Polska Akademia Nauk • Instytut Badań Systemowych

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Książka jubileuszowa
z okazji
70-lecia urodzin

PROFESORA KAZIMIERZA MAŃCZAKA

pod redakcją
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COORDINATION BY PRICE INSTRUMENTS; NETWORK CONGESTION CONTROL AND LOCAL PROBLEMS

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Abstract: *Coordination by price instruments represents a family of well-established techniques for optimization and control of complex systems. General properties of the Price Method have been investigated in a number of seminal books and various modifications of the coordinating conditions and strategies were developed with standard formulation of the local problems of the Price Method being commonly accepted. Recently, price instruments were found to be important for both understanding and improving Internet Congestion Control (ICC) and a number of ICC schemes were proposed to understand of how the data networks – a fundamental resource of our time – are or could be controlled. Both the scale and the complexity of the Internet make this control problem beyond the reach of most analytical modeling tools and feedback control techniques and require the usage of distributed, decentralized decision mechanisms. This paper summarizes basic properties of the Price Method and then focuses on price-based ICC, and on possible representation of the local (traffic source) problems.*

Keywords: *price method, coordination, network congestion control, utility function, flow optimization.*

1. Price Method; the basic facts

The Price Method, also known under the name of the Dual Method or the Interaction Balance Method (Mesarovic et al. 1970, Lasdon 1970, Findeisen et al. 1980, Tamura and Yoshikawa 1990) represents, in broad terms, a particular approach to modeling, optimization and control of complex systems. It follows the natural role played by prices on various markets - to achieve the balance between the demand and the supply. The method allows for various problem formulations and price adjustment (price coordination) strategies. Problem formulation may either result from a formal partitioning (decomposition) of a large-scale optimization problem or it may result from more practical, application case oriented considerations. Likewise, price adjustment strategies can be derived from dual function optimization, from solving sets of the coordinating conditions or from practical possibilities offered by a particular application.

Consider first the following, fairly general, deterministic problem related to static complex system (or dynamic system at equilibrium state) optimization:

$$\max_{x,u,y} \sum_i U_i(x_i, u_i, y_i) \tag{1}$$

subject to

$$x_i \in I_i, \ \ i = 1,\ldots,n \tag{2}$$

$$y_i \ = \ F_i(x_i, u_i) \text{ and } \ u_i = \sum_j H_{ij} y_j, \ i, j = 1,\ldots,N \tag{3}$$

$$\sum_i r_{ij}(x_i) \le c_l, \ \ l = 1,\ldots,L \tag{4}$$

In the above formulation the objective is to maximize the aggregate utility across N involved entities (system elements). x_i is a vector comprising local decision variables (of dimension n_{xi}), u_i is the local (vector) interaction input (dim $u_i = n_{ui}$) and y_i (dim $y_i = n_{yi}$) is the local interaction output of the i-th system element. In eqn. (3) it is assumed that the interaction output y_i is a unique deterministic function of x_i and u_i and that the *interaction coupling constraints* $u_i = \sum_j H_{ij} y_j$ are linear. Obviously, these relations can be made more general. Similarly, local decision constraints (represented by the sets I_i) can be imposed also on u_i. On the other hand sets I_i are often given in form of the box constraints $I_i = [x_{i,\min}, x_{i,\max}]$ (with $x_{i,\min}$ being the vector of lower bounds and $x_{i,\max}$ the vector of upper bounds on the components of x_i). Constraint (4) is the global resource constraint; there are assumed to be L resources which may be required by the local entities; $r_i(x_i)$ represents the consumption of the l-th resource by system element i. It is assumed that the optimization problem (1-4) has a solution.

The above optimization problem can be decomposed into N parallel Local Problems by introducing prices (in mathematical terms Lagrange multipliers) λ_i (dim $\lambda_i = n_{ui}$), $i = 1,\ldots,N$, associated with the coupling constraints $u_i = \sum_j H_{ij} y_j$, and p_l, $l = 1,\ldots,L$, representing prices of the global resources.

The i-th Local Problem is then defined – for given prices – as follows:

$$\max_{x_i, u_i} [U_i(x_i, u_i, y_i) - \lambda_i^T u_i + \sum_{j=1}^{N} \lambda_j^T H_{ji} y_i - \sum_{l=1}^{L} p_l r_{li}(x_i)] \tag{5}$$

where $\quad y_i = F_i(x_i, u_i)$ and subject to $x_i \in I_i$. (6)

