ANALYSIS OF LAMINATES WITH THE USE OF 2-D COSSERAT CONSTITUTIVE MODEL

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1. Introduction

The paper presents the analysis of laminates within the framework of 6-parameter (6p) non-linear shell theory with the use of 2-D Cosserat constitutive model. This theory is specially dedicated to the modelling of irregular shells with intersections, since it takes into account the drilling rotation at material point naturally. As the direct consequence, the unsymmetrical in-plane strain and stress measures arise and reduced shell body is a Cosserat type surface [1]. The constitutive relation for such specific kinematics is non-trivial, especially if laminated fibre reinforced material is considered. Up till now the Authors have utilized the constitutive law expressed in terms of 5 independent engineering constants. In such case the drilling rotation stiffness was in a sense an arbitrary chosen quantity [2]. The approach was successfully used in the analysis of laminates undergoing large displacements [2], first-ply failure [3] and progressive failure [4]. Now a new attempt is made and the Cosserat material law is employed.

2. Cosserat law for fiber reinforced layer

According to [5], where the Cosserat material law for isotropic continuum is presented, we propose an analogical relation for a fibre reinforced layer, similarly as in [6]:

(1)
$$\begin{Bmatrix}\sigma_{aa}\\ \sigma_{bb}\\ \sigma_{ab}\\ \sigma_{ba}\\ \hline \sigma_a\\ \sigma_b\\ \hline m_a\\ m_b\end{Bmatrix} = \left[\begin{array}{cccc|cc|cc} \frac{E_a}{1-\nu_{ab}\nu_{ba}} & \frac{\nu_{ab}E_b}{1-\nu_{ab}\nu_{ba}} & 0 & 0 & 0 & 0 & 0 & 0\\ \frac{\nu_{ba}E_a}{1-\nu_{ab}\nu_{ba}} & \frac{E_b}{1-\nu_{ab}\nu_{ba}} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & G_{ab}+G_C & G_{ab}-G_C & 0 & 0 & 0 & 0\\ 0 & 0 & G_{ab}-G_C & G_{ab}+G_C & 0 & 0 & 0 & 0\\ \hline 0 & 0 & 0 & 0 & \alpha_s G_{ac} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha_s G_{bc} & 0 & 0\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 2G_{ab}l^2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2G_{ab}l^2 \end{array}\right] \begin{Bmatrix}\varepsilon^{aa}\\ \varepsilon^{bb}\\ \varepsilon^{ab}\\ \varepsilon^{ba}\\ \hline \varepsilon^{a}\\ \varepsilon^{b}\\ \hline \kappa_a\\ \kappa_b\end{Bmatrix},$$

where σ_{ab}, ε^{ab} are, respectively, plane stress and strain components, m_j, κ_i, ($j = a,b$) are the coupling stresses and strains, E_a, E_b are the longitudinal and transverse Young moduli, ν_{ab} and ν_{ba} are the Poisson ratios, G_{ab} is the shear moduli; α_s is the shear correction factor, l is the characteristic length and G_c is the Cosserat shear modulus $G_c = N^2/(1-N^2)G_{ab}$ where $0 < N < 1$ is the so-called coupling number. Equation (1) is integrated in the through-the-thickness direction using equivalent single layer approach. Progressive failure analysis is performed with Hashin criterion used as the failure condition. The algorithm is based on the stiffness reduction parameter SRC as described in [4].

3. Numerical example

Consider a C-shaped column which was investigated numerically and experimentally in [7]. The scheme of the experimental setup and present FEM model is shown in Figure 1. The edges of the top cross-section are totally fixed whereas the bottom edges are pinned. Such boundary conditions provide the best agreement with the experimental results [4]. The laminate is composed of 8 0.26 mm thick layers $[0°/-45°/+45°/90°]_s$ which are made of the material with following properties: E_1 = 38.5 GPa, E_2 = 8.1 GPa, G_{12} = 2 GPa, ν_{12} = 0.27, X_t = 792 MPa, X_c = 679 MPa, Y_t = 39 MPa, Y_c = 71 MPa, S_L = 108 MPa. The fibers orientation is measured with reference to the *y*-axis (Fig. 1). To impose the two half waves buckling mode observed during the experiment, additional imperfection forces $P_i = 0.002\,P$ are applied (Fig. 1). The column is discretized with 16 node fully integrated elements, in particular 30 elements along the column's height, 6 elements along the width of flanges and 12 elements along the web's width. The stiffness reduction parameter is taken as SRC = 0.01. All nodes along the top edge are kinematically coupled with respect to the axial displacement *v* (Fig. 1), which is chosen as the path control parameter. In the present model only the constitutive law is exchanged. In the computations the Cosserat coupling number was set as $N = \sqrt{2}/2$. This value ensures that the constitutive relation for in-plane shear components is the same as in [4]. Two values of characteristic length were chosen, i.e. l = 0.26 mm and l = 2.08 mm, which correspond to the thickness of a single layer and the total thickness of the laminate. As it can be observed in Figure 1 the obtained results are in good agreement with the previous solution [4].

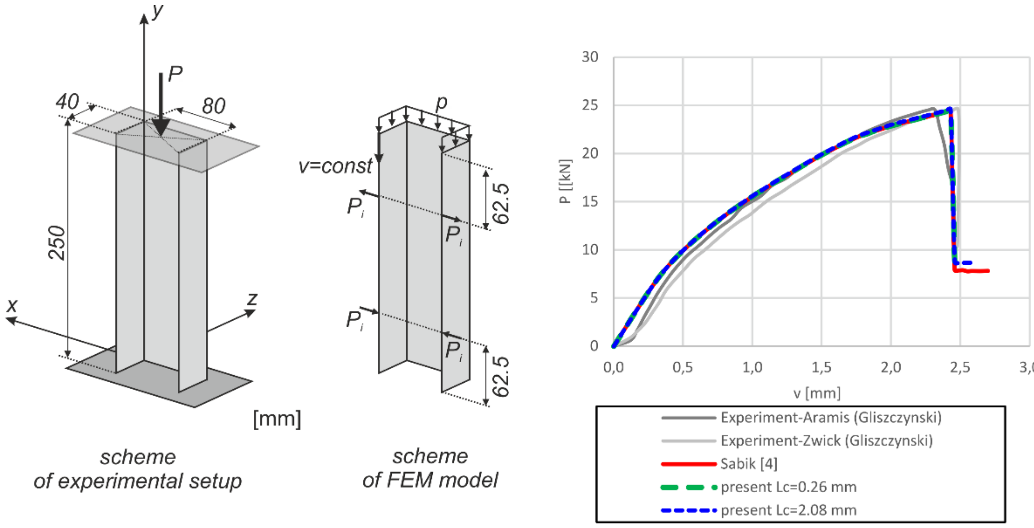

Figure 1: C-shaped column - data and results.

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