61/2012

Raport Badawczy
Research Report

RB/33/2012

Negotiation strategies of programmable agents in Continuous Double Auctions

Z. Nahorski, W. Radziszewska

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 3810100

fax: (+48) (22) 3810105

Kierownik Zakładu zgłaszający pracę:
Prof. zw. dr hab. inż. Zbigniew Nahorski

Warszawa 2012

SYSTEMS RESEARCH INSTITUTE
POLISH ACADEMY OF SCIENCES

Zbigniew Nahorski, Weronika Radziszewska

Negotiation strategies of programmable agents in Continuous Double Auction

Warszawa 2012

Contents

Chapter 1

Introduction

Auctions as a method of selling and buying goods have a long history, initially there were only ascending auctions with simple rules (now known as English auctions) but with time a variety of types of auctions has emerged. Now, auctions have become a very popular method of trading popularized by on-line auctions as Ebay or Allegro (a big Polish auction platform).

According to definition made by McAfee and McMillan in 1987: "an auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants".

A special type of auctions, maybe not the most popular in an on-line internet auctions but interesting from point of view of computer simulation, are so called *double auctions*. In double auctions, there are multiple buyers and sellers on the market that place their offer simultaneously.

In this work we review strategies of agents participating in a double auction. There are a lot of different categories of strategies: some consider history, others are reacting on the last placed bid or apply learning algorithms. Some strategies, as ZI, GD, and AA, have been already reviewed in an earlier publication of the present authors [21]. They are repeated here to make a possibly full compendium of strategies proposed in the literature.

The practical context of this research is the double auction for trading emissions of pollutants. Emission, in this context, is the short name for "permission to emit a unit of greenhouse gas"; its unit is either one tonne of carbon dioxide or the mass of another greenhouse gas which is recalculated to so-called carbon dioxide equivalent (tCO2e) emissions. This is expressed in units like Certified Emission Reductions (CERs) or carbon credits. This concept was introduced in the Kyoto Protocol, which entered into force in

16 February 2005, obliging countries that ratified it to limit their greenhouse gases (GHG) emissions below the levels of 1990.

The protocol introduced so called "flexible" market-based mechanisms (Emission Trading, Joint Implementation and Clean Development), which are meant to achieve the common reduction target with minimal costs, without knowledge of the parties cost functions. The emission trading market is still not mature and it is still under the process of adjusting the rules and protocols to make it efficient and resistant to collapsing. The Chicago Climate Exchange market ceased operations in 2010 because the legislation was refused by the US Senate and companies were no longer interested in trading this commodity.

There are different schemes developed for this type of market. In report [26], the English auction trading scheme for emission permit trading was considered. In the present work the double auction mechanism for emission trading is defined, as it is a very popular method of creating efficient markets.

This work summarizes the most well known strategies, that present the evolution of automated negotiation strategies: from simple and intuitive approaches as ZI, PS and ZIP, to more forecasting like GD and adapting as AA strategy. None of the general issues of on-line auctions are discussed here. An interested reader is referred to recent reviews of these matters [12, 17, 24].

The structure of the paper is as follows. In chapter 2 the current state of research on the Continuous Double Auction, emission trading and agent strategies are shortly reviewed. In the following chapter the concept of negotiations and different ways of trading is described. In chapter 4 some informations on double auction are presented. Chapter 5 discusses the formal model of the auction double market used in this paper. The following chapters contain the description of the existing strategies for participants in the continuous double auction, they are divided to strategies using only current information, GD strategies, AA strategies and FL-strategy, that uses fuzzy rules to determine the value of next shout. The general architecture of the implemented software is located in the chapter 10, followed by description of its implementation. In chapter 11 some preliminary results are presented. Conclusions summarizes the whole report. Also future works are sketched there.

Chapter 9

The FL-strategy

9.1 Preliminaries

The FL-strategy is a fuzzy logic based strategy. It has been proposed by He et al. [14]. It uses some notions, which are defined in this section.

A *round* is the time period between two successive deals or the time from the beginning of the auction to the first deal. The rounds are numbered and the round number r is the number of concluded deals.

The *price history* H_l in the round r is a series of l last successful deal prices

$$H_l = \{p_{r-l}, \ldots, p_{r-1}\}$$

where p_i is the price of the i-th deal.

The *reference price* P_r in the round r is the median value of the prices from the price history.

All the variables and structures related to the seller (s-agent) will be marked with the letter s, and those related the buyer (b-agent) with the letter b.

