Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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On the distributive equation $\mathcal{I}(x, \mathcal{S}_1(y, z)) = \mathcal{S}_2(\mathcal{I}(x, y), \mathcal{I}(x, z))$ for t-representable t-conorms generated from nilpotent or strict t-conorms

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Abstract

Recently, in [5] we have examined the solutions of the following distributive functional equation $I(x, S_1(y, z)) = S_2(I(x, y), I(x, z))$, when S_1, S_2 are either both strict or nilpotent t-conorms and I is an unknown function. On the other side, in [3] and [4], we have discussed the distributive equation of implications $\mathcal{I}(x, \mathcal{T}_1(y, z)) = \mathcal{T}_2(\mathcal{I}(x, y), \mathcal{I}(x, z))$ over t-representable t-norms generated from strict or nilpotent t-norms in interval-valued fuzzy sets theory. In this work we continue these investigations, but for the following distributive functional equation $\mathcal{I}(x, \mathcal{S}_1(y, z)) = \mathcal{S}_2(\mathcal{I}(x, y), \mathcal{I}(x, z))$, when S_1, S_2 are t-representable t-conorms generated from either both strict or nilpotent t-conorms and $\mathcal{I}$ is an unknown function.

Keywords: interval-valued fuzzy sets; intuitionistic fuzzy sets; distributivity; fuzzy implication; triangular conorm; functional equations.

1 Introduction

Distributivity of fuzzy implications over different fuzzy logic connectives has been studied in the recent past by many authors (see [1], [15], [6], [13], [14], [5],

[2]). These equations have a very important role to play in efficient inferencing in approximate reasoning, especially fuzzy control systems (see [7]).

Recently, in [5] we have examined the solutions of the following distributive functional equation $I(x, S_1(y, z)) = S_2(I(x, y), I(x, z))$, when S_1, S_2 are either both strict or nilpotent t-conorms and I is an unknown function. On the other side, in [3] and [4], we have discussed the distributive equation of implications $\mathcal{I}(x, \mathcal{T}_1(y, z)) = \mathcal{T}_2(\mathcal{I}(x, y), \mathcal{I}(x, z))$ over t-representable t-norms generated from strict or nilpotent t-norms in interval-valued and intuitionistic fuzzy sets theories. In this work we continue these investigations, but for the following distributive functional equation

$$\mathcal{I}(x, \mathcal{S}_1(y, z)) = \mathcal{S}_2(\mathcal{I}(x, y), \mathcal{I}(x, z))$$

when S_1, S_2 are t-representable t-conorms generated from either both strict or nilpotent t-conorms and $\mathcal{I}$ is an unknown function.

We assume that the reader is familiar with the notion of intuitionistic (by Atanassov) fuzzy sets theory and interval-valued fuzzy sets theory. In [8] it is shown that both theories are equivalent from the mathematical point of view, thus in this article we discuss main results in the language of interval-valued fuzzy sets, but they can be easily transformed to the intuitionistic fuzzy case. Let us define

$$L^I = \{(x_1, x_2) \in [0, 1]^2 \ : \ x_1 \leq x_2\},$$
$$(x_1, x_2) \leq_{L^I} (y_1, y_2) \iff x_1 \leq y_1 \wedge x_2 \leq y_2.$$

In the sequel, if $x \in L^I$, then we denote it by $x = [x_1, x_2]$. One can easily observe that $\mathcal{L}^I = (L^I, \leq_{L^I})$ is a complete lattice with units $0_{\mathcal{L}^I} = [0, 0]$ and $1_{\mathcal{L}^I} = [1, 1]$. An interval-valued fuzzy set on X is a mapping $A\colon X \to L^I$.

2 Basic fuzzy connectives

We assume that the reader is familiar with the classical results concerning basic fuzzy logic connectives, but we briefly mention some of the results employed in the rest of the work.

Definition 1. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. An associative, commutative operation $\mathcal{S}\colon L^2 \to L$ is called a t-conorm if it is increasing and $0_{\mathcal{L}}$ is the neutral element of $\mathcal{S}$.

