arriving at an optimal partitioning. A comparative study of its performance is made with related partitive algorithms. The effectiveness of the algorithm is demonstrated on real and synthetic datasets, including microarray gene expression data from Bioinformatics. In the same study, the author noted that the parameter threshold measures the relative distance of an object X_k from a pair of clusters having centroids cen_imi and cen_j. The smaller the value of threshold, the more likely is X_k to lie within the rough boundary (between upper and lower approximations) of a cluster. This implies that only those points which definitely belong to a cluster (lie close to the centroid) occur within the lower approximation. A large value of threshold implies a relaxation of this criterion, such that more patterns are allowed to belong to any of the lower approximations. The parameter w_{low} controls the importance of the objects lying within the lower approximation of a cluster in determining its centroid. A lower w_{low} implies a higher w_{up}, and hence an increased importance of patterns located in the rough boundary of a cluster towards the positioning of its centroid.

The Rough $C-K-$ mean is a new challenge for use in MR and Mammogram image segmentation as an aid to small lesion diagnosis and can effectively remove the influence of tiny details and noise.

1.4 Rough Sets in feature reduction and image classification

In image processing, raw images represented by the gray levels of the pixels are usually transformed to features which can better capture the characteristics of the images in the preprocessing phase. Texture features are such features that are often used in image classification and segmentation. In particular, texture features proposed by Haralick [2] are typically computed from the gray-level co-occurrence matrices, and then used to classify each pixel for its type. Feature selection is an important step in the pre-processing, since there are numerous potential features, some of which might be irrelevant or unimportant. Not only can reducing features speed up the processing time and possibly improve the classification accuracy, it also allows us to use classification methods which are not good at processing high dimensional data, such as neural networks and support vector machines. Feature selection aims to determine a minimal feature subset from a problem domain while retaining a suitably high accuracy in representing the original features. Rough set theory (RST) enables the discovery of data dependencies and the reduction of the number of features (features) contained in a dataset or extracted from images using the data alone, requiring no additional information. (see [75])

The computation of the core and reducts from a rough set decision table is a way of selecting relevant features [102, 15, 19, 23, 103]. It is a global method in the sense that the resultant reducts represent the minimal sets of features which are necessary to maintain the same classification power given by the original and complete set of features. A straighter manner for selecting relevant features is to assign a measure of relevance to each feature and choose the features with higher values. Based on the reduct system, we can generate the list of rules that will be used for building the classifier model for the new objects. Reduct is an important concept in rough set theory and data reduction is a main application of rough set theory in pattern recognition and data mining. As it has been proven that finding the minimal reduct of an information system is a NP hard problem.

Since rule induction methods generate rules whose lengths are the shortest for discrimination between given classes, they tend to generate rules too short for medical experts. Thus, these rules are difficult for the experts to interpret from the viewpoint of domain knowledge. Shusaku Tsumoto [113] introduced the characteristics of experts' rules are closely examined and proposed a new approach to generate diagnostic rules using rough sets and medical diagnostic model. The proposed approach focused on the hierarchical structure of differential diagnosis and consists of three procedures; (1) the characterization of decision features (given classes) is extracted from databases and the classes are classified into several generalized groups with respect to the characterization, (2) two kinds of sub-rules, classification rules for each generalized group and rules for each class within each group are induced, and (3) those two parts are integrated into one rule for each decision feature. His proposed approach was evaluated on a medical database, the experimental results of which show that induced rules correctly represent experts' decision processes.

Many researchers have endeavored to develop efficient algorithms to compute useful feature extraction and reduction of information systems and besides mutual information and discernibility matrix based feature reduction methods. These techniques have been successfully applied to medical domain [111, 130].

Zbigniew Wojcika [133] approached the nature of a feature recognition process through the description of image features in terms of the rough sets. Since the basic condition for representing images must be satisfied by any recognition result, elementary features are defined as equivalence classes of possible occurrences of specific fragments existing in images. The names of the equivalence classes (defined through specific numbers of objects and numbers of background parts covered by a window) constitute the best lower approximation of window contents (i.e., names of recognized features). The best upper approximation is formed by the best lower approximation, its features, and parameters, all referenced to the object fragments situated in the window. The rough approximation of shapes is resistant to accidental changes in the width of contours and lines and to small discontinuities and, in general, to possible positions or changes in shape of the same feature. The rough sets are utilized also on the level of image processing for noiseless image quantization. This initiative study is very interest in many area of medical image processing including filtering, segmentation and classification.