9.2 The decision sets

The *decision set* (acceptable offers) for a trading agent is a segment. For the seller it stretches from his unit cost of the good c_s and the current outstanding ask a_0. However, as it is not reasonable to ask lower than the outstanding bid b_0, the decision set of a seller is

$$\max(b_0, c_s) \leq a < a_0$$

For the buyer it streches from the current outstanding bid and his valuation of a unit of the good. For her/him, it is not reasonable to bid higher than the outstanding ask, so the decision set of a buyer is

$$b_0 < b \leq \min(a_0, v_b)$$

As special cases, the above sets may be empty.

9.3 Strategy rules for the crisp values

9.3.1 Logic rules

To submit an offer an agent maximizes his profit in the decision set. It is based on a number of heuristic rules, which give propositions of tentative offers. The rules depend on a common relation of three current values, the reference price P_r, the outstanding bid b_0, and the outstanding ask a_0. Three cases are discussed below; first for crisp values, to better understand the idea, and then for fuzzy ones.

The case $P_r \leq b_0 < a_0$

The heuristic rule for the **seller** is:

IF b_0 is *much bigger* than P_r
THEN accept b_0
ELSE ask is $a_0 - \beta_{s,1}$

where $\beta_{s,1}$ a constant dependent of the agent's attitude to risk. That is, the rule is either to accept the big outstanding bid, or to diminish the outstanding ask, if the outstanding bid is only slightly bigger than the reference price.

The rule for the **buyer** is:

IF b_0 is *much bigger* than P_r
THEN no new bid
ELSE bid is $b_0 + \beta_{b,1}$

where $\beta_{b,1}$ is a constant dependent of the agent's attitude to risk. The rule is either to stop bidding, if the outstanding bid is already big enough, or outbid the outstanding bid, if it is acceptably low.

The case $b_0 < a_0 \leq P_r$

The rules in this case are symmetric to the previous case, with the change of roles. Thus, the rule for the **seller** is:

IF a_0 is *much smaller* than P_r
THEN no ask
ELSE ask is $a_0 - \beta_{s,2}$

where $\beta_{s,2}$ is a constant dependent on the agent's attitude to risk. Just, the seller refuses to ask, if the outstanding ask is too small. Otherwise, it steps down from the outstanding ask.

The rule for the **buyer** is:

IF a_0 is *much smaller* than P_r
THEN accept a_0
ELSE bid is $b_0 + \beta_{b,2}$

where $\beta_{b,2}$ has a similar meaning as other βs before.

The case $b_0 \leq P_r \leq a_0$

In this case the rules are more complicated. For the **seller** we have:

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *far from* P_r)
THEN ask is $a_0 - \lambda_{s,1}$

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *medium to* P_r)
THEN ask is $a_0 - \lambda_{s,2}$

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *close to* P_r)
THEN ask is $a_0 - \lambda_{s,3}$

IF b_0 is *close to* P_r
THEN ask is $P_r + \lambda_{s,4}$

Now, if the outstanding bid is much or fairly smaller than the reference price, the buyer steps down from the outstanding ask. If the outstanding bid is close to the reference price, the buyer bids slightly higher than the reference price. The constants *lambda* has here the meaning similar to this for βs before.

And symmetrically for the **buyer**:

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *far from* P_r)
THEN ask is $b_0 + \lambda_{b,1}$

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *medium to* P_r)
THEN ask is $b_0 + \lambda_{b,2}$

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *close to* P_r)
THEN ask is $b_0 + \lambda_{b,3}$

IF a_0 is *close to* P_r
THEN ask is $P_r - \lambda_{b,4}$

9.3.2 Final decision

To decide finally on submitting an offer, three cases are considered. For the **seller** they are as follows.

IF the tentative ask is outside the decision set
THEN no submitted ask
ELSE
IF b_0 belongs to the decision set and it is *close to* the tentative ask
THEN submitted ask $= b_0$

ELSE submitted ask = tentative ask

Notice that b_0 belongs to the decision set when $c_s \leq b_0$.
Now, for the buyer the decision rules are:

IF the tentative bid is outside the decision set
THEN no submitted bid
ELSE
IF a_0 belongs to the decision set and it is *close to* the tentative bid
THEN submitted bid = a_0
ELSE submitted bid = tentative bid

9.4 Strategy rules for the fuzzy values

9.4.1 Preliminaries

Fuzzy linguistic variables

In the above rules, there are notions which are note precisely defined, like *much bigger, far from, close to*, etc. In the fuzzy logic these linguistic variables are modelled using the fuzzy sets, and the fuzzy reasoning is applied to obtain the results. A short introduction to the fuzzy sets is given in the appendix to this chapter.

The linguistic variables connected with location of the outstanding ask and bids with respect to P_r, are depicted in Figure 9.1. The variables *close to, medium to, far from* are shown in Figure 9.2. The parameters of the membership functions are decided by intuition and experience.

