37/2009

Raport Badawczy
Research Report

RB/33/2009

Compliance and emission trading rules for asymmetric emission uncertainty estimates

Z. Nahorski, J. Horabik

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 3810100

fax: (+48) (22) 3810105

Kierownik Pracowni zgłaszający pracę:
Prof. dr hab. inż. Zbigniew Nahorski

Warszawa 2009

INSTYTUT BADAŃ SYSTEMOWYCH
POLSKIEJ AKADEMII NAUK

Zbigniew Nahorski, Joanna Horabik

COMPLIANCE AND EMISSION TRADING RULES FOR ASYMETRIC EMISSION UNCERTAINTY ESTIMATES

Praca poprawiona po recenzjach, zaakceptowana do druku w *Climatic Change*.

Warszawa 2009

An addendum to the paper "Compliance and emission trading rules for asymmetric emission uncertainty estimates"

Zbigniew Nahorski, Joanna Horabik
Systems Research Institute, Polish Academy of Sciences
Newelska 6, 01-477 Warsaw
(Zbigniew.Nahorski, Joanna.Horabik)@ibspan.waw.pl

Abstract. This note contains a prove of the expression for the efficient emission with fuzzy asymmetric distributions in the paper (Nahorski & Horabik, 2009) and is an appendix to this paper. It includes some technical derivations, which place outside the main paper according to the rules of the *Climate Policy* editors.

Keywords: greenhouse gases emission inventories, uncertainty, fuzzy sets, emission permit trading.

1. Derivation of the expression for the efficient emission with fuzzy asymmetric distributions

From equation (27) of the paper (Nahorski & Horabik, 2009) repeated below for convenience

$$\hat{x}_c + \left\{1 - \left[(1 + \frac{d^l_{bc}}{d^u_{bc}} \frac{1+\gamma^u_{bc}}{1+\gamma^l_{bc}})\alpha\right]^{\frac{1}{1+\gamma^u_{bc}}}\right\} d^u_{bc} \leq (1-\delta)\hat{x}_b \tag{1}$$

the compliance condition of the buying party, after simple manipulation, is

$$\hat{x}^B_c + d^{uB}_{bc} - \left[\left((d^{uB}_{bc})^{1+\gamma^{uB}_{bc}} + (d^{uB}_{bc})^{\gamma^{uB}_{bc}} d^{lB}_{bc} \frac{1+\gamma^{uB}_{bc}}{1+\gamma^{lB}_{bc}}\right)\alpha\right]^{\frac{1}{1+\gamma^{uB}_{bc}}} \leq (1-\delta^B)\hat{x}^B_b$$

After buying $\hat{E}$ emission units from the selling party, the compliance condition becomes

$$\hat{x}^B_c - \hat{E}^S + d^{uB}_{bc} + \hat{E}^S R^{uS}_c - \left[\left((d^{uB}_{bc} + \hat{E}^S R^{uS}_c)^{1+\gamma^{uB}_{bc}} + \right.\right.$$
$$\left.\left.(d^{uB}_{bc} + \hat{E}^S R^{uS}_c)^{\gamma^{uB}_{bc}} (d^{lB}_{bc} + \hat{E}^S R^{lS}_c) \frac{1+\gamma^{uB}_{bc}}{1+\gamma^{lB}_{bc}}\right)\alpha\right]^{\frac{1}{1+\gamma^{uB}_{bc}}} \leq$$
$$\leq (1-\delta^B)\hat{x}^B_b \tag{2}$$

In approximations used in the sequel the following approximate equation will be applied

$$(1+z)^{\xi} \approx 1 + \xi z \tag{3}$$

which can be easily obtained from the first order Taylor series expansion of the left hand side. Good approximations are obtained for $|z| << 1$.

Let us consider the expression in the square brackets in equation (2). Applying (3) we get

$$(d_{bc}^{uB} + \hat{E}^S R_c^{uS})^{1+\gamma_{bc}^{uB}} = (d_{bc}^{uB})^{1+\gamma_{bc}^{uB}}(1 + \frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}})^{1+\gamma_{bc}^{uB}} \approx$$

$$\approx (d_{bc}^{uB})^{1+\gamma_{bc}^{uB}}(1 + (1+\gamma_{bc}^{uB})\frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}})$$

and similarly

$$(d_{bc}^{uB} + \hat{E}^S R_c^{uS})^{\gamma_{bc}^{uB}} \approx (d_{bc}^{uB})^{\gamma_{bc}^{uB}}(1 + \gamma_{bc}^{uB}\frac{\hat{E}^S R_C^{uS}}{d_{bc}^{uB}})$$

