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**Compliance and emission
trading rules for nonsymmetric
emission uncertainty estimates**

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An addendum to the paper
 "Compliance and emission trading rules
 for nonsymmetric emission uncertainty estimates"

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Abstract. This note contains a prove of the expression for the efficient emission with fuzzy nonsymmetric distributions in the paper (Nahorski & Horabik, 2009) and is an appendix to this paper. It includes some technical derivations, which place outside the main paper according to the rules of the *Climate Policy* editors.

Keywords: greenhouse gases emission inventories, uncertainty, fuzzy sets, emission permit trading.

1. Derivation of the expression for the efficient emission
 with fuzzy nonsymmetric distributions

From equation (27) of the paper (Nahorski & Horabik, 2009) repeated below for convenience

$$\hat{x}_c + \left\{ 1 - \left[\left(1 + \frac{d_{bc}^u}{d_{bc}^u} \frac{1 + \gamma_{bc}^u}{1 + \gamma_{bc}^u} \right) \alpha \right]^{\frac{1}{1 + \gamma_{bc}^u}} \right\} d_{bc}^u \leq (1 - \delta) \hat{x}_b \quad (1)$$

the compliance condition of the buying party, after simple manipulation, is

$$\hat{x}_c^B + d_{bc}^{uB} - \left[\left((d_{bc}^{uB})^{1 + \gamma_{bc}^{uB}} + (d_{bc}^{uB})^{\gamma_{bc}^{uB}} d_{bc}^{lB} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right) \alpha \right]^{\frac{1}{1 + \gamma_{bc}^{uB}}} \leq (1 - \delta^B) \hat{x}_b^B$$

After buying \hat{E} emission units from the selling party, the compliance condition becomes

$$\begin{aligned} \hat{x}_c^B - \hat{E}^S + d_{bc}^{uB} + \hat{E}^S R_c^{uS} - \left[\left((d_{bc}^{uB} + \hat{E}^S R_c^{uS})^{1 + \gamma_{bc}^{uB}} + \right. \right. \\ \left. \left. (d_{bc}^{uB} + \hat{E}^S R_c^{uS})^{\gamma_{bc}^{uB}} (d_{bc}^{lB} + \hat{E}^S R_c^{lS}) \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right) \alpha \right]^{\frac{1}{1 + \gamma_{bc}^{uB}}} \leq \\ \leq (1 - \delta^B) \hat{x}_b^B \end{aligned} \quad (2)$$



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In approximations used in the sequel the following approximate equation will be applied

$$(1+z)^\xi \approx 1 + \xi z \quad (3)$$

which can be easily obtained from the first order Taylor series expansion of the left hand side. Good approximations are obtained for $|z| \ll 1$.

Let us consider the expression in the square brackets in equation (2). Applying (3) we get

$$\begin{aligned} (d_{bc}^{uB} + \hat{E}^S R_c^{uS})^{1+\gamma_{bc}^{uB}} &= (d_{bc}^{uB})^{1+\gamma_{bc}^{uB}} \left(1 + \frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}}\right)^{1+\gamma_{bc}^{uB}} \approx \\ &\approx (d_{bc}^{uB})^{1+\gamma_{bc}^{uB}} \left(1 + (1+\gamma_{bc}^{uB}) \frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}}\right) \end{aligned}$$

and similarly

$$(d_{bc}^{uB} + \hat{E}^S R_c^{uS}) \gamma_{bc}^{uB} \approx (d_{bc}^{uB}) \gamma_{bc}^{uB} \left(1 + \gamma_{bc}^{uB} \frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}}\right)$$

Let us notice that $\frac{\hat{E}^S R_c^{uS}}{d_{bc}^{uB}} = \frac{\hat{E}^S d_c^{uS}}{\hat{x}_c d_{bc}^{uB}}$. As we can expect that the emission sold will be much smaller than that emitted by the selling party, then $\frac{\hat{E}^S}{\hat{x}_c} \ll 1$, and therefore approximation should be good. Thus, dropping the expressions with the squares of $\frac{\hat{E}^S}{\hat{x}_c}$ we can transform the expression in the square brackets in (2) to

$$\begin{aligned} &(d_{bc}^{uB})^{1+\gamma_{bc}^{uB}} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}\right) + \\ &+ (1 + \gamma_{bc}^{uB}) \hat{E}^S R_c^{uS} (d_{bc}^{uB}) \gamma_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} + \frac{R_c^{lS}}{R_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}}\right) \end{aligned}$$

and then to

$$\begin{aligned} &(d_{bc}^{uB})^{1+\gamma_{bc}^{uB}} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}\right) \times \\ &\times \left(1 + (1 + \gamma_{bc}^{uB}) \hat{E}^S R_c^{uS} \frac{1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} + \frac{d_c^{lS}}{d_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}}}{d_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}\right)}\right) \end{aligned}$$

