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Simulation of an uncertain emission market for greenhouse gases using agent-based methods

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Simulation of an uncertain emission market for greenhouse gases using agent-based methods\*

<sup>\*</sup>Praca zaakceptowana po recenzjach do Climatic Change.



#### Supplementary material to the paper

#### Simulation of an uncertain emission market for greenhouse gases

#### using agent-based methods

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#### The central planner view on the optimization of a completely known market

The central planner perceives the use of a emission trading system as a minimization of the social cost function

$$F(x) = \sum_{n=1}^{N} c^{P_n}(x^{P_n}), \quad x = (x^{P_1}, ..., x^{P_N})$$
 (s1)

where  $c^{P_n}(x^{P_n})$  is the abatement cost function of the *n*-th party to reduce the emission to a level of  $x^{P_n}$ , and subject to

$$\sum_{n=1}^{N} x^{P_n} = K_0$$

where  $K_0$  is the total allowable emission of the market participants<sup>4</sup>. The costs typically grow with a raising of reduced quotas, so it can be assumed that  $\frac{dc^P n(x)}{dx} < 0$ . Using the Lagrange method, an optimal solution satisfies the necessary conditions

$$\lambda = -\frac{dc^{P_n}(x^{P_n})}{dx}, \quad n = 1, ...N; \qquad \sum_{n=1}^{N} x^{P_n} = K_0.$$
 (s3)

The solution is optimal globally, if the functions  $c^{P_n}$  are convex. Thus, at the optimal emissions the marginal costs of each party are equal to  $\lambda$ , which is the optimal price. In this

<sup>&</sup>lt;sup>4</sup> In actual fact, the total emission is bounded from above, i.e. it holds that  $\sum_{n=1}^{N} x^{P_n} \leq K_0$ , but it is easy to notice that it is not optimal to keep the emissions below the boundary.

manner the market can be optimized centrally, but only if the functions  $c^{P_n}(x)$  are known to the central planner.

Figure S1. An illustration of full compliance and compliance with the risk  $\alpha$ .

#### Proof of assertion (21)

To prove property  $(21)^5$ , let us first consider a party which only sells its emission permits. From (19), after the t-th transaction we have

$$\hat{x}_{t}^{S} = \hat{x}_{0}^{S} + \frac{\sum_{i=1}^{t} E_{eff,i}^{S}}{1 - u^{S}(\alpha)}.$$

Consequently, from (11) we get

$$\widehat{\mathbf{x}}_t^S = \widehat{\mathbf{x}}_0^S + \sum_{i=1}^t \widehat{E}_i^S.$$

As a result, it is simply the sum of the initial emission and any permits that have been sold. All bear an association with the actual seller's uncertainty. Thus, the compliance with a risk  $\alpha$  condition, see (9), is

$$\hat{x}_t^S[1+u^S(\alpha)] \leq K^S.$$

By using definition (10) we now get (21). This way (21) is true for the seller.

Let us now consider a party which only buys the permits. Similarly, as above, we have

<sup>5</sup> All numbers in parentheses pertain to the equations in the main paper.

$$\hat{x}^B_t = \hat{x}^B_0 - \frac{\sum_{i=1}^t E^B_{eff,i}}{1 + u^B(\alpha)} = \frac{[1 + u^B(\alpha)]\hat{x}^B_0 - \sum_{i=1}^t [1 - u^{S_i}(\alpha)]\hat{E}^{S_i}_i}{1 + u^B(\alpha)}.$$

a situation where  $S_i$  are the parties that have sold permits to the buyer B. Now, proceeding in an iterative way, analogously to the left hand side of (12), we can see that the numerator on the upper right hand side is the value used in checking the compliance with the risk  $\alpha$ . Thus

$$\hat{x}_t^B \le \frac{K^B}{1 + u^B(\alpha)} = \widetilde{K}^B.$$

So, (21) is also true for the buyer.

In general, we can order the selling transactions as the first  $j \le t$  transactions, without loosing any generality. Then, when considering only the first j transactions we realise that (21) is true. Were we to treat the estimated emissions and uncertainty spreads after the first j transactions as a new starting point, and to then consider the buying transactions, it may be concluded with reference to the first proven stage that (21) is true. This completes the proof of the assertion (21).