Definition 2. A t-conorm S on $([0, 1], \leq)$ is said to be

(i) strict, if S is continuous and strictly monotone, i.e., $S(x, y) < S(x, z)$ whenever $x < 1$ and $y < z$,

(ii) nilpotent, if S is continuous and if for each $x \in (0,1)$ there exists $n \in \mathbb{N}$ such that $x_S^{[n]} = 1$, where $x_S^{[n]} := \begin{cases} x, & \text{if } n = 1, \\ S(x, x_S^{[n-1]}), & \text{if } n > 1. \end{cases}$

The following characterizations of strict (nilpotent) t-conorms are well-known in the literature.

Theorem 1 ([12])**.** *A function $S\colon [0,1]^2 \to [0,1]$ is a strict t-conorm if and only if there exists a continuous, strictly increasing function $s\colon [0,1] \to [0,\infty]$ with $s(0) = 0$ and $s(1) = \infty$, which is uniquely determined up to a positive multiplicative constant, such that*

$$S(x,y) = s^{-1}(s(x) + s(y)), \qquad x, y \in [0,1].$$

Theorem 2 ([12])**.** *A function $S\colon [0,1]^2 \to [0,1]$ is a nilpotent t-conorm if and only if there exists a continuous, strictly decreasing function $s\colon [0,1] \to [0,\infty)$ with $s(0) = 0$ and $s(1) < \infty$, which is uniquely determined up to a positive multiplicative constant, such that*

$$S(x,y) = s^{-1}(\min(s(x) + s(y), s(1))), \qquad x, y \in [0,1].$$

In our article we shall consider the following special class of t-conorms.

Definition 3 (see [9])**.** A t-conorm $\mathcal{S}$ on $\mathcal{L}^I$ is called t-representable if there exist t-conorms S_1 and S_2 on $([0,1], \leq)$ such that $S_1 \leq S_2$ and

$$\mathcal{S}([x_1,x_2],[y_1,y_2]) = [S_1(x_1,y_1), S_2(x_2,y_2)], \qquad [x_1,x_2],[y_1,y_2] \in L^I.$$

It should be noted that not all t-conorms on $\mathcal{L}^I$ are t-representable (see [9]).

One possible definition of an implication on $\mathcal{L}^I$ is based on the well-accepted notation introduced by Fodor and Roubens [11] (see also [10]).

Definition 4. Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. A function $\mathcal{I}\colon L^2 \to L$ is called a fuzzy implication on $\mathcal{L}$ if it is decreasing with respect to the first variable, increasing with respect to the second variable and fulfills the following conditions: $\mathcal{I}(0_\mathcal{L}, 0_\mathcal{L}) = \mathcal{I}(1_\mathcal{L}, 1_\mathcal{L}) = \mathcal{I}(0_\mathcal{L}, 1_\mathcal{L}) = 1_\mathcal{L}$ and $\mathcal{I}(1_\mathcal{L}, 0_\mathcal{L}) = 0_\mathcal{L}$.

3 Some results pertaining to functional equations

In this section we show two results related to functional equations, which are crucial in presenting main results.

Proposition 1. *Let* $L_\infty = \{(u_1, u_2) \in [0,\infty]^2 \;:\; u_1 \le u_2\}$. *For a function* $f\colon L^\infty \to [0,\infty]$ *the following statements are equivalent:*

(i) f *satisfies the functional equation*

$$f(u_1+v_1, u_2+v_2) = f(u_1,u_2) + f(v_1,v_2), \quad (u_1,u_2),(v_1,v_2) \in L_\infty. \tag{A}$$

(ii) Either

$$f = 0, \tag{SA1}$$

or

$$f = \infty, \tag{SA2}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_1 = 0, \\ \infty, & \text{if } u_1 > 0, \end{cases} \tag{SA3}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_1 < \infty, \\ \infty, & \text{if } u_1 = \infty, \end{cases} \tag{SA4}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_2 = 0, \\ \infty, & \text{if } u_2 > 0, \end{cases} \tag{SA5}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_1 = u_2 < \infty, \\ \infty, & \text{if } u_1 = \infty \text{ or } u_1 < u_2, \end{cases} \tag{SA6}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_1 = 0 \text{ and } u_2 < \infty, \\ \infty, & \text{if } u_1 > 0 \text{ or } u_2 = \infty, \end{cases} \tag{SA7}$$

or

$$f(u_1,u_2) = \begin{cases} 0, & \text{if } u_2 < \infty, \\ \infty, & \text{if } u_2 = \infty, \end{cases} \tag{SA8}$$