Fuzzy reasoning

The fuzzy reasoning goes along the established rules. We refer only to the rules needed in our reasoning.

Now, the fuzzy logic rules for calculating a tentative ask or a tentative bid use the fuzzy linguistic variables in the logical consequence premises.

The membership function of the intersection $A(x) \cap B(x)$ (representing the linguistic operator *and*) of two fuzzy sets $A(x)$ and $B(y)$ with the membership

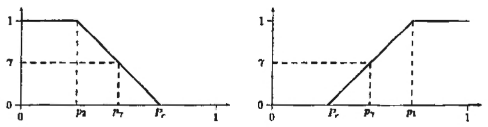

Figure 9.1: Membership functions of the fuzzy sets representing the linguistic variables 'an offer is *much smaller* than the reference price P_r' (left) and 'an offer is *much bigger* than the reference price P_r' (right).

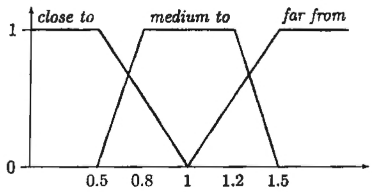

Figure 9.2: Membership functions of fuzzy sets representing the linguistic variables '*close to*' (left), '*medium to*' (middle), and '*far from*' (right).

functions $\mu_A(x)$ and $\mu_B(x)$, respectively, is calculated as

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(y)\}$$

The membership function of the union $A(x) \cup B(x)$ (representing the linguistic operator *or*) of two fuzzy sets $A(x)$ and $B(y)$ is calculated as

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(y)\}$$

The conclusion is a triangular fuzzy set, see Figure 9.3

$$(m, \theta, \chi)$$

where m is the center of the fuzzy set, θ is its left and χ is its right spread. In [14] it is suggested that the spreads may be equal to double price step in the auction.

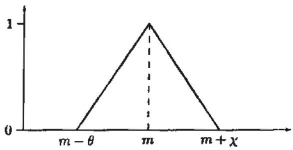

Figure 9.3: Membership functions of a triangular fuzzy set.

9.4.2 Fuzzy logic rules

The case $P_r \leq b_0 < a_0$

The heuristic rule for the **seller** is:

IF b_0 is *much bigger* than P_r
THEN accept b_0
ELSE ask is $(a_0 - \beta_{s,1}, \theta, \chi)$

where $\beta_{s,1}$ a constant dependent of the agent's attitude to risk. Similar as in the crisp case, the first part of the rule is acceptation of the big outstanding bid, if it is much bigger than the reference price P_r, but *much bigger* is now expressed in a soft way. The second part is diminishing the outstanding ask, but the conclusion is now a triangular fuzzy set.

The rule for the **buyer** is:

IF b_0 is *much bigger* than P_r
THEN no new bid
ELSE bid is $(b_0 + \beta_{b,1}, \theta, \chi)$

where $\beta_{b,1}$ is a constant dependent of the agent's attitude to risk.

The case $b_0 < a_0 \leq P_r$

Also now, the rules are symmetric to the previous case. Thus, the rule for the **seller** is:

IF a_0 is *much smaller* than P_r
THEN no ask
ELSE ask is $(a_0 - \beta_{s,2}, \theta, \chi)$

where $\beta_{s,2}$ is a constant dependent on the agent's attitude to risk.

The rule for the **buyer** is:

IF a_0 is *much smaller* than P_r
THEN accept a_0
ELSE bid is $(b_0 + \beta_{b,2}, \theta, \chi)$

where $\beta_{b,2}$ has a similar meaning as βs before.

The case $b_0 \leq P_r \leq q_0$

In this case the rules are more complicated and require applications of the fuzzy *and* and *or* operators. For the **seller** we have:

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *far from* P_r)
THEN ask is $(a_0 - \lambda_{s,1}, \theta, \chi)$

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *medium to* P_r)
THEN ask is $(a_0 - \lambda_{s,2}, \theta, \chi)$

IF (b_0 is *far from* or *medium to* P_r) and (a_0 is *close to* P_r)
THEN ask is $(a_0 - \lambda_{s,3}, \theta, \chi)$

IF b_0 is *close to* P_r
THEN ask is $(P_r + \lambda_{s,4}, \theta, \chi)$

And symmetrically for the **buyer**:

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *far from* P_r)
THEN ask is $(b_0 + \lambda_{b,1}, \theta, \chi)$

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *medium to* P_r)

THEN ask is $(b_0 + \lambda_{b,2}, \theta, \chi)$

IF (a_0 is *far from* or *medium to* P_r) and (b_0 is *close to* P_r)
THEN ask is $(b_0 - \lambda_{b,3}, \theta, \chi)$

IF a_0 is *close to* P_r
THEN ask is $(P_r - \lambda_{b,4}, \theta, \chi)$

9.4.3 Conclusions of rules

To fire a rule, a firing level is used. This means that the rule is fired only if the value of its premise membership function is greater than a predefined value γ, e.g. $\gamma = 0.5$.

