

**Developments in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics.
Volume I: Foundations**

**Recent Advances in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics
Volume I: Foundations**

Editors

Editors
Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Krassimir T. Atanassov
Janusz Kacprzyk
Michał Baczyński
Maciej Krawczak
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrozny

SRI PAS



IBS PAN

**Recent Advances in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics
Volume I: Foundations**



Systems Research Institute
Polish Academy of Sciences

**Recent Advances in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics
Volume I: Foundations**

Editors

Krassimir T. Atanassov

Michał Baczyński

Józef Drewniak

Janusz Kacprzyk

Maciej Krawczak

Eulalia Szmidt

Maciej Wygralak

Sławomir Zadrozny

IBS PAN



SRI PAS

© **Copyright by Systems Research Institute**
Polish Academy of Sciences
Warsaw 2011

All rights reserved. No part of this publication may be reproduced, stored in retrieval system or transmitted in any form, or by any means, electronic, mechanical, photocopying, recording or otherwise, without permission in writing from publisher.

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl
ISBN 9788389475350

Intuitionistic Fuzzy Interpretations of Conway's Game of Life. Part 3: Modal and extended modal transformations of the game field

Lilija Atanassova¹ and Krassimir Atanassov²

¹ Institute of Information and Communication Technologies
Bulgarian Academy of Sciences
“Acad. G. Bonchev” Str., Block 2, Sofia 1113, Bulgaria
l.c.atanassova@gmail.com

² Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
krat@bas.bg

Abstract

Conway's Game of Life is a popular heuristic zero-player game, devised by John Horton Conway in 1970. Its “universe” is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive or dead. Every cell interacts with its eight neighbours, which are the cells that are directly horizontally, vertically, or diagonally adjacent. In a stepwise manner, the state of each cell in the grid preserves or alternates with respect to a given list of rules.

In a previous authors' research, for each cell its Intuitionistic Fuzzy (IF) estimation was defined as a pair consisting of the degrees l_p and l_a , namely degrees of presence and absence of life, where $l_p + l_a \leq 1$.

The article proposes intuitionistic fuzzy approaches to modifying of the Game of Life cells' states. These approaches are based on the IF modal and extended modal operators.

Keywords: Conway's game of life, intuitionistic fuzzy set, modal operator.

1 Introduction

Conway's Game of Life is devised by John Horton Conway in 1970, and already 40 years it is an object of research, software implementations and modifications. In [8] there is a list of many papers devoted to Conway's Game of Life. In [5], the authors introduced a modification of this game, based on the idea of the intuitionistic fuzziness (see [1]).

Here, in continuation of the ideas from [5], we will discuss the possibility to use some of the standard or extended modal operators, defined over Intuitionistic fuzzy sets (IFS; see, e.g., [1]).

The standard Conway's Game of Life has a "universe" which is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive or dead, or as we learned the game in the middle of 1970s, in the square there is an asterisk, or not.

Every cell interacts with its eight neighbours, namely the cells that are directly adjacent either in horizontal, vertical, or diagonal direction. In a stepwise manner, the state of each cell in the grid preserves or alternates with respect to a given list of rules.

Here, we will discuss some versions of the game in which we will keep the condition for the necessary number of existing neighbours asterisks for birth or dying of an asterisk in some square. For our aims, we will use elements of intuitionistic fuzzy set theory (see, [1]).

2 Previous results: Intuitionistic fuzzy criteria of existing, birthing and dying of an asterisk, and roles for changing of the game-field

Following [5], we will introduce some definitions.

Let us have a plane tessellated with squares. Let in some of these squares there be symbols "*", meaning that the squares are "alive". Now we will extend this construction of the Game of Life to some new forms.

Let us assume that the square $\langle i, j \rangle$ is assigned a pair of real numbers $\langle \mu_{i,j}, \nu_{i,j} \rangle$, so that $\mu_{i,j} + \nu_{i,j} \leq 1$. We can call the numbers $\mu_{i,j}$ and $\nu_{i,j}$ degree of presence and degree of absence of symbol "*" in square $\langle i, j \rangle$. Therefore, $\pi(i, j) = 1 - \mu_{i,j} - \nu_{i,j} \leq 1$ will correspond to the degree of uncertainty, e.g., lack of information about either presence or absence of an asterisk in the respective square.