or there exists unique $c \in (0,\infty)$ *such that*

$$f(u_1,u_2) = cu_1, \tag{SA9}$$

or

$$f(u_1,u_2) = \begin{cases} cu_2, & \text{if } u_1 = u_2, \\ \infty, & \text{if } u_1 < u_2, \end{cases} \tag{SA10}$$

or

$$f(u_1,u_2)=\begin{cases} cu_1, & \textit{if } u_2<\infty,\\ \infty, & \textit{if } u_2=\infty, \end{cases} \tag{SA11}$$

or

$$f(u_1,u_2)=\begin{cases} cu_2, & \textit{if } u_1=0,\\ \infty, & \textit{if } u_1>0, \end{cases} \tag{SA12}$$

or

$$f(u_1,u_2)=\begin{cases} c(u_2-u_1), & \textit{if } u_1<\infty,\\ \infty, & \textit{if } u_1=\infty, \end{cases} \tag{SA13}$$

or

$$f(u_1,u_2)=cu_2, \tag{SA14}$$

or there exist unique $c_1,c_2\in(0,\infty)$*,* $c_1\neq c_2$ *such that*

$$f(u_1,u_2)=\begin{cases} c_1(u_2-u_1)+c_2u_1, & \textit{if } u_1<\infty,\\ \infty, & \textit{if } u_1=\infty, \end{cases} \tag{SA15}$$

for all $(u_1,u_2)\in L_\infty$*.*

Proof. It is enough to define function $g(u_1,u_2):=f(u_2,u_1)$ and use the solutions described in [3, Proposition 3.2]. □

Proposition 2. *Fix real* $a,b>0$*. Let* $L_a=\{(u_1,u_2)\in[0,a]^2\ :\ u_1\le u_2\}$*. For a function* $f\colon L_a\to[0,b]$ *the following statements are equivalent:*

(i) f *satisfies the functional equation*

$$f(\min(u_1+v_1,a),\min(u_2+v_2,a))=\min(f(u_1,u_2)+f(v_1,v_2),b), \tag{B}$$

for all $(u_1,u_2),(v_1,v_2)\in L_a$*.*

(ii) Either

$$f=0, \tag{SB1}$$

or

$$f=b, \tag{SB2}$$

or

$$f(u_1,u_2)=\begin{cases} 0, & \textit{if } u_1=0,\\ b, & \textit{if } u_1>0, \end{cases} \tag{SB3}$$

or

$$f(u_1,u_2)=\begin{cases}0, & \textit{if } u_2=0,\\ b, & \textit{if } u_2>0,\end{cases} \tag{SB4}$$

or there exists unique $c\in\left[\frac{b}{a},\infty\right)$ *such that*

$$f(u_1,u_2)=\min(cu_1,b), \tag{SB5}$$

or

$$f(u_1,u_2)=\begin{cases}\min(cu_2,b), & \textit{if } u_1=u_2,\\ b, & \textit{if } u_1<u_2,\end{cases} \tag{SB6}$$

or

$$f(u_1,u_2)=\begin{cases}\min(cu_2,b), & \textit{if } u_1=0,\\ b, & \textit{if } u_1>0,\end{cases} \tag{SB7}$$

or

$$f(u_1,u_2)=\min(cu_2,b), \tag{SB8}$$

or there exist unique $c_1,c_2\in\left[\frac{b}{a},\infty\right)$, $c_1\neq c_2$ *such that*

$$f(u_1,u_2)=\begin{cases}\min(c_1(u_2-u_1)+c_2u_1,b), & \textit{if } u_1<a,\\ b, & \textit{if } u_1=a,\end{cases} \tag{SB9}$$

for all $(u_1,u_2)\in L_a$.

Proof. It is enough to define function $g(u_1,u_2):=f(u_2,u_1)$ and use the solutions described in [4, Proposition 3.2]. □

4 Distributive equation for t-representable t-conorms

In this section we will show how we can use solutions presented in the previous section to obtain all solutions, in particular fuzzy implications, of our main distributive equation

$$\mathcal{I}(x,\mathcal{S}_1(y,z))=\mathcal{S}_2(\mathcal{I}(x,y),\mathcal{I}(x,z)), \qquad x,y,z\in L^I, \tag{D}$$

where I is an unknown function and the t-conorms $\mathcal{S}_1,\mathcal{S}_2$ on $\mathcal{L}^I$ are t-representable and generated from both strict or nilpotent t-conorms.