Swiniarski and Skowron [117] presented applications of rough set methods for feature selection in pattern recognition. They emphasize the role of the basic constructs of rough set approach in feature selection, namely reducts and their approximations, including dynamic reducts. They algorithm for feature selection is based on an application of a rough set method to the result of principal components analysis (PCA) used for feature projection and reduction. The paper presents many experiments including face and mammogram recognition experiments. In their study, 144 mammogram images has been selected for recognition experiments. The MIAS MiniMammographic Database with 1024 x 1024 pixels images has been used [56] The database contains three types of class-labeled images: normal, benign (abnormal), and malignant (abnormal). For each abnormal image the coordinates of centre of abnormality and proximate radius (in pixels) of a circle enclosing the abnormality, have been given. For classifications the centre locations and radii apply to clusters rather than to the individual classifications. They have provided an experiment of recognition of normal and abnormal images (two category classification). This set was divided into 128 case training set and 16 case test set. From the original 1024 pixel gray scale mammographic image, they have extracted a 64 x 64 pixels sub-image around the center of abnormality (or at the average coordinate for normal cases. They concluded that the rough set methods have shown ability to reduce significantly the pattern dimensionality and have proven to be viable image mining techniques as a front end of neural network classifiers.

Wang [18, 19] studied the relationship of the definitions of rough reduction in algebra view and information view. Some relationships such as inclusion relationship under some conditions and equivalence relationship under some other conditions are presented. The inclusion relationship between the feature importance defined in algebra view and information view is presented also. For example, the reduction under algebra view will be equivalent to the reduction under information view if the decision table is consistent. Otherwise, the reduction under information view will include the reduction under algebra view. These results will be useful for designing further heuristic reduction algorithms.

Qinghua Hu et.al.[20] proposed an information measure to computing discernibility power of a crisp equivalence relation or a fuzzy one, which is the key concept in classical rough set model and fuzzy-rough set model. Based on the information measure, a general definition of significance of nominal, numeric and fuzzy features is presented. They redefine the independence of hybrid feature subset, reduct, and relative reduct. Then two greedy reduction algorithms for unsupervised and supervised data dimensionality reduction based on the proposed information measure are constructed. It is reported that the reducts founded by the proposed algorithms get a better performance compared with classical rough set approaches.

Lymphoma is a broad term encompassing a variety of cancers of the lymphatic system. Lymphoma is differentiated by the type of cell that multiplies and how the cancer presents itself. It is very important to get an exact diagnosis regarding lymphoma and to determine the treatments that will be most effective for the patient's condition. Milan et. al [57] focused on the identification of lymphomas by finding follicles in microscopy images provided by the Laboratory of Pathology in the University Hospital of Tenerife, Spain. The study contains two stages: in the first stage they did image pre-processing and feature extraction, and in the second stage they used different rough set approaches for pixel classification. These results were compared to decision tree results. The results they got are very promising and show that symbolic approaches can be successful in medical image analysis applications. Pham et. al [97] presented a rough set based medical decision support systems for TNM (tumor characteristics, lymph node involvement, and distant metastatic lesions) classification aiming to divide cancer patients to low and high risk patients. In addition, the system also explained the decision in the form of IF-THEN rules and in this manner performed data mining and new knowledge discovery. The introduced system was reported to have the best classification performance, robust and not dependent on the database size and the noise. The accuracy was almost 80% which is comparable with the accuracy of physicians and much better then obtained with more conventional discriminant analysis (62% and 67%).

Microcalcification on x-ray mammogram is a significant mark for early detection of breast cancer. Texture analysis methods can be applied to detect clustered microcalcification in digitized mammograms. In order to improve the predictive accuracy of the classifier, the original number of feature set is reduced into smaller set using feature reduction techniques. Thangavel, et.al [119] introduced rough set based reduction algorithms such as Decision Relative Discernibility based reduction, Heuristic approach, Hus algorithm, Quick Reduct (QR), and Variable Precision Rough Set (VPRS) to reduce the extracted features. The performance of all the introduced algorithms is compared. The Gray Level Co-occurrence Matrix (GLCM) is generated for each mammogram to extract the Haralick features as feature set. The rough reduction algorithms are tested on 161 pairs of digitized mammograms from Mammography Image Analysis Society (MIAS) database [56].

Krzysztof et. al [37] showed how rough sets can be applied to improve the classification ability of a hybrid pattern recognition system. The system presented consists of a feature extractor based on a computer-generated hologram (CGH). Features extracted are shift, rotation, and scale invariant and although they can be optimized. This article presented an original method of optimizing the feature extraction abilities of a CGH. The method uses rough set theory (RST) to measure the amount of essential information contained in the feature vector. This measure is used to define an objective function in the optimization process. Since RST-based factors are not differentiable, they use a nongradient approach for a search in the space of possible solutions. Finally, RST is used to determine decision rules for the classification of feature vectors. The proposed method is illustrated by a system recognizing the class of speckle pattern images indicating the class of distortion of optical fibers.