In [5], first, we formulated a series of different criteria for correctness of the

intuitionistic fuzzy interpretations that will include as a particular case the standard game.

We will suppose that there exists an asterisk in square $\langle i, j \rangle$ if:

- **(1.1)** $\mu_{i,j} > 0.5$. Therefore $\nu_{i,j} < 0.5$. In the particular case, when $\mu_{i,j} = 1 > 0.5$ we obtain $\nu_{i,j} = 0 < 0.5$, i.e., the standard existence of the asterisk.
- **(1.2)** $\mu_{i,j} \geq 0.5$. Therefore $\nu_{i,j} \leq 0.5$. Obviously, if case (1.1) is valid, then case (1.2) also will be valid.
- **(1.3)** $\mu_{i,j} > \nu_{i,j}$. Obviously, case (1.1) is particular case of this one, but case (1.2) is not included in the currently discussed case for $\mu_{i,j} = 0.5 = \nu_{i,j}$.
- **(1.4)** $\mu_{i,j} \geq \nu_{i,j}$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of this one.
- **(1.5)** $\mu_{i,j} > 0$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of this one, but case (1.4) is not included in the currently discussed case for $\mu_{i,j} = 0.0 = \nu_{i,j}$.
- **(1.6)** $\nu_{i,j} < 1$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of this one, but case (1.5) is not included in the currently discussed case for $\mu_{i,j} = 0.0$.

From these criteria it follows that if one is valid – let it be s -th criterion ($1 \leq s \leq 6$) then we can assert that the asterisk exists with respect to s -th criterion and therefore, it will exist with respect to all other criteria, the validity of which follows from the validity of the s -th criterion.

On the other hand, if s -th criterion is not valid, then we will say that the asterisk is absent with respect to s -th criterion. It is very important that in this case the square may not be empty. It is suitable to tell that the square $\langle i, j \rangle$ is totally empty, if its degrees of presence and absence are $\langle 0, 1 \rangle$.

The square is s -full if it contains an asterisk with respect to the s -th criterion and that the square is s -empty if it does not satisfy the s -th criterion.

For the aims of the game-method for modelling, it will be suitable to use (with respect to the type of the concrete model) one of the first four criteria for existence of an asterisk. Let us say for each fixed square $\langle i, j \rangle$ that therein is an asterisk due to the s -th criterion for $1 \leq s \leq 4$, if this criterion confirms the existence of an asterisk.

In [5], four criteria for the birth of an asterisk and four criteria for the death of an asterisk are given. Also, seven intuitionistic fuzzy rules for changing the

game-field are introduced there. Two of these rules are extensions of the standard game-rules, while the rest five rules do not have analogues in the standard game.

3 Transformations of the game field by modal and extended modal operators

Let a set E be fixed. An IFS A in E is an object of the following form (see [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in E\},$$

where the functions $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let A be an IFS and let $\alpha, \beta \in [0, 1]$.

We shall define the following modal operators (see, e.g., [1]):

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E\};$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x) \rangle | x \in E\};$$

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x) \rangle | x \in E\};$$

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x) \rangle | x \in E\},$$

where $\alpha + \beta \leq 1$;

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x) \rangle | x \in E\}.$$

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x) \rangle | x \in E\},$$

$$H_{\alpha,\beta}^*(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x)) \rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x) \rangle | x \in E\},$$

$$J_{\alpha,\beta}^*(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x) \rangle | x \in E\}.$$

The first two operators are analogous of the operators in the modal logic (see, e.g., [7]). The next seven operators are extensions of the first two. It is important to denote that they do not have analogues in the modal logic.

3.1 Global transformations of the game field by modal and extended modal operators

Now, we can apply each of the nine modal operators over the game field F . Having in mind the above discussed IF-interpretations of the asterisk, we see, that we can construct the set

$$A = \{\langle\langle i, j \rangle, \mu_{i,j}, \nu_{i,j} \rangle \mid \langle i, j \rangle \in F\}.$$

On one hand, it corresponds to the game-field, while, on the other, it is an IFS. Now, if we apply some of the modal operators over IFS A , we will change the game-field in this way. For this aim, we must determine in a suitable way the values of arguments α and β of the extended modal operators. Thus, we will globally transform the game field. The first two (standard) modal operators can be used only for this type of transformations.