Thus, in simple cases, when both a_0 and b_0 are on the same side of P_r, the rule is fired if the value of the membership function of the fuzzy set representing the *much bigger* or *much smaller* variable is equal or greater than the respective value of γ.

In the third case, when P_r is between a_0 and b_0, elaboration of the final tentative offer is more complicated. First of all, calculation of the fuzzy set in the premises requires first application of the linguistic operators *and* and *or*. As in the earlier cases, to fire the rule the premise membership function has to be greater than γ. However, in elabration of the final tentative offer the conclusions of all four rules are taken into account. This is achieved using the Sugeno expression. Let us denote the (fuzzy) conclusions of each of the four component rules as a_i for the seller and b_i for the buyer, where $i = 1, 2, 3, 4$. Then, the Sugeno expression is

$$a = \frac{\Sigma_{i=1}^{4} \alpha_i a_i}{\Sigma_{i=1}^{4} \alpha_i}$$

where α_i is the value of the membership functions of the corresponding premise at the ask a_i. The membership function values which are smaller than γ are taken as equal to 0.

In calculating the above expression, the algebraic rules for the fuzzy sets are needed. We deal with the triangular fuzzy sets. Let us consider two triangular fuzzy sets $A_1 = (m_1, \theta_1, \chi_1)$ and $A_2 = (m_2, \theta_2, \chi_2)$. Then, we have

$$A_1 \pm A_2 = (m_1 \pm m_2, \theta_1 + \theta_2, \chi_1 + \chi_2)$$

$$kA_1 = (km_1, k\theta_1, k\chi_1)$$

where k is a crisp number, see Appendix.

9.4.4 Final decision

The final decision on submitting or not submitting an offer is a fuzzy version of the earlier one, for the crisp values. For the seller they are as follows.

IF the tentative ask is outside the decision set
THEN no submitted ask
ELSE
IF b_0 belongs to the decision set and to the π_a-cut of the tentative ask a
THEN submitted ask $= b_0$
ELSE submitted ask $=$ argument of the maximum of the membership function of the tentative ask in the decision set.

Notice that b_0 belongs to the decision set when $c_s \leq b_0$. π_a is a threshold of decision for the tentative ask. It may be decided from the attitude to risk of the agent.

Now, for the **buyer** the decision rules are:

IF the tentative bid is outside the decision set
THEN no submitted bid
ELSE
IF a_0 belongs to the decision set and to the π_b-cut of the tentative bid
THEN submitted bid $= a_0$
ELSE submitted bid $=$ argument of the maximum of the membership function of the tentative bid in the decision set.

Similarly to before, a_0 belongs to the decision set, if it is not greater than the valuation of the good by the agent. π_b is the threshold of decision.

9.5 Adaptive agents

9.5.1 Attitude towards risk

Agents can be risk-neutral, risk-averse, or risk-seeking. These notions can be explained using the utility functions of the agents. Let us look at three utility functions of the selling agents on Figure 9.4. The utilities functions grow with the ask values (the gain rises, provided the deal with the price is closed). The linear utility function $U_s^N(a)$ is attributed to the risk-neutral agent. The risk-averse agent is characterized by a utility function $U_s^A(a) \geq U_s^N$, while the risk-seeking agent has the utility function $U_s^S(a) \leq U_s^N(a)$. Thus, a risk-averse agent is quickly satisfied with growing price, a risk-seeking agent is slowly satisfied, waiting for higher prices. In effect, it holds

$$U_s^S(a) \leq U_s^N(a) \leq U_s^A(a)$$

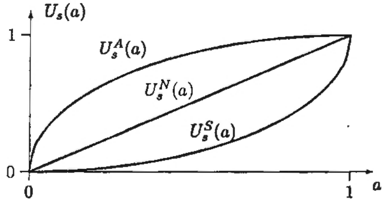

Figure 9.4: Utility functions of risk-averse, $U_s^A(a)$, risk-neutral, $U_s^N(a)$, and risk-seeking, $U_s^S(a)$, selling agents.

The attitude toward risk can be expressed by parameters of the strategy used by the agent. Three groups of parameters define a strategy. The parameters $\beta_{s,1}$ and $\beta_{s,2}$ are the values of the price step-up and step-down in the negotiation in simple situations when the outstanding bid is not big or the outstanding ask is not small. The parameters $\lambda_{s,i}, i = 1,\ldots 4$ are also price steps in the four component rules in the more difficult situations when the reference price is between the outstanding bid and the outstanding ask. The parameters $\gamma_{s,1}$ and $\gamma_{s,2}$ are the cut levels, see Figs. 9.1 and 9.2.