Jiang et al. [35] developed a joining associative classifier (JAC)algorithm using the rough set theory to mining digital mammography. The experimental results showed that the joining associative classifier performance at 77.48% of classifying accuracy which is higher than 69.11% using associative classifier only. At the same time, the number of rules decreased distinctively.

Krzysztof et al. [33] discussed a process of analysing medical diagnostic data by means of the combined rule induction and rough set approach. The first step of this analysis includes the use of various techniques for discretization of numerical features. Rough sets theory is applied to determine feature importance for the patients' classification. The novel contribution concerns considering two different algorithms inducing either minimum or satisfactory set of decision rules. Verification of classification abilities of these rule sets is extended by an examination of sensitivity and specificity measures. Moreover, a comparative study of these composed approaches against other learning systems is discussed. The approach is illustrated on a medical problem concerning anterior cruciate ligament (ACL) rupture in a knee. The patients are described by features coming from anamnesis, MR examinations and verified by arthroscopy. It i s reported that the clinical impact of this research is indicating two features (PCL index, age) and their specific values that could support a physician in resigning from performing arthroscopy for some patients.

1.5 Joint Rough Sets with Other Intelligent Approach

Intelligent systems comprise various paradigms dedicated to approximately solving real-world problems, e.g., in decision making, classification or learning; among these paradigms are fuzzy sets, neural networks, decision tree, and rough sets, algorithms. Combination of kinds of computational intelligence techniques in application area of pattern recognition and in particulars in medical imaging problems has become one of the most important ways of research of intelligent information processing [109].

Neural Networks with Rough sets

Neural network shows us its strong ability to solve complex problems for medical image processing. But neural network can't tell the redundant information from huge amount of data, which will easily lead to some problems such as too complex network structure, long training time, low converging speed and much computation. Focusing on these problems. Many successful work towered this issue has been addressed and discussed. For example, Hassanien and Selzak [26] introduced a rough neural approach for rule generation and image classification. Hybridization of intelligent computing techniques has been applied to see their ability and accuracy to classify breast cancer images into two outcomes: malignant cancer or benign cancer. Algorithms based on fuzzy image processing are first applied to enhance the contrast of the whole original image; to extract the region of interest and to enhance the edges surrounding that region. Then, they extract features characterizing the underlying texture of the regions of interest by using the gray-level co-occurrence matrix. Then, the rough set approach to feature reduction and rule generation is presented. Finally, rough neural network is designed for discrimination of different regions of interest to test whether they represent malignant cancer or benign cancer. Rough neural network is built from rough neurons, each of which can be viewed as a pair of sub-neurons, corresponding to the lower and upper bounds. To evaluate performance of the presented rough neural approach, they run tests over different mammogram images. In their experiments, results show that the overall classification accuracy offered by rough neural approach is high compared with other intelligent techniques.

The introduced rough neural networks [43, 88, 89] used in their study, consist of one input layer, one output layer and one hidden layer. The input layer neurons accept input from the external environment. The outputs from input layer neurons are fed to the hidden layer neurons. The hidden layer neurons feed their output to the output layer neurons which send their output to the external environment.

The number of hidden neurons is determined by the following inequality [11, 31].

$$N_{hn} \leq \frac{N_{ts} * T_e * N_f}{N_f + N_o} \tag{1.18}$$

N_{hn} is the number of hidden neurons, N_{ts} is the number of training samples, T_e is the tolerance error, N_f is the number of features, and N_o is the number of the output.

The output of a rough neuron is a pair of upper and lower bounds, while the output of a conventional neuron is a single value. Rough neuron was introduced in 1996 by Lingras [43]. It was defined relative to upper bound (U_n), lower bound (L_n), and inputs were assessed relative to boundary values. Rough neuron has three types of connections:

Step 1. Input-Output connection to U_n

Step 2. Input-Output connection to L_n

Step 3. Connection between U_n and L_n

DEFINITION 1.15 *(Rough neuron) A rough neuron R_n is a pair of usual rough neurons $R_n = (U_n, L_n)$, where U_n and L_n are the upper rough neuron and the lower rough neuron, respectively.*

