Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

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The inclusion-exclusion principle on some algebraic structures

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Abstract

Recently P. Grzegorzewski ([3]) has generalized the well known inclusion - exclusion principle for IF-events. In the paper the abstract formulation of the principle is presented.

Keywords: inclusion-exclusion principle, algebraic structures.

1 Introduction

Let's recall the well-known inclusion-exclusion principle: May $\mathcal{R}$ be a lattice of subsets of a nonempty set $\Omega, A_i \in \mathcal{R}(i = 1, \ldots, n), m : \mathcal{R} \to R$ be a set function, such that $m(A \cup B) = m(A) + m(B) - m(A \cap B)$. Then

$$m\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} m(A_i) - \sum_{i<j}^{n} m(A_i \cap A_j) + \ldots + (-1)^{n+1} m\left(\bigcap_{i=1}^{n} A_i\right).$$

The principle can be generalized for some other objects as well. Grzegorzewski ([3]) works with IF-sets. An IF-set A on Ω is an ordered pair $A = (\mu_A, \nu_A)$, where $\mu_A, \nu_A : \Omega \to [0, 1]$ and following condition is satisfied:

$$\mu_A + \nu_A \leq 1.$$

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μ_A is called a membership function, ν_A is a nonmembership function. In addition, if μ_A and ν_A are Borel measurable, then A is an IF-event. The union and intersection of IF-sets are defined by this way:

$$A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B),\ A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B). \tag{1}$$

Grzegorzewski defines a probability of an IF-event A like a number from interval or the whole interval

$$\mathcal{P}(A) = \left[\int_\Omega \mu_A dP, 1 - \int_\Omega \nu_A dP\right], \tag{2}$$

where P is a probability measure on Ω. We consider $\mathcal{P}(A)$ to be the whole interval (see also [6]) and thus wee need to apply interval arithmetics. For addition and substraction will we use following formulas:

$$[\alpha, \beta] + [\gamma, \delta] = [\alpha + \gamma, \beta + \delta]$$

$$[\alpha, \beta] - [\gamma, \delta] = [\alpha - \delta, \beta - \gamma]$$

$\forall \alpha, \beta, \gamma, \delta \in R$. Within this approach, the following theorem holds:

Theorem 1 *May $A_i = (\mu_{A_i}, \nu_{A_i})$ be an IF-events. Then*

$$\mathcal{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mathcal{P}(A_i) - \sum_{i<j}^{n} \mathcal{P}(A_i \cap A_j) + \ldots + (-1)^{n+1} \mathcal{P}\left(\bigcap_{i=1}^{n} A_i\right).$$

Riečan ([5]) introduced more general, axiomatic approach to the probability on IF-sets and Ciungu and Riečan ([2]) have shown, that each probability on IF-sets has a form

$$\mathcal{P}(A) = \left[\mathcal{P}^\flat(A), \mathcal{P}^\sharp(A)\right], \tag{3}$$

$$\mathcal{P}^\flat(A) = \int_\Omega \mu_A dP + \alpha\left(1 - \int_\Omega (\mu_A + \nu_A) dQ\right),$$

$$\mathcal{P}^\sharp(A) = \int_\Omega \mu_A dR + \beta\left(1 - \int_\Omega (\mu_A + \nu_A) dS\right),$$

where P, Q, R, S are certain probability measures on Ω and $\alpha, \beta \in R$. There was shown, that the inclusion-exclusion principle works for this general form as well in [4]. Moreover, the formula (2) is a special case of (3), where $R = S$, $\alpha = 0$ and $\beta = 1$. In this paper we bring an abstract form of the inclusion-exclusion principle.

2 The abstract form of inclusion-exclusion principle

Theorem 2 *Let* $(A, +, \cdot)$ *be an algebraic structure, where* A *is a nonempty set,* $+$ *and* $\cdot$ *be commutative and associative binary operations defined on* A*,* m *be a mapping (*$m : A \to [0, 1]$*) and the following conditions hold:*

$\forall c \in A : c \cdot c = c,$

$\forall a, b \in A : m(a + b) = m(a) + m(b) - m(a \cdot b).$

Then

$$m(a_1 + \ldots + a_n) =$$

$$= \sum_{i=1}^{n} m(a_i) - \sum_{i<j}^{n} m(a_i \cdot a_j) + \sum_{i<j<k}^{n} m(a_i \cdot a_j . a_k) - \ldots + (-1)^{n+1} m \left(\prod_{i=1}^{n} a_i \right)$$

Proof. We shall use induction. The assertion holds for $n = 2$.

Let the assertion holds for $n \in N$. Then

$$m(a_1 + \ldots + a_n + a_{n+1}) =$$

$$= m(a_1 + \ldots + a_n) + m(a_{n+1}) - m((a_1 + \ldots + a_n).a_{n+1}) =$$

$$= m(a_1 + \ldots + a_n) + m(a_{n+1}) - m(a_1.a_{n+1} + \ldots + a_n.a_{n+1}) =$$

$$= \sum_{i=1}^{n} m(a_i) - \sum_{i<j}^{n} m(a_i.a_j) + \sum_{i<j<k}^{n} m(a_i.a_j.a_k) - \ldots + (-1)^{n+1} m \left(\prod_{i=1}^{n} a_i \right) +$$

$$+ m(a_{n+1}) - \sum_{i=1}^{n} m(a_i.a_{n+1}) + \sum_{i<j}^{n} m(a_i.a_j.a_{n+1}) - \ldots +$$

$$+ (-1)^{n} \sum_{i=1}^{n} m \left(\left(\prod_{k \neq i}^{n} a_k \right) .a_{n+1} \right) + (-1)^{n+1} m \left(\left(\prod_{i=1}^{n} a_i \right) .a_{n+1} \right) =$$

$$= \sum_{i=1}^{n+1} m(a_i) - \sum_{i<j}^{n+1} m(a_i.a_j) + \sum_{i<j<k}^{n+1} m(a_i.a_j.a_k) - \ldots +$$

$$+ (-1)^{n+1} \sum_{i=1}^{n+1} m \left(\prod_{k \neq i}^{n+1} a_k \right) + (-1)^{n+2} m \left(\prod_{i=1}^{n+1} a_i \right). \ \square$$

Example 1 *Let* $(L, \vee, \wedge)$ *be a distributive lattice and* $m : L \to [0, 1]$ *be a mapping such that*

$$m(a \vee b) + m(a \wedge b) = m(a) + m(b)$$

for any $a, b \in L$. *Put*

$$+ = \vee, \cdot = \wedge.$$

Then all assumptions Theorem 2 are satisfied.

Example 2 *May* F *be the family of all IF subsets of* Ω, *and may* $A \cup B$ *and* $A \cap B$ *be defined as in (1). May* $m : F \to [0, 1]$ *be a Gödel state, i.e.*

$$m(A \vee B) + m(A \wedge B) = m(A) + m(B).$$

Then again all assumptions of Theorem 2 are satisfied.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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