Now, raising above to the power $\frac{1}{1+\gamma_{bc}^{uB}}$ and using (3) we get

$$d_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}\right)^{\frac{1}{1+\gamma_{bc}^{uB}}} \times$$

$$\begin{aligned}
& \times \left(1 + \hat{E}^S R_c^{uS} \frac{1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} + \frac{d_c^{lS}}{d_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}}}{d_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right)} \right) = \\
& = d_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right)^{\frac{1}{1 + \gamma_{bc}^{uB}}} + \\
& + \hat{E}^S R_c^{uS} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} + \frac{d_c^{lS}}{d_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}} \right) \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right)^{-\frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{uB}}}
\end{aligned}$$

The last but one term can be transformed as follows

$$\begin{aligned}
\left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} + \frac{d_c^{lS}}{d_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}} \right) & \approx \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right) \left(1 + \frac{d_c^{lS}}{d_c^{uS}} \frac{1}{1 + \gamma_{bc}^{lB}} \right) \approx \\
& \approx \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \right)^{\frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}} \left(1 + \frac{d_c^{lS}}{d_c^{uS}} \right)^{\frac{1}{1 + \gamma_{bc}^{lB}}}
\end{aligned}$$

and similarly the last term

$$\left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right)^{-\frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{uB}}} \approx \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \right)^{-\frac{\gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}}}$$

Finally, the expression in the brackets in (2) approximately equals to

$$d_{bc}^{uB} \left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right)^{\frac{1}{1 + \gamma_{bc}^{uB}}} + \hat{E}^S R_c^{uS} \left(1 + \frac{d_c^{lS}}{d_c^{uS}} \right)^{\frac{1}{1 + \gamma_{bc}^{lB}}}$$

Quality of approximations above depend on the ratios $\frac{d_{bc}^{lB}}{d_{bc}^{uB}}$ and $\frac{d_c^{lS}}{d_c^{uS}}$. When they are small, the approximation is good. When they are close to 1, the approximation should be worse. The final formula (5) is, however, also true for $\frac{d_c^{lS}}{d_c^{uS}} = 1$, as can be seen from an independent derivation of (??). This may suggest that actually the formula (5) might be perhaps derived without using approximation (3), at least in the final part of the derivation.

Thus, condition (2) can be approximately written as

$$\begin{aligned}
\hat{x}_c^B - \hat{E}^S + d_{bc}^{uB} + \hat{E}^S R_c^{uS} - d_{bc}^{uB} \left[\left(1 + \frac{d_{bc}^{lB}}{d_{bc}^{uB}} \frac{1 + \gamma_{bc}^{uB}}{1 + \gamma_{bc}^{lB}} \right) \alpha \right]^{\frac{1}{1 + \gamma_{bc}^{uB}}} - \\
- \hat{E}^S R_c^{uS} \left[\left(1 + \frac{d_c^{lS}}{d_c^{uS}} \right) \alpha \right]^{\frac{1}{1 + \gamma_{bc}^{lB}}} \leq (1 - \delta^B) \hat{x}_b^B \quad (4)
\end{aligned}$$

Comparing (4) with (1) we find the difference

$$E_{eff} = \hat{E}^S \left\{ 1 - \left[1 - \hat{E}^S R_c^{uS} \left[\left(1 + \frac{d_c^{uS}}{d_c^{uS}} \alpha \right)^{\frac{1}{1+\gamma_{bc}}} \right] R_c^{uS} \right] \right\} \quad (5)$$

which is exactly equation (29) of the paper ((Nahorski & Horabik, 2009)).

References

- Nahorski Z, Horabik J (2009) Compliance and emission trading rules for nonsymmetric emission uncertainty estimates, *Climatic Change*, submitted.

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the 1990s, the number of people in the UK who are aged 65 and over has increased from 10.5 million to 13.5 million, and the number of people aged 75 and over has increased from 4.5 million to 6.5 million (Office for National Statistics 2000).

There is a growing awareness of the need to address the needs of older people, and the UK Government has set out a strategy for the 21st century (Department of Health 1999). The strategy is based on the following principles: (1) to improve the health and quality of life of older people; (2) to ensure that older people are able to live independently; (3) to ensure that older people are able to participate in society; and (4) to ensure that older people are able to live in their own homes.

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