Let (Ir_{L_n}, Or_{L_n}) be the input/output of a lower rough neuron and (Ir_{U_n}, Or_{U_n}) be the input/output of an upper rough neuron. Calculation of the input/output of the lower/upper rough neurons is given as follows:

$$Ir_{L_n} = \sum_{j=1}^{n} w_{L_{nj}} On_j \tag{1.19}$$

$$Ir_{U_n} = \sum_{j=1}^{n} w_{U_{nj}} On_j \tag{1.20}$$

$$Or_{L_n} = min(f(Ir_{L_n}), f(Ir_{U_n})) \tag{1.21}$$

$$Or_{U_n} = max(f(Ir_{L_n}), f(Ir_{U_n})) \tag{1.22}$$

The output of the rough neuron (O_{rn}) will be computed as follows:

$$O_{rn} = \frac{Or_{U_n} - Or_{L_n}}{avarge(Or_{U_n}, Or_{L_n})} \tag{1.23}$$

Algorithm 1 Rule Generation

Input: Decision system (U, C, D)
Decision reduct $R \subseteq C$; $R = \{a_1, ...a_m\}$; $m = |R|$
Output: The set of decision rules $RULES(R)$ generated for R

1: **for** $u \in U$ **do**
2: **for** $a_i \in R$ **do**
3: $v_i = a_i(u)$;
4: **end for**
5: $v_d = d(u)$;
6: $RULES(R) = RULES(R) \cup \{a_1 = v_1 \wedge ... \wedge a_m = v_m \rightarrow d = v_d\}$;
7: **end for**
8: Return $RULES(R)$;

In their experiments,the segmentation performance is measured by the value of accuracy as defined below and the average of segmentation accuracy achieved by the reported algorithm is 97%, which means that it is robust enough.

$$S_A = \frac{M_P}{T_{NP}} \tag{1.24}$$

Where S_A, M_P and T_{NP} are the segmentation accuracy, number of misclassified pixel and total number of pixel, respectively.

The rule importance measure R_I were used as an evaluation to study the quality of the generated rule. It is defined by:

$$R_I = \frac{\tau_r}{\rho_r} \tag{1.25}$$

Where τ_r is the number of times a rule appears in all reduct and ρ_r is the number of reduct sets. The quality of rules is related to the corresponding reduct(s) They are generating rules which cover largest parts of the universe U. Covering U with more general rules implies smaller size of a rule set. They use the importance rule criteria introduced in [42] to study the rules' importance (cf. [3]).

Swiniarski and Hargis [116] described an application of rough sets method to feature selection and reduction as a front end of neural-network-based texture images recognition. The methods applied include singular-value decomposition (SVD) for feature extraction, principal components analysis (PCA) for feature projection and reduction, and rough sets methods for feature selection and reduction. For texture classification the feedforward backpropagation neural networks were applied. The numerical experiments showed the ability of rough sets to select reduced set of pattern's features, while providing better generalization of neural-network texture classifiers, see also [111].

Jiang et al. [36] proposed rough neural network that integrates neural network with reduction of rough set theory to classify digital mammography. It is reported that such combined method performed better than neural network alone in terms of complexity time, and it can get 92.37% classifying accuracy which is higher than 81.25% using neural network only.

Fuzzy with Rough sets

Mao et al. [12] proposed a new fuzzy Hopfield-model net based on rough-set reasoning for the classification of multispectral images. The main purpose is to embed a rough-set learning scheme into the fuzzy Hopfield network to construct a classification system called a rough-fuzzy Hopfield net (RFHN). The classification system is a paradigm for the implementation of fuzzy logic and rough systems in neural network architecture. Instead of all the information in the image being fed into the neural network, the upper- and lower-bound gray levels, captured from a training vector in a multispectal image, are fed into a rough-fuzzy neuron in the RFHN. Therefore, only 2/N pixels are selected as the training samples if an N-dimensional multispectral image was used. In the simulation results, the proposed network not only reduces the consuming time but also reserves the classification performance.

Wang et al. [125] proposed a new nearest neighbor clustering classification algorithm based on fuzzy-rough set theory (FRNNC). First, they make every training sample fuzzy-roughness and use edit nearest neighbor algorithm to remove training sample points in class boundary or overlapping regions, and then use Mountain Clustering method to select representative cluster center points, then Fuzzy-Rough Nearest neighbor algorithm (FRNN) is applied to classify the test data. The new algorithm is applied to hand gesture image recognition, the results show that it is more effective and performs better than other nearest neighbor methods.The introduced algorithm is recommended to use in any type of medical image application. such as medical images sequences or finding similar tumor shapes.