Assume that projection mappings on $\mathcal{L}^I$ are defined as the following:

$$pr_1([x_1,x_2])=x_1, \qquad pr_2([x_1,x_2])=x_2, \qquad \text{for } [x_1,x_2]\in L^I.$$

One can easily see that if $\mathcal{S}_1$, $\mathcal{S}_2$ on $\mathcal{L}^I$ are t-representable and generated from S_1, S_2 and S_3, S_4, respectively, then

$$g^1_{[x_1,x_2]}([S_1(y_1,z_1),S_2(y_2,z_2)]) = S_3(g^1_{[x_1,x_2]}([y_1,y_2]),g^1_{[x_1,x_2]}([z_1,z_2])),$$
$$g^2_{[x_1,x_2]}([S_1(y_1,z_1),S_2(y_2,z_2)]) = S_4(g^2_{[x_1,x_2]}([y_1,y_2]),g^2_{[x_1,x_2]}([z_1,z_2])),$$

where $[x_1,x_2] \in L^I$ is arbitrarily fixed and functions $g^1_{[x_1,x_2]}, g^2_{[x_1,x_2]} \colon L^I \to L^I$ are defined by

$$g^1_{[x_1,x_2]}(\cdot) := pr_1 \circ \mathcal{I}([x_1,x_2],\cdot), \qquad g^2_{[x_1,x_2]}(\cdot) := pr_2 \circ \mathcal{I}([x_1,x_2],\cdot).$$

Let us assume firstly that $S_1 = S_2 = T$ is a strict t-conorm. Using the representation theorem of strict t-conorms (Theorem 1) we can transform our problem to the following equation (for a simplicity we deal only with g^1 now):

$$\begin{aligned} &g^1_{[x_1,x_2]}([s^{-1}(s(y_1)+s(z_1)),s^{-1}(s(y_2)+s(z_2))]) \\ &\qquad = s^{-1}(s(g^1_{[x_1,x_2]}([y_1,y_2])) + s(g^1_{[x_1,x_2]}([z_1,z_2]))). \end{aligned}$$

Hence

$$\begin{aligned} s \circ g^1_{[x_1,x_2]}([s^{-1}(s(y_1)+s(z_1)),&s^{-1}(s(y_2)+s(z_2))]) \\ &= s \circ g^1_{[x_1,x_2]}([y_1,y_2]) + s \circ g^1_{[x_1,x_2]}([z_1,z_2]). \end{aligned}$$

This equation can be written in the following form:

$$\begin{aligned} s\circ g^1_{[x_1,x_2]}&([s^{-1}(s(y_1)+s(z_1)),s^{-1}(s(y_2)+s(z_2))]) \\ &= s \circ g^1_{[x_1,x_2]}([s^{-1}(s(y_1)),s^{-1}(s(y_2))]) \\ &\quad + s \circ g^1_{[x_1,x_2]}([s^{-1}(s(z_1)),s^{-1}(s(z_2))]). \end{aligned}$$

Let us put $s(y_1) = u_1$, $s(y_2) = u_2$, $s(z_1) = v_1$ and $s(z_2) = v_2$. Of course $u_1,u_2,v_1,v_2 \in [0,\infty]$. Moreover $[y_1,y_2],[z_1,z_2] \in L^I$, thus $y_1 \le y_2$ and $z_1 \le z_2$. The generator s is strictly increasing, so $u_1 \le u_2$ and $v_1 \le v_2$. If we put

$$f_{[x_1,x_2]}(a,b) := s \circ pr_1 \circ \mathcal{I}([x_1,x_2],[s^{-1}(a),s^{-1}(b)]),$$

for $a,b \in [0,\infty]$ and $a \le b$, then we get the following functional equation

$$f_{[x_1,x_2]}(u_1+v_1,u_2+v_2) = f_{[x_1,x_2]}(u_1,u_2) + f_{[x_1,x_2]}(v_1,v_2), \tag{1}$$

where $(u_1,u_2),(v_1,v_2) \in L_\infty$. In a same way we can repeat all the above calculations but for the function g^2, to obtain the following functional equation

$$f^{[x_1,x_2]}(u_1+v_1,u_2+v_2) = f^{[x_1,x_2]}(u_1,u_2) + f^{[x_1,x_2]}(v_1,v_2), \tag{2}$$

where

$$f^{[x_1,x_2]}(a,b) := s \circ pr_2 \circ \mathcal{I}([x_1,x_2],[s^{-1}(a),s^{-1}(b)]).$$