The following inequalities hold between parameters deciding on the strategy of the three kind of agents

$$\beta^A_{s,i} > \beta^N_{s,i} > \beta^S_{s,i}, \quad i = 1, 2$$

$$\lambda^A_{s,i} > \lambda^N_{s,i} > \lambda^S_{s,i}, \quad i = 1, 2, 3$$

$$\lambda^A_{s,4} > \lambda^N_{s,4} > \lambda^S_{s,4}$$

$$\gamma^A_{s,1} < \gamma^N_{s,1} < \gamma^S_{s,1}$$

$$\gamma^A_{s,2} > \gamma^N_{s,2} > \gamma^S_{s,2}$$

The directions of the inequalities are obvious. For example, a risk-averse selling agent reduces his asks more (with bigger $\beta_{s,i}, i = 1, 2$ or $\lambda_{s,i}, i = 1, 2, 3$) than a risk-neutral agent. Similarly, a risk-averse agent raises his asks (with smaller $\lambda_{s,4}$) the a risk-neutral one. Also, a risk-averse agent earlier decides that the outstanding bid is much bigger than the reference price, than the risk-neutral agent. And a risk-averse agent later decides that the outstanding ask is much bigger than the reference price, than the risk-neutral one.

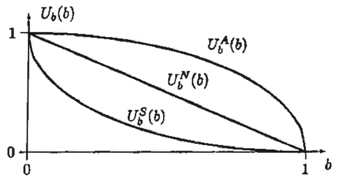

Figure 9.5: Utility functions of risk-averse, $U_b^A(b)$, risk-neutral, $U_b^N(b)$, and risk-seeking, $U_b^S(b)$, buying agents.

For analogous three kinds of the buying agents the similar inequalities hold, see Figure 9.5

$$U_s^S(a) \le U_s^N(a) \le U_s^A(a)$$

with the same dependencies between the parameters

$$\beta^A_{b,i} > \beta^N_{b,i} > \beta^S_{b,i}, \quad i = 1, 2$$

$$\lambda^A_{b,i} > \lambda^N_{b,i} > \lambda^S_{b,i}, \quad i = 1,2,3$$
$$\lambda^A_{b,4} > \lambda^N_{b,4} > \lambda^S_{b,4}$$
$$\gamma^A_{b,1} < \gamma^N_{b,1} < \gamma^S_{b,1}$$
$$\gamma^A_{b,2} > \gamma^N_{b,2} > \gamma^S_{b,2}$$

The parameters π_a and π_b are not considered in [14] in the set of parameters characterizing the attitude to risk. But the following inequalities hold for both selling and buying agents

$$\pi^A < \pi^N < \pi^S$$

9.5.2 Learning

The risk attitude of the agent can be conveniently used to design learning rules. The principle is as follows. If an agent makes transactions frequently, it means that it should become more risk-seeking, in hope of increasing its profit by negotiating better prices. In contrary, if an agent waits long for transactions, it should be more risk-averse. The notions of *transacting frequently* and *waiting long* are modeled using fuzzy sets with membership functions depicted in Figure 9.6. The transaction rate is calculated as a ratio of the transactions of the agent to all transactions made in the market after the last change of the agent parameters.

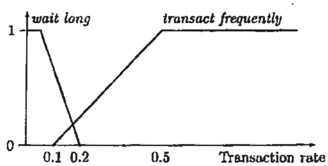

Figure 9.6: Memberships of the fuzzy set functions representing linguistic variables *wait long* and *transact frequently*.

In the learning algorithm the attitude toward risk A_{at} is normalized between -1, meaning the most averse attitude, through 0, the neutral attitude, until +1, the most risk-seeking attitude. Thus, $A_{at} \in [-1, 1]$. The attitude A_{at} is related to the parameters βs, λs, and γs. This relation is not specified in [14]. The simplest way is to normalize the values of the parameters beween -1 and 1 and assume that every parameter is an element of a vetor value of the attitude, which may take values between -1 and +1. Another, more complicated relation could be introduction of a norm in the parameter space, which is normalized to keep the norm between -1 and +1. Then the norm may be made eqivalent to A_{at}. Other options are possible as well.

Now, the learning rules are as follows

IF agent *waits long* to transact THEN $A = A - r\delta$
IF agent *transacts frequently* THEN $A = A + r\delta$

where δ is the minimum step, and $r > 0$ is the learning rate.

