FINITE ELEMENT BASED ON A REDUCED KIRCHHOFF-LOVE SHELL MODEL FOR SIMULATION OF SOFT BILAYERS

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1. Background

Depending on geometrical and material properties thin-film/soft-substrate bilayers may, when subjected to compressive loading, loose its stability and bifurcate from a homogeneous state of compression to either global or local deformation modes. In the case of a stiff and thin surface, bonded to a thick and compliant substrate, several wrinkling modes (e.g. dimple, labyrinthine or hybrid) are the modes preferred over the global buckling in early post-critical regime. Further increase in the load pushes the structure deeper into the post-critical regime and typically induces secondary instabilities, showing that these types of structures are often multi-stable. Several interesting phenomena can be observed, such as pattern switching, wrinkle-to-fold transitions, creasing, etc., which make this problem worth exploring, especially in search for advanced functionalities. As such, researchers used surface wrinkling as a platform to study tuneable adhesion [1], wetting to attain hydro-phobicity/hydro-philicity [2], for fabrication of micro-lens arrays [3], micro-gears [4], etc.

It turns out that theoretical solutions of this problem are very difficult to find due to the strong non-linearities (large deformations, discrete material distribution, multi-stable states, etc.). With the exception of a few studies (see e.g. Xu and Potier-Ferry [5,6]), no efficient (fast and reliable) numerical procedures are available. Here, we present our attempt to develop such procedure based on a finite element method and a reduced model of the Kirchhoff-Love shell.

2. Finite element based on a reduced Kirchhoff-Love shell model

The derivation of the finite element procedure we show in this contribution is based on the Kirchhoff-Love kinematics. Moreover, we apply a reduced Kirchhoff-Love kinematic model, similar to that used in [7], where the tangential components of the displacement vector were considered to be small when compared to the normal component. The simplification calls for the use of the classical treatment of the shell displacement field (with the displacement components resolved with respect to the curvilinear covariant basis in the finite element formulation). By neglecting the tangential strains, the standard Kirchhoff-Love shell Green-Lagrange strain tensor is significantly simplified. As for the material model, we use the St. Venant-Kirchhoff shell hyperelastic model. The substrate is modelled as a linear spring, with constant stiffness k in units of N/m^2.

We derived quadrilateral and triangle finite elements, denoted as DKQ-4 and DKT-3, respectively. In order to approximate the Kirchhoff-Love shell kinematic constraint, three degrees of freedom, w, $\partial w/\partial x$ and $\partial w/\partial y$, were introduced at each node, where w is normal displacement and x, y are orthogonal coordinates in the tangent plane at the node. By using linked interpolation of w and its derivatives, similar to the one applied in discrete Kirchhoff (DK) quadrilateral (Q) and tringle (T) plate elements [8], the Kirchhoff kinematic constraint becomes fulfilled along the edge of each element in the direction of the edge.

The results shown in the next section were computed by either standard Newton-Raphson iterative procedure or by the path-following methods presented in [9]. We note that the problems presented in the next section, computed by DKQ-4 and DKT-3, were too difficult for standard Assumed Natural Strain (ANS) quadrilateral with Reissner-Mindlin kinematics (both static analysis with enhanced path-following method and implicit dynamics analysis failed).

4. Results

Figure 1 shows deformed configurations of a spherical and cylindrical shell on elastic substrates. Material and geometrical parameters are as follows: thickness t = 0.48 mm, radius R = 20.0 mm, elastic modulus E = 2 MPa, k = 0.22, Poisson's ratio v = 0.48 for both shells, height h = 10 mm for cylindrical shell.

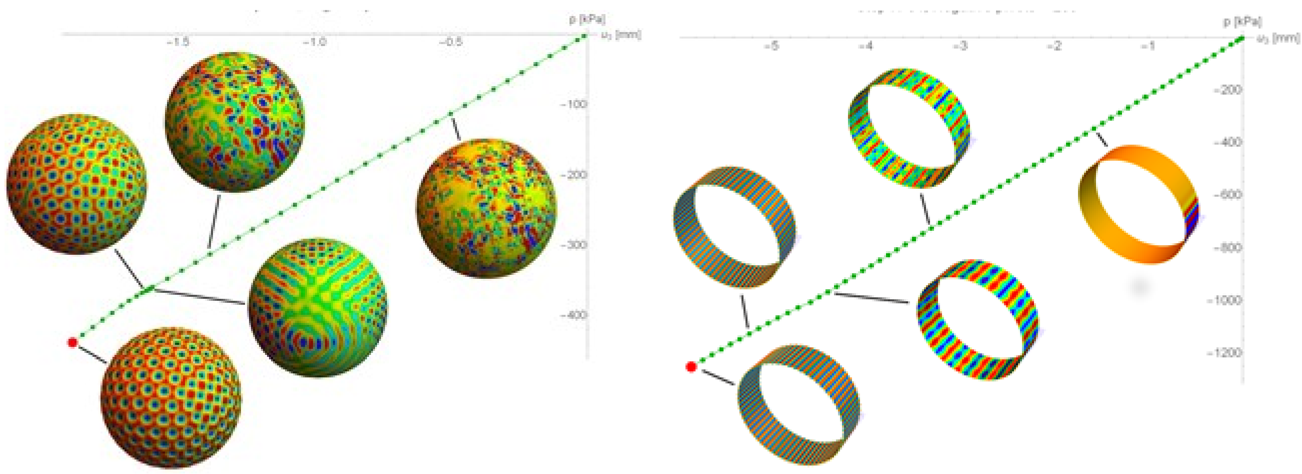

Figure 1: Deformed configurations of a spherical (left) and cylindrical (right) shell.

The solution procedure converged within 10 mins on an average laptop. The drawback of the current setup is that the solution is only valid for moderate displacements. Generalization of the procedure is underway.

References

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