Observe that (1) and (2) are exactly our functional equation (A). Therefore, using solutions of Proposition 1, we are able to obtain the description of the vertical section $\mathcal{I}([x_1,x_2],\cdot)$ for a fixed $[x_1,x_2] \in L^I$. Since in this proposition we have 15 possible solutions, we should have 225 different solutions of (D). Observe now that some of these solutions are not good, since the range of $\mathcal{I}$ is L^I.

Let us assume now that $S_1 = S_2$ and $S_3 = S_4$ are nilpotent t-conorms generated from additive generators s_1 and s_3, respectively. Using the representation theorem of nilpotent t-conorms (Theorem 2) we can transform our problem to the following equation (for a simplicity we deal only with g^1 now):

$$\begin{aligned} g^1_{[x_1,x_2]}([s_1^{-1}(\min(s_1(y_1)+s_1(z_1),s_1(1))),s_1^{-1}(\min(s_1(y_2)+s_1(z_2),s_1(1)))]) \\ = s_3^{-1}(\min(s_3(g^1_{[x_1,x_2]}([y_1,y_2]))+s_3(g^1_{[x_1,x_2]}([z_1,z_2])),s_3(1))). \end{aligned}$$

Similarly as earlier let us put $s_1(y_1) = u_1$, $s_1(y_2) = u_2$, $s_1(z_1) = v_1$ and $s_1(z_2) = v_2$. Of course $u_1,u_2,v_1,v_2 \in [0,s_1(1)]$. Moreover $[y_1,y_2],[z_1,z_2] \in L^I$, thus $y_1 \le y_2$ and $z_1 \le z_2$. The generator s_1 is strictly increasing, so $u_1 \le u_2$ and $v_1 \le v_2$. If we put

$$f_{[x_1,x_2]}(a,b) := s_3 \circ pr_1 \circ \mathcal{I}([x_1,x_2],[s_1^{-1}(a),s_1^{-1}(b)]),$$

for $a,b \in [0,s_1(1)]$ such that $a \le b$, then we get the following functional equation

$$\begin{aligned} f_{[x_1,x_2]}(\min(u_1+v_1,s_1(1)),\min(u_2+v_2,s_1(1))) \\ = \min(f_{[x_1,x_2]}(u_1,u_2)+f_{[x_1,x_2]}(v_1,v_2),s_3(1)), \end{aligned} \tag{3}$$

where $(u_1,u_2),(v_1,v_2) \in L_{s_1(1)}$ and $f_{[x_1,x_2]}\colon L_{s_1(1)} \to [0,s_3(1)]$ is an unknown function. In a same way we can repeat all the above calculations, but for the function g^2, to obtain the following functional equation

$$\begin{aligned} f^{[x_1,x_2]}(\min(u_1+v_1,s_1(1)),\min(u_2+v_2,s_1(1))) \\ = \min(f^{[x_1,x_2]}(u_1,u_2)+f^{[x_1,x_2]}(v_1,v_2),s_3(1)), \end{aligned} \tag{4}$$

where

$$f^{[x_1,x_2]}(a,b) := s_3 \circ pr_2 \circ \mathcal{I}([x_1,x_2],[s_1^{-1}(a),s_1^{-1}(b)]).$$

Observe that (3) and (4) are exactly our functional equation (B). Therefore, using solutions of Proposition 2, we are able to obtain the description of the vertical section $\mathcal{I}([x_1,x_2],\cdot)$ for a fixed $[x_1,x_2] \in L^I$. Since in this proposition we have 9 possible solutions, we should have 81 different solutions of (D). Observe now that some of these solutions are not good, since the range of $\mathcal{I}$ is L^I.

Finally, we need to notice that not all obtained vertical solutions in $\mathcal{L}^I$ in both cases can be used for obtaining fuzzy implications on $\mathcal{L}^I$ in the sense of Definition 4. We will investigate this problem in our future works.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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