Hassanien [22] introduced a hybrid scheme that combines the advantages of fuzzy sets and rough sets in conjunction with statistical feature extraction techniques. An application of breast cancer imaging has been chosen and hybridization scheme have been applied to see their ability and accuracy to classify the breast cancer images into two outcomes: cancer or non-cancer. The introduced scheme starts with fuzzy image processing as pre-processing techniques to enhance the contrast of the whole image; to extracts the region of interest and then to enhance the edges surrounding the region of interest. A subsequently extract features from the segmented regions of the interested regions using the gray-level co-occurrence matrix is presented. Rough sets approach for generation of all reducts that contains minimal number of features and rules is introduced. Finally, these rules can then be passed to a classifier for discrimination for different regions of interest to test whether they are cancer or non-cancer. To measure the similarity, a new rough set distance function is presented. The experimental results showed that the hybrid scheme applied in this study perform well reaching over 98% in overall accuracy with minimal number of generated rules.

Image clustering analysis is one of the core techniques for image indexing, classification, identification and segmentation for medical image processing. Mitra et. al. [55] introduced a hybrid clustering architecture, in which several subsets of patterns can be processed together with an objective of finding a common structure. A detailed clustering algorithm is developed by integrating the advantages of both fuzzy sets and rough sets, and a measure of quantitative analysis of the experimental results is provided for synthetic and real-world data. The structure revealed at the global level is determined by exchanging prototypes of the subsets of data and by moving prototypes of the corresponding clusters toward each other. Thereby, the required communication links are established at the level of cluster prototypes and partition matrices, without hampering the security concerns.

Petrosino et al. [96] presented a multi-scale method based on the hybrid notion of rough fuzzy sets, coming from the combination of two models of uncertainty like vagueness by handling rough sets and coarseness by handling fuzzy sets. Marrying both notions lead to consider, as instance, approximation of sets by means of similarity relations or fuzzy partitions. The most important features are extracted from the scale spaces by unsupervised cluster analysis, to successfully tackle image processing tasks. Here, we report some results achieved by applying the method to multi-class image segmentation and edge detection, but it can be shown to be successfully applied to texture discrimination problem too. Mitra et al. [55] and Petrosino et al. [96] approaches can be applied in many medical imaging clustering problems such as image segmentation in abdomen medical images and cluster filter bank response vectors to obtain a compact representation of the image structures found within an image quality verification of color retina images in diabetic retinopathy screening, for example.

Genetic Algorithm with Rough sets

Lingras [46] proposed an unsupervised rough set classification using Genetic algorithm and how genetic algorithms can be used to develop rough sets. The proposed rough set theoretic genetic encoding will be especially useful in unsupervised learning. A rough set genome consists of upper and lower bounds for sets in a partition. The partition may be as simple as the conventional expert class and its complement or a more general classification scheme. Lingers provides a complete description of design and implementation of rough set genomes. The introduced was reported to have the best classification performance which will be usefulness in classification and segmentation of medical images.

Mitra et al. [54] described a way of designing a hybrid system for detecting the different stages of cervical cancer. Hybridisation includes the evolution of knowledge-based subnetwork modules with GAs using rough set theory and the ID3 algorithm. Crude subnetworks for each module are initially obtained via rough set theory and the ID3 algorithm. These subnetworks are then combined, and the final network is evolved using genetic algorithms. The evolution uses a restricted mutation operator which utilises the knowledge of the modular structure, already generated, for faster convergence. The GA tunes the network weights and structure simultaneously. The aforesaid integration enhances the performance in terms of classification score, network size and training time, as compared to the conventional MLP. This methodology also helps in imposing a structure on the weights, which results in a network more suitable for rule extraction.

Bayesian and Particle Swarm Optimization with Rough Sets

In many applications in computer vision and signal processing, it is necessary to assimilate data from multiple sources. This is a particularly important issue in medical imaging, where information on a patient may be available from a number of different modalities. The original Rough Set model is concerned primarily with algebraic properties of approximately defined sets. The Variable Precision Rough Set (VPRS) model extends the basic rough set theory to incorporate probabilistic information. As a result, there has been much recent research interest in this area. Many successful work towered this issue has been addressed and discussed. For example, Slezak et al. [13] presented a non-parametric modification of the Variable Precision Rough Set (VPRS) model called the Bayesian Rough Set (BRS) model, where the set approximations are defined by using the prior probability as a reference. Mathematical properties of BRS are investigated. It is shown that the quality of BRS models can be evaluated using probabilistic gain function, which is suitable for identification and elimination of redundant features. This is a promising algorithm for generates a segmented classification concurrently with improving reconstructions of a set of registered medical images or fusing computed tomography (CT) and single photon emission computed tomography (SPECT) brain scans.

Swiniarski [115] described an application of rough sets and Bayesian inference to a breast cancer detection using electro-potentials. The statistical principal component analysis and the rough sets methods were applied for feature extraction, reduction and selection. The quadratic discriminant was applied as a classifier for a breast cancer detection.