Three adjustment methods has been considered in [14]:

1. an agent adjust the attitude at the constant minimum rate δ, that is $r = 1$;

2. an agent adjust the attitude at a bigger step than the minimum rate δ, that is $r = m\delta$, $m > 1$;

3. an agent adjust the attitude randomly, that is r is an independent random number uniformly distributed in the range $[1, \tau]$, where τ is the maximum adjustment number.

The simulations performed in [14] indicate that the best agent performance is for the constant learning rate $r = 1$.

Appendix. Fuzzy sets and fuzzy numbers

To introduce the notion of a fuzzy set let us first consider a classical set A from an universe U. It can be conveniently described by the characteristic function χ_A defined as

$$\chi_A(u) = \begin{cases} 1 & \text{if } u \in A \\ 0 & \text{if } u \notin A \end{cases}$$

which say that a point $u \in U$ belongs to the set, if $\chi_A(u) = 1$, or does not belong, if $\chi_A(u) = 0$.

In a fuzzy set the characteristic function χ_A is generalized to take any value from the interval $[0, 1]$. It is then called *a membership function* and is denoted μ_A. The value of a membership function $\mu_A(u)$ reflects the degree of acceptance of the point u to the set. Thus, *a fuzzy set* is characterized by the set A and the membership function μ_A. Then, an usual set is a special fuzzy set with the membership function being the characteristic function. A comparison of a membership function and a characteristic function of a set is shown in Figure 9.7.

A fuzzy set can be also fully characterized by a family of so called γ-cuts[1] denoted by A_γ, i. e. points of U, for which the value $\mu_A(u)$ assumes at least the value γ, see Figure 9.7, where an example of a γ-cut for $\gamma = 0.5$ is depicted.

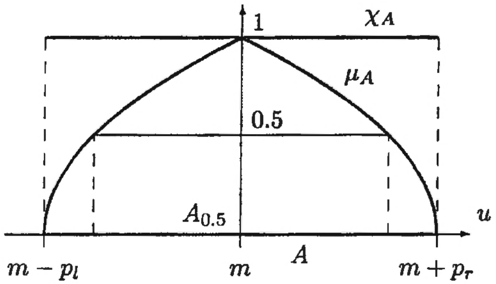

Figure 9.7: The characteristic function and a membership functions of the set A.

Two additional notions connected with a fuzzy set are worth to mention. One is *the support*, called supp A, which is the set of points u, for which the membership function is positive, i. e.:

$$\text{supp } A = \{u \in U : \mu_A(u) > 0\}$$

Another definition of the support may be formulated using γ-cuts, as

$$\text{supp } A = \lim_{\gamma \to 0} A_\gamma$$

[1]Here we call as the γ-cut of a fuzzy set A the notion usually called the α-cut, i.e. the set $A_\gamma = \{x \in \text{supp } A | \mu_A(x) \geq \gamma\}$, for $\gamma \in (0, 1]$.

The second notion is *the core* of the fuzzy set, called core A, which is the set of points, for which the membership function is equal 1, i. e.:

$$\text{core } A = \{u \in U : \mu_A(u) = 1\}$$

Using the notion of γ-cuts we may also write

$$\text{core } A = A_1$$

A special case of a fuzzy set A is called *a fuzzy number*, if it satisfies three additional conditions:

1. core A consists of only one point.
2. The membership function does not increase starting from the core point towards both sides.
3. Every γ-cut is a (connected) close interval.

The γ-cuts for a fuzzy number form a family of intervals. Each interval can be interpreted as our conviction in precision of knowledge of the core value. Values of the level γ close to 1 mean that we are well convinced that the core value is precise. Small values of γ, close to 0, mean that our conviction is small. Calculations performed on fuzzy numbers allow us to process whole this knowledge in common.

Technically, two functions defined for nonnegative arguments may be introduced, L and R, such that they have the unique value 1 at 0, $L(0) = R(0) = 1$, equal zero for arguments greater or equal 1, $L(u) = R(u) = 0$ for $u \geq 1$, and are not increasing. Then, given core $A = \{m\}$, the membership function of a fuzzy number may be constructed using the above functions as its left and right branches

$$\mu_A^l(u) = L\Big(\frac{m-u}{p_l}\Big) \quad \text{for } u \leq m \tag{9.1}$$

$$\mu_A^r(u) = R\Big(\frac{u-m}{p_r}\Big) \quad \text{for } u \geq m \tag{9.2}$$

where p_l and p_r are scale parameters, see Figure 9.7. Let us denote the fuzzy number constructed this way as $A = (m, p_l, p_r)_{LR}$.