Let for a fixed cell $\langle i, j \rangle$ the subset of A that contains its neighbouring cells be

$$A_{(i,j)} = \{\langle\langle p, q \rangle, \mu_{p,q}, \nu_{p,q} \rangle \mid \langle p, q \rangle \in F \ \& \ p \in \{i-1, i, i+1\} \\ \& \ q \in \{j-1, j, j+1\} \ \& \ \langle p, q \rangle \neq \langle i, j \rangle\}.$$

3.2 Local transformations of the game field by the extended modal operators

These transformations are related to the way of determining arguments α and β of the extended modal operators. In the first case (previous subsection), they were given independently on the game. Here, they will be generated on the basis of the IF-values of the asterisks.

The transformations can be of two types. In the first case, the values of the asterisks that occupy the eight neighbours of a fixed asterisk will determine the values of the α and β -parameters. There are three cases for the calculation of these parameters.

6.1 (pessimistic parameter values):

$$\alpha = \min_{\langle p, q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \max_{\langle p, q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

In this paper we continue the numeration of [5, 6]

6.2 (average parameter values):

$$\alpha = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

6.3 (optimistic parameter values):

$$\alpha = \max_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \min_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

The α and β -values determined by (6.1) - (6.3) are used as parameters of a previously fixed extended modal operator that is applied over the IF-values of the fixed asterisk.

In the second case, the values of the asterisks that occupy the eight neighbours of a fixed asterisk will determine the values of the μ^* and ν^* -parameters for the asterisk in cell $\langle i, j \rangle$, while its original values $\mu_{i,j}$ and $\nu_{i,j}$ will be used for α and β -parameters, i.e, the extended modal operators that will be used for the transformation of the game field for cell $\langle i, j \rangle$ will have values $\alpha = \mu_{i,j}$ and $\beta = \nu_{i,j}$.

The μ^* and ν^* parameters will be calculated by the following three ways.

7.1 (pessimistic parameter values):

$$\mu_{i,j}^* = \min_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu_{i,j}^* = \max_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

7.2 (average parameter values):

$$\mu_{i,j}^* = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu_{i,j}^* = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

7.3 (optimistic parameter values):

$$\mu_{i,j}^* = \max_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu_{i,j}^* = \min_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

4 Conclusion

In the next authors' research new modifications of this game will be described. We will modify the rules of the game, as we did this in our previous research, e.g. [2, 3, 4].

Acknowledgement

The authors are grateful for the support provided by the projects DID-02-29 "Modelling processes with fixed development rules" and BIn-2/09 "Design and development of intuitionistic fuzzy logic tools in information technologies" funded by the National Science Fund, Bulgarian Ministry of Education, Youth and Science and by the Ministry of Science and Higher Education of Poland, Grant Nr N N519 384936.

References

- [1] Atanassov K. *Intuitionistic Fuzzy Sets*, Heidelberg, Springer, 1999.
- [2] Atanassov K., On a combinatorial game-method for modelling, *Advances in Modelling & Analysis*, AMSE Press, Vol. 19, 1994, No. 2, 41-47.
- [3] Atanassov K., L. Atanassova, A game method for modelling. Third International School "Automation and Scientific Instrumentation" (Ed. L. Antonov) Varna, 1984, 229-232.
- [4] Atanassov K., Atanassova L., Sasselov D., On the combinatorial game-method for modelling in astronomy, *Comptes Rendus de l'Academie bulgare des Sciences*, Tome 47, 1994, No. 9, 5-7.
- [5] Atanassova L., K. Atanassov, Intuitionistic fuzzy interpretations of Conway's Game of Life
- [6] Atanassova L., K. Atanassov, Intuitionistic fuzzy interpretations of Conway's Game of Life Part 2: Topological transformations of the game field. Proc. of 11th Workshop on Generalized Nets, Intuitionistic Fuzzy Sets and Knowledge Engineering, 9 July 2010, London (in press).
- [7] R. Feys, *Modal logics*, Gauthier-Villars, Paris, 1965.
- [8] <http://en.wikipedia.org/wiki/Conway>

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

<http://www.ibspan.waw.pl/ifs2010>

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475350
ISBN 838947535-9



9 788389 475350