Das et al. [100] presented a framework to hybridize the rough set theory with a famous swarm intelligence algorithm known as Particle Swarm Optimization (PSO). The hybrid rough-PSO technique has been used for grouping the pixels of an image in its intensity space. Medical images become corrupted with noise very often. Fast and efficient segmentation of such noisy images (which is essential for their further interpretation in many cases) has remained a challenging problem for years. In there work, they treat image segmentation as a clustering problem. Each cluster is modelled with a rough set. PSO is employed to tune the threshold and relative importance of upper and lower approximations of the rough sets. Davies-Bouldin clustering validity index is used as the fitness function, which is minimized while arriving at an optimal partitioning.

Support vector machines with Rough sets

Support Vector Machines (SVMs) are a general algorithm based on guaranteed risk bounds of statistical learning theory. They have found numerous applications in image processing and pattern recognition and, in particulars in medical imaging problems such as in classification of brain PET images, detection of microcalcification (MC) clusters in digital mammograms, lung cancer Nodules extraction and classification, etc., and are now established as one of the standard computational intelligence tools. Support vector machines (SVMs) have good classification performances and good capabilities of fault-tolerance and generalization. To inherit the merits of both RST and SVMs, a hybrid classifier called rough set support vector machines (RS-SVMs) is proposed by Gexiang et al. [16] to recognize radar emitter signals. RST is used as preprocessing step to improve the performances of SVMs. A large number of experimental results showed that RS-SVMs achieve lower recognition error rates than SVMs and RS-SVMs have stronger capabilities of classification and generalization than SVMs, especially when the number of training samples is small. RS-SVMs are superior to SVMs greatly.

Support vector machines (SVMs) are essentially binary classifiers. To improve their applicability, several methods have been suggested for extending SVMs for multi-classification, including one-versus-one (1-v-1), one-versus-rest (1-v-r) and Decision Directed Acyclic Graph Support Vector Machines (DDAGDAGSVM). Lingras and Butz [48]described how binary classification with SVMs can be interpreted using rough sets and how rough set theory may help in reducing the storage requirements of the 1-v-1 approach in the operational phase. The introduced rough set approach to SVM classification removes the necessity of exact classification and is especially useful when dealing with noisy data. Next, by utilizing the boundary region in rough sets, they suggested two new approaches, extensions of (1-v-r) and (1-v-1), to SVM multi-classification that allow for an error rate. They explicitly demonstrate how their extended 1-v-r may shorten the training time of the conventional (1-v-r) approach. In addition, they showed that our (1-v-1) approach may have reduced storage requirements compared to the conventional (1-v-1) and DAGSVM techniques. Their techniques provided better semantic interpretations of the classification process. The theoretical conclusions are supported by experimental findings involving a synthetic dataset. The presented work is useful for soft margin classifiers in solving medical imaging problems especially a multi-class classification system for medical images [6].

1.6 Other Application with Rough Sets in Medical Imagining

In addition to the areas mentioned above, Rough sets has also been applied to other relevant areas such as medical image retrieval, knowledge discovery and data mining.

Content-based medical image retrieval

Content-based image retrieval (CBIR) consists of retrieving the most visually similar images to a given query image from a database of images. CBIR from medical image databases does not aim to replace the physician by predicting the disease of a particular case but to assist him/her in diagnosis. The visual characteristics of a disease carry diagnostic information and oftentimes visually similar images correspond to the same disease category. By consulting the output of a CBIR system, the physician can gain more confidence in his/her decision or even consider other possibilities.

Zhao et al. [14, 15] proposed an interactive image retrieval system that uses a novel relevance feedback method called group-based relevance feedback. In the proposed system, the user divides the relevant images into multiple groups according to his/her perspective. Using each user's perspective, the retrieval intention of the user will be captured more accurately than is possible by conventional methods, which use only images for retrieval. Moreover, the retrieval results are shown as grouped images, which facilitates the understanding of the user as to why such results were produced by the system. In order to implement the proposed system, they introduced an efficient learning method that uses the Reduct from the Rough Set Theory to learn the retrieval intention of the user.

Wang [129] proposed a relevance feedback mechanism that can express objectively human perception by using rough set theory in retrieval system. The mechanism makes full use of the inherent advantages of rough set to solve the difficulty that the retrieval system cannot express human perception. Wang developed rough set-based image retrieval system called Basestar to illustrate the retrieval performance.

Li et al. [51] developed a retrieval method of rough set-based low-level features selection and feedback-based semantic-level features annotation. Their experimental results showed that the introduced method is more user-adaptive, and can achieve better performance compared with another retrieval method which is only based on low-level features.