Although operations on fuzzy sets or fuzzy numbers can be defined in a more general context, they will be restricted here only to fuzzy numbers described in the above LR form. For two fuzzy numbers $A = (m, p_l, p_r)_{LR}$ and $B = (n, q_l, q_r)_{LR}$ the following operations are defined:

1. Addition
$$A + B = (m + n, p_l + q_l, p_r + q_r)_{LR} \tag{9.3}$$

2. Multiplication by a positive real number c
$$cA = (cm, cp_l, cp_r)_{LR} \tag{9.4}$$

3. Multiplication by a negative real number c
$$cA = (cm, |c|p_r, |c|p_l)_{LR} \tag{9.5}$$

with interchange of the function L and R in (9.1) and (9.2)

$$\mu^l_{cA}(u) = R\Big(\frac{cm - u}{|c|p_r}\Big) \quad \text{for } u \leq cm$$

$$\mu^r_{cA}(u) = L\Big(\frac{u - cm}{|c|p_l}\Big) \quad \text{for } u \geq cm$$

Chapter 12

Conclusions

Emission permits are a new commodity that can have a very uncertain volume. Moreover, uncertainties for different types of greenhouse gases differ considerably. For example, uncertainty of emission of CO_2 from a power plant may be few percents, while that of N_2O from agricultural activities may be close to 100%. Thus, a risk for traders to realy reach the imposed emission level is much different when buing one or another emissions. Trading under such conditions requires new rules, but also provides a unique base to develop new strategies that are able to fulfill the requirements. Before it will be possible to include uncertainties in the agents behavior, the market scheme has to be designed and tested.

Given the tool as the *multi-agent system*, it is possible to design a market that is be simple, dynamic and that allows participants to adjust their desired profit and the time of placing an offer. The continuous double auction chosen in the report has simple rules and does not impose limitations on neither the number of participants nor their strategies.

The aim of the present report is to go through the most well-known strategies for this type of market, to classify them and to summarize their properties. The existing strategies can be divided into few groups: simple and reactive strategies (e.g. TT, ZI, ZIP); strategies that are using historical data to predict the prices (e.g. GD) and strategies that are exploiting features of agents and market configuration (e.g. Kaplan, AA). Most of the strategies (except for the very simple ones) result in the market price converging to equilibrium price and generally in most participants reaching profit.

The next step is to create agents that will dynamically adjust or even change their strategies depending on the situation on the market. After that, specific features of the emission market will be added to check how agents behave. Limit price will become a function of traded permits and participants would have to consider the level of uncertainty of the traded permit.

Bibliography

[1] K. Cai, J. Niu, and S. Parsons. Using evolutionary game-theory to analyse the performance of trading strategies in a continuous double auction market. In *Adaptive Agents and Multi-Agent Systems III. Adaptation and Multi-Agent Learning*, pages 44–59, 2007.

[2] D. Cliff. Minimal-intelligence agents for bargaining behaviors in market-based environments. Technical report, School of Cognitive and Computing Sciences, University of Sussex, 1997.

[3] D. Cliff. Zip60: Further explorations in the evolutionary design of online auction market mechanisms. Technical report, School of Cognitive and Computing Sciences, University of Sussex, 2005.

[4] E. Drabik. Wykorzystanie reguł aukcyjnych do handlu energią w polsce. *Przegląd statystyczny*, 57(4):70–88, 2010.

[5] Y. Ermoliev, M. Michalevich, and A. Nentjes. Markets for tradeable emission and ambient permits: A dynamic approach. *Environmental & Resource Economics*, 15(1):39–56, January 2000.

[6] T. Ermolieva, Y. Ermoliev, G. Fischer, M. Jonas, and M. Makowski. Cost effective and environmentally safe emission trading under uncertainty. *Lecture Notes in Economics and Mathematical Systems*, 633(2):79–99, 2010.

[7] T. Ermolieva, Y. Ermoliev, M. Jonas, G. Fischer, M. Makowski, F. Wagner, and W. Winiwater. A model for robust emission trading under uncertainties. *3rd International Workshop on Uncertainty in Greenhouse Gas Inventories*, pages 57–64, September 2010.

[8] D.P. Friedman and J. Rust. *The Double Auction Market, Institutions, Theories, and Evidence: Proceedings of the Workshop on Double Auction Markets, Held June, 1991 in Santa Fe, New Mexico.* Proceedings Volume, Santa Fe Institute Studies in the Scienc. Basic Books, 1993.

[9] S. Gjerstad and J. Dickhaut. *Price Formation in Double Auctions.* Computer science/mathematics. IBM T.J. Watson Research Center, 2000.

[10] O. Godal, Y. Ermoliev, G. Klaassen, and M. Obersteiner. Carbon trading with imperfectly observable emissions. *Environmental & Resource Economics*, 25(2):151–169, June 2003.

[11] D. K. Gode and S. Sunder. Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of Political Economy*, 101(1):119–137, 1993.