Table S1. Data for the case study: parameters of cost reduction function and spreads of emission uncertainties.

Party	BAU emission $\hat{x}_0^{P_n}$ [MtC/y]	Cost function parameter $a^{P_n}$ [MUSD/(MtC/y) <sup>2</sup> ]	Kyoto limit  K <sup>Pn</sup> [MtC/y]	$d^l = d^u$
USA	1820.3	0.2755	1251	0.13
EU	1038.0	0.9065	860	0.10
Japan	350.0	2.4665	258	0.15
CANZ	312.7	1.1080	215	0.20
EEFSU	898.6	0.7845	1314	0.30

Table S2. Final results for several values of  $\alpha$ . Prices in USD/MtC/y, costs in USD.

	price	transaction	of traded	emission	reduction	of traded
		price	permits	<u> </u>	cost	permits
		Sealed		se auctions		
			α=0.5			,
USA	143.3	143.5	309.3	1560.3	18 623.80	+
EU	141.4	141.1	100.0	960.0	5 5 15.15	16 216.40
Japan	144.0	141.1	62.8	320.8	2 103.04	11 307.24
CANZ	144.0	146.2	32.7	247.7	4 681.30	2 275.47
EEFSU	140.3	139.6	-504.8	809.2	6 270.01	-84 630.57
			$\alpha=0.3$			
USA	245.3	246.2	124.0	1375.0	60 755.86	41 027.40
EU	241.8	241.6	44.6	904.6	14 894.66	15 735.65
Japan	244.1	246.2	42.5	300.5	5 353.36	12 456.14
CANZ	240.9	242.9	-11.0	204.0	11 197.25	1 280.45
EEFSU	238.5	241.6	-567.4	746.6	23 316.02	-70 499.64
			α=0.1			
USA	319.9	318.6	-11.4	1239.6	115 585.61	29 660.62
EU	318.7	323.4	2.2	862.2	33 038.58	15 596.07
Japan	315.9	321.6	28.0	286.0	8 025.75	15 365.88
CANZ	315.3	321.6	-44.6	170.4	31 670.74	-2 274.78
EEFSU	308.2	310.8	-611.8	702.2	52 389.28	-58 347.79
		Bilat	eral trans	actions		
			α=0.5			
USA	142.6	142.6	310.5	1561.5	18 452.61	66 364.93
EU	142.6	142.6	99.3	959.3	5 608.05	17 667.54
Japan	142.6	142.6	63.1	321.1	2 061,10	12 581.13
CANZ	142.6	142.6	33.3	248.3	4 588.17	-2 400.45
EEFSU	142.6	142.6	-506.3	807.7	6 480.17	-94 213.16
			α=0.3			
USA	250.8	250.8	114.2	1365.2	63 502.10	51 723.90
EU	250.8	250.8	39.7	899.7	18 819.85	19 925.29
Japan	250.8	250.8	41.2	299.2	7 214.22	20 466.53
CANZ	250.8	250.8	-15.5	199.5	16 765.29	1 697.77

Volume

Final

Last

Party Marginal

EEFSU

USA

EU

Japan

CANZ

EEFSU

250.8

336.4

336.4

336.4

336.4

336.4

250.8

336.4

336.4

336.4

336.4

336.4

-575.2

-41.2

-7.5

23.8

-54.1

-629.8

α=0.1

738.8

1209.8

852.5

281.8

160.9

684.2

25 880.23 -93 813.48

127 907.68 29 874.72

14 811.72 29 369.90

62 435.51 -82 647.62

21 193,41

2 209.59

36 872.98

36 187.08

Emission Total cost

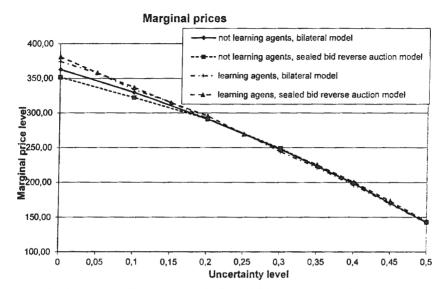


Figure S2. Dependence of marginal price, in USD/MtC/y, on the uncertainty level.