Rough sets in medical data mining

With increasing data in medical databases, medical data mining is growing in popularity. Some of these analysis including inducing propositional rules from databases using rough sets, then using these rules in an expert system. Shusaku Tsumoto [114, 113] presented a knowledge discovery system based on rough sets and feature-oriented generalization and its application to medicine. Diagnostic rules and information on features are extracted from clinical databases on diseases of congenital anomaly. In his results, he showed that the proposed method extracts experts' knowledge correctly and it also discovers that symptoms observed in six positions (eyes, noses, ears, lips, fingers and feet) play important roles in differential diagnosis.

Hassanien [24] presented a rough set approach to feature reduction and generation of classification rules from a set of medical datasets. This work was first introduced a rough set reduction technique to find all reducts of the data that contain the minimal subset of features associated with a class label for classification. To evaluate the validity of the rules based on the approximation quality of the features, the author introduced a statistical test to evaluate the significance of the rules. a set of data samples of patients with suspected breast cancer were used and evaluated. In addition, the rough set classification accuracy is also compared to the well-known ID3 classifier algorithm. This study showed that the theory of rough sets is a useful tool for inductive learning and a valuable aid for building expert systems.

Huang and Zhang [126] presented a new application of rough set to ECG recognition. Firstly the recognition rules for characteristic points in ECG are reduced by using rough set theory. Then the reduced rules are used as restriction conditions of eigenvalue determination arithmetic which is presented in this paper and employed in recognizing characteristic points in ECG. There are some discussions about correlative arithmetic such as sizer method, difference method and how to choose difference parameters. Also in their paper, an example of this new application is discussed. They adopted MIT-BIH data to verify R wave recognition and the detection rate is proved higher than the one of routine recognition method.

Recently Independent Component Analysis (ICA) [30] has gained popularity as a promising method for discovering the statistically independent variables (sources) for data, blind source separation, as well as for feature extraction from images. Roman et al. [118] studied several hybrid methods for feature extraction/reduction, feature selection, and classifier design for breast cancer recognition in mammograms recognition. The methods included independent component analysis (ICA), principal component analysis (PCA) and rough set theory. Three classifiers were designed and tested: a rough sets rule-based classifier, an error back propagation neural network, and a Learning Vector Quantization neural network. They provided comparative study for two different data sets of mammograms. In both data sets, the rough sets rule-based classifier performed with a significantly better level of accuracy than the other classifiers. Therefore, the use of ICA or PCA as a feature extraction technique in combination with rough sets for feature selection and rule-based classification is an improved solution for mammogram recognition in the detection of breast cancer.

Rough sets in decision support system for medical diagnosis

In the medical diagnosis process can be interpreted as a decision-making process, during which the physician induces the diagnosis of a new and unknown case from an available set of clinical data and from his/her clinical experience. This process can be computerized in order to present medical diagnostic procedures in a rational, objective, accurate and fast way. In fact, in the last two or three decades, diagnostic decision support systems have become a well-established component of medical technology.

Podraza et. al [74] presented an idea of complex data analysis and decision support system for medical staff based on the rough set theory. The main goal of their system is to provide an easy to use, commonly available tool for quick diagnosing diseases, suggesting possible further treatment and deriving unknown dependences between different data coming from various patient's examinations. The sketch of possible architecture of such system is presented including some example algorithms and suggested solutions, which may be applied during implementation. The unique feature of the system relies on removing some data from rough set decision tables to enhance the quality of generated rules. Usually such a data is discarded, because it is useless in knowledge acquisition. In their approach the improper data (excluded from the data used for drawing conclusions) is carefully taken into considerations. This methodology can be very important in medical applications. A case not fitting to the general classification cannot be neglected, but should be examined with a special care.

Mitra et al. [52] implemented a rule-based rough-set decision system for the development of a disease inference engine for ECG classification. An offline-data-acquisition system of paper electrocardiogram (ECG) records is developed using image-processing techniques. The ECG signals may be corrupted with six types of noise. Therefore, at first, the extracted signals are fed for noise removal. A QRS detector is also developed for the detection of R-R interval of ECG waves. After the detection of this R-R interval, the P and T waves are detected based on a syntactic approach. The isoelectric-level detection and base-line correction are also implemented for accurate computation of different features of P, QRS, and T waves. A knowledge base is developed from different medical books and feedbacks of reputed cardiologists regarding ECG interpretation and essential time-domain features of the ECG signal. Finally, a rule-based rough-set decision system is generated for the development of an inference engine for disease identification from these time-domain features.