[12] M. He, N. R. Jennings, and H. Leung. On agent-mediated electronic commerce. *IEEE Transactions on Knowledge and Data Engineering*, 15:985 1003, 2003.

[13] M. He, N. R. Jennings, and H. Leung. On agent-mediated electronic commerce. *IEEE Trans on Knowledge and Data Engineering*, 15(4):985–1003, 2003.

[14] M. He, H. Leung, and N. R. Jennings. A fuzzy-logic based bidding strategy for autonomous agents in continuous double auctions. *IEEE Transactions on Knowledge and Data Engineering*, 15:1345–1363, 2003.

[15] Ch. Hood. Reviewing existing and proposed emissions trading systems information paper, November 2010.

[16] G. Klaassen, A. Nentjes, and M. Smith. Testing the dynamic theory of emissions trading: Experimental evidence for global carbon trading. Technical Report IR-01-063, International Institute for Applied Systems Analysis, November 2001.

[17] F. Lopes, M. Wooldridge, and A. Q. Novais. Negotiation among autonomous computational agents: principles, analysis and challenges. *Artif. Intell. Rev.*, 29(1):1–44, March 2008.

[18] H. Mizuta and Y. Yamagata. Agent-based simulation and greenhouse gas emissions trading. In *Winter Simulation Conference*, pages 535 540, 2001.

[19] Z. Nahorski and J. Horabik. Compliance and emission trading rules for asymmetric emission uncertainty estimates. *Climatic Change*, 103:303–325, 2010. 10.1007/s10584-010-9916-4.

[20] Z. Nahorski, J. Horabik, and M. Jonas. Compliance and emissions trading under the kyoto protocol: Rules for uncertain inventories. *Water, Air and Soil Pollution: Focus*, 7(4-5):539–558, September 2007.

[21] Z. Nahorski and W. Radziszewska. Price formation strategies of programmable agents in continuous double auctions. In M. Busłowicz and K. Malinowski, editors, *Advances in Control Theory and Automation*, pages 181 194. Komitet Automatyki PAN, Oficyna Wyd. Politechniki Białostockiej, 2012.

[22] Z. Nahorski, J. Stańczak, and P. Pałka. Multi-agent approach to simulation of the greenhouse gases emission permits market. *3rd International Workshop on Uncertainty in Greenhouse Gas Inventories*, pages 183 194, September 2010.

[23] S. Phelps, S. Parsons, and P. Mcburney. Automated trading agents verses virtual humans: An evolutionary game-theoretic comparison of two double-auction market designs, 2004.

[24] E. J. Pinker, A. Seidmann, and Y. Vakrat. Managing online auctions: Current business and research issues. *Management Science*, 49:2003, 2003.

[25] Ch. Preist and M. van Tol. Adaptive agents in a persistent shout double auction. Technical Report HPL-2003-242, Hewlett-Packard, December 2003.

[26] W. Radziszewska. Auction-based market for ghg permits. Technical Report RB/16/2011, IBS PAN, 2011.

[27] J. Rust, J. H. Miller, and R. Palmer. Behavior of trading automata in a computerized double auction market. *The Double Auction Market: Institutions, Theories, and Evidence*, pages 155–198, 1991.

[28] J. Rust, J. H. Miller, and R. Palmer. Characterizing effective trading strategies: Insights from a computerized double auction tournament. *Journal of Economic Dynamics and Control*, 18(1):61 – 96, 1994. <ce:title>Special Issue on Computer Science and Economics</ce:title>.

[29] V. Smith. An experimental study of comparative market behavior. *Journal of Political Economy*, 70:111 137, 1962.

[30] D.T. Spreng, T. Flüeler, D.L. Goldblatt, and J. Minsch. *Tackling Long-Term Global Energy Problems: The Contribution of Social Science*. Environment and Policy Series. Springer London, Limited, 2011.

[31] J. Stańczak. Application of an evolutionary algorithm to simulation of the co2 emission permits market with purchase prices. *Operations Research and Decisions*, 4:94–108, 2009.

[32] J. Stańczak and P. Bartoszczuk. Co2 emission trading model with trading prices. *Climatic Change*, 103:291–301, 2010.

[33] P. Vytelingum. *The Structure and Behaviour of the Continuous Double Auction*. PhD thesis, University of Southampton, December 2006.

[34] P. Vytelingum, D. Cliff, and N. R. Jennings. Strategic bidding in continuous double auctions. *Artificial Intelligence Journal*, 172(14):1700–1729, 2008.

[35] P. Vytelingum, R.K. Dash, M. He, and N. R. Jennings. A framework for designing strategies for trading agents. In *IJCAI Workshop on Trading Agent Design and Analysis*, pages 7–13, 2005.