Let us notice that $\frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}} = \frac{\hat{E}^S d_c^{uS}}{\hat{x}_c d_{bc}^{uB}}$. As we can expect that the emission sold will be much smaller than that emitted by the selling party, then $\frac{\hat{E}^S}{\hat{x}_c} << 1$, and therefore approximation should be good. Thus, dropping the expressions with the squares of $\frac{\hat{E}^S}{\hat{x}_c}$ we can transform the expression in the square brackets in (2) to

$$(d_{bc}^{uB})^{1+\gamma_{bc}^{uB}}(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})+$$

$$+(1+\gamma_{bc}^{uB})\hat{E}^S R_c^{uS}(d_{bc}^{uB})^{\gamma_{bc}^{uB}}\left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}} + \frac{R_c^{lS}}{R_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}\right)$$

and then to

$$(d_{bc}^{uB})^{1+\gamma_{bc}^{uB}}(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})\times$$

$$\times\left(1 + (1+\gamma_{bc}^{uB})\hat{E}^S R_c^{uS}\frac{1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}} + \frac{d_c^{lS}}{d_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}}{d_{bc}^{uB}(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})}\right)$$

Now, raising above to the power $\frac{1}{1+\gamma_{bc}^{uB}}$ and using (3) we get

$$d_{bc}^{uB}(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})^{\frac{1}{1+\gamma_{bc}^{uB}}}\times$$

$$\times\Big(1+\hat{E}^S R_c^{uS}\frac{1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}+\frac{d_c^{lS}}{d_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}}{d_{bc}^{uB}(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})}\Big)=$$

$$=d_{bc}^{uB}(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})^{\frac{1}{1+\gamma_{bc}^{uB}}}+$$

$$+\hat{E}^S R_c^{uS}\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}+\frac{d_c^{lS}}{d_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}\Big)\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}\Big)^{-\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{uB}}}$$

The last but one term can be transformed as follows

$$\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}+\frac{d_c^{lS}}{d_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}\Big)\approx\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}\Big)\Big(1+\frac{d_c^{lS}}{d_c^{uS}}\frac{1}{1+\gamma_{bc}^{lB}}\Big)\approx$$

$$\approx\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\Big)^{\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}}\Big(1+\frac{d_c^{lS}}{d_c^{uS}}\Big)^{\frac{1}{1+\gamma_{bc}^{lB}}}$$

and similarly the last term

$$\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}\Big)^{-\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{uB}}}\approx\Big(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\Big)^{-\frac{\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}}}$$

Finally, the expression in the brackets in (2) approximately equals to

$$d_{bc}^{uB}(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})^{\frac{1}{1+\gamma_{bc}^{uB}}}+\hat{E}^S R_c^{uS}\Big(1+\frac{d_c^{lS}}{d_c^{uS}}\Big)^{\frac{1}{1+\gamma_{bc}^{lB}}}$$

Quality of approximations above depend on the ratios $\frac{d_{bc}^{lB}}{d_{bc}^{uB}}$ and $\frac{d_c^{lS}}{d_c^{uS}}$. When they are small, the approximation is good. When they are close to 1, the approximation should be worse. The final formula (5) is, however, also true for $\frac{d_c^{lS}}{d_c^{uS}}=1$, as can be seen from an independent derivation of (??). This may suggest that actually the formula (5) might be perhaps derived without using approximation (3), at least in the final part of the derivation.

Thus, condition (2) can be approximately written as

$$\hat{x}_c^B-\hat{E}^S+d_{bc}^{uB}+\hat{E}^S R_c^{uS}-d_{bc}^{uB}\Big[(1+\frac{d_{bc}^{lB}}{d_{bc}^{uB}}\frac{1+\gamma_{bc}^{uB}}{1+\gamma_{bc}^{lB}})\alpha\Big]^{\frac{1}{1+\gamma_{bc}^{uB}}}-$$

$$-\hat{E}^S R_c^{uS}\Big[\Big(1+\frac{d_c^{lS}}{d_c^{uS}}\Big)\alpha\Big]^{\frac{1}{1+\gamma_{bc}^{lB}}}\leq(1-\delta^B)\hat{x}_b^B \tag{4}$$

Comparing (4) with (1) we find the difference

$$E_{eff} = \hat{E}^S \Big\{ 1 - \{ 1 - \hat{E}^S R_c^{uS} \Big[\Big(1 + \frac{d_c^{lS}}{d_c^{uS}} \Big) \alpha \Big]^{\frac{1}{1+\gamma_{bc}^{lB}}} \} R_c^{uS} \Big\} \tag{5}$$

which is exactly equation (29) of the paper ((Nahorski & Horabik, 2009)).

References

Nahorski Z, Horabik J (2009) Compliance and emission trading rules for asymmetric emission uncertainty estimates, *Climatic Change*, submitted.