Wakulicz-Deja and Paszek [1] implemented an example of application of Rough Set Theory to decision making - diagnosing Mitochondrial Encephalomyopathies (MEM) for children. The resulting decision support system maximally limits the indications for invasive diagnostic methods (puncture, muscle and/or nerve specimens). Moreover, it shortens the time necessary for making diagnosis. The system has been developed on the basis of data obtained from the II Clinic Department of Pediatrics of the Silesian Academy of Medicine

1.7 Challenges and Future Directions

Rough set theory is a fairly new intelligent technique that has been applied to the medical domain and is used for the discovery of data dependencies, evaluates the importance of features, discovers the patterns of data, reduces all redundant objects and features, and seeks the minimum subset of features. Moreover, it is being used for the extraction of rules from databases. Most of the current literature on rough sets based methods for medical imaging addresses the classification and feature reduction issues. A few papers deal with medical imaging problems such as image segmentation, image filtering, voxel representation, etc. Therefore, medical imaging is another challenge and fruitful area for rough sets to play crucial roles in resolving problems and providing solutions to medical image processing that understand, represent and process the images, their segments and features as rough sets.

Near sets [67, 68, 69, 70, 28, 71, 72] represent a generalization of the rough set approach to the classification of objects introduced by Zdzisław Pawlak [73]. Near sets and rough sets are very much like two sides of the same coin. From a rough set point-of-view, the focus is on the approximation of sets with non-empty boundaries. By contrast, in a near set approach to set approximation, the focus is on the discovery of near sets in the case where there is either a non-empty or an empty approximation boundary. There are a number of practical outcomes of the near set approach, *e.g.*, feature selection [71], object recognition in images [28, 70], image processing [7], granular computing [85, 81] and in various forms of machine learning [78, 85, 94, 80, 83, 84, 82, 93]. Near sets-based methods is another challenge which offer a generalization of traditional rough set theory and a approach to classifying perceptual objects by means of features could be lead to new and will be useful in solving object recognition, in particulars in solving medical imaging problems.

sets [67, 68, 69, 70, 28, 71, 72] offer a generalization of traditional rough set theory [?, ?, 73, 75, 76] and a new approach to classifying perceptual objects by means of features [77, 78, 83, 84, 85, 86]. The near set approach can be used to classify images that are qualitatively but not necessary quantitatively close to each other. This is essentially the idea expressed in classifying images in [70, 28]. If one adopts the near set approach in image processing, a byproduct of the approach is the separation of images into non-overlapping sets of images that are similar (*descriptively* near to) each other. This has recently led to an application of the near set approach in 2D and 3D interactive gaming with a vision system that learns and serves as the backbone for an adaptive telerehabilitation system for patients with finger, hand, arm and balance disabilities (see, *e.g.*, [?, ?]). Each remote node in the telerehabilitation system includes a vision system that learns to track the behaviour of a patient. Images deemed to be "interesting" (*e.g.*, im

A combination of kinds of computational intelligence techniques in application area of pattern recognition and in particulars in medical imaging problems has become one of the most important ways of research of intelligent information processing. Neural network shows us its strong ability to solve complex problems for medical image processing. From the perspective of the specific rough sets approaches that need to be applied, explorations into possible applications of hybridize rough sets with other intelligent systems like neural networks, genetic algorithms, fuzzy approaches, etc. to image processing and pattern recognition, in particulars in medical imaging problems could lead to new and interesting avenues of research and it is always a challenge for the CI researchers. Finally, even though many rough-based approaches are being proposed for various applications in medical filed, their impact is mostly confined to academic circles. These methods are yet to find wide acceptance in industrial circles, and get incorporated in many industrial products. This trend is also evident from the very small number of industrial patents in this direction. Hence, the main challenge of rough sets researchers is to provide the industry leaders a convincing demonstration of the superiority of their approaches over the traditional methods.

In conclusion, many successful algorithms applied in medical imaging have been reported in the literature and the applications of rough sets in medical image processing have to be analyzed individually. Rough sets is a new challenge to deal with the issues that can not be addressed by traditional image processing algorithms or by other classification techniques. By introducing rough sets, algorithms developed for medical imaging and pattern recognition often become more intelligent and robust that provides a human-interpretable, low cost, exact enough solution, as compared to traditional techniques. While this chapter provided a focused survey on a range of rough sets and their applications to medical imaging including object representation, image segmentation, classification and feature extraction, etc. Finally, the main purpose here is to present to the rough sets and medical imaging research communities the state of the art in rough sets applications to image processing and pattern recognition, and in particular in medical imaging, and to inspire further research and development on new applications and new concepts in new trend-setting directions and in exploiting rough sets.

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