Assuming *unique solutions* to the above N problems, $x_i(\lambda, p)$, $u_i(\lambda, p)$ and $y_i(\lambda, p)$ (where $y_i(\lambda, p) = F_i(x_i(\lambda, p))$, $u_i(\lambda, p)$ and $\lambda^T = (\lambda_1^T, \ldots, \lambda_N^T)$, $p^T = (p_1, \ldots, p_L)$; symbol T denotes transposition) one can seek such coordinating values λ^C and p^C for which the coupling constraints (3) and the resource constraints (4) are satisfied, that is

$$y_i(\lambda^C, p^C) = F_i(x_i(\lambda^C, p^C), u_i(\lambda^C, p^C)) \tag{7}$$

and

$$u_i(\lambda^C, p^C) = \sum_j H_i y_i(\lambda^C, p^C), i = 1,\ldots,N, \tag{8}$$

and

$$\sum_i r_{li}(x_i(\lambda^C, p^C)) \le l = 1,\ldots,L, \tag{8a}$$

and, in addition,

$$p_{\mathrm{l}}^C \ge 0,\ p_{\mathrm{l}}^C [\sum_i r_{li}(x_i(\lambda^C, p^C)) - c_l] = 0,\ \ l = 1,\ldots,L. \tag{9}$$

The above conditions (9) result from the requirement to satisfy overall optimality conditions. It simple terms they state that optimal prices of the resources must be nonnegative and that a positive price can be charged for the usage of commonly available resource only when this resource is fully utilized, i. e. when the respective resource constraint is active. On the other hand it should be observed that eventually some values of the components of λ_i^C *may be negative.*

The assumption about uniqueness of the local problem solutions – for given price values – is an essential one (Findeisen et al. 1980) to assure the existence of the coordinating prices λ^C, p^C. If these solutions do not have this property, then the coordinating prices satisfying eqns. (7-9) may easily not exist. One could expect that the lack of uniqueness of $x_i(\lambda, p), u_i(\lambda, p)$ for an isolated point $(\lambda^0, p^0), p^0 \geq 0$ should not matter too much. Alas, the Murphy's law says that if something may go wrong it will; and true enough it is easy to demonstrate with many important examples that if the local solutions are not unique for some pair (λ^0, p^0), they are also not unique for any pair (λ, p) that could possibly satisfy the coordinating conditions. Now, the simple sufficient conditions guaranteeing the uniqueness of $x_i(\lambda, p), u_i(\lambda, p)$ (and hence of $y_i(\lambda, p)$) for any feasible pair (λ, p) are: the strict concavity of the function $U_i(x_i, u_i, F_i(x_i, u_i))$ with respect to its arguments, linearity of F_i and the convexity of r_{li} and of the set I_i. More general conditions can also be given (e.g. Findeisen et al. 1980, Malinowski 1977) but in general nonlinear case the desirable uniqueness property of the local problem solutions is, unfortunately, not easily achievable.

Assuming the existence of λ^C, p^C one can propose a number of algorithms for iterating the values of λ, p. The most basic strategy is to use the following gradient method:

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} + \gamma[u_i(\lambda^{(k)}, p^{(k)}) - \sum_j H_{ij} y_i(\lambda^{(k)}, p^{(k)})] \tag{10}$$

$$p_l^{(k+1)} = \{p_l^{(k)} + \gamma[\sum_i r_{li}(x_i(\lambda^{(k)}, p^{(k)})) - c_l]\}_+ \tag{11}$$

for $i = 1, \ldots, N, l = 1, \ldots, L,$ where γ is a positive step size, $\{z\}_+ = \max\{0, z\}$ and k is the iteration index. The algorithm defined by (10,11) is a gradient strategy – with price projection on the feasible range – since, assuming the uniqueness of $x_i(\lambda, p), u_i(\lambda, p)$ for every feasible pair (λ, p) and each i, the expressions in square brackets in (10,11) are the slopes (with minus sign) of the dual function defined as the sum of the maxima of the local performance values (5) minus the term $\sum_l p_l^{(k)} c_l$. The dual function attains its minimum at λ^C, p^C.

The above price adjustment strategy will converge for sufficiently small value of γ if all local utility functions are strongly concave, functions F_i are linear, functions η_{li} are convex, sets I_i are convex and there exist a feasible point satisfying all the constraints, such that all the inequality constraints are inactive.

Algorithm (10, 11) has distributed character. In particular, adjustment of the resource prices p_l can be performed for each price independently from the other price adjustments. This property may appear to be very useful.

It should be observed that in the considered system optimization problem both the subsystem input-output equations (3), i.e. $y_i = F_i(x_i, u_i)$, and the performance functions U_i may often represent not simple function relations but rather statistical relationships between input resources and the output effects; these relations need then to be properly identified by statistical methods (Mańczak 1969, 1971). We will explicitly address this issue later, when discussing local problem formulations in case of the network congestion control with the use of price instruments.

In Section 2 a number of price-based Internet Congestion Control schemes will be presented and discussed. Then, in Section 3, possible extensions of the local problem formulation will be considered.

2. Network congestion control and price-based schemes

In recent years it was observed that the use of the price instruments could be made both to propose the new techniques for congestion control in data networks, in particular for Internet Congestion Control (ICC), and to better explain the existing congestion control mechanisms (the current TCP congestion control protocols). It is claimed in Low et al. (2002) that "As the Internet continues to expand in size, diversity, and reach, playing an ever-increasing role in the integration of other networks (transportation, finance, ...), having a solid understanding of how this fundamental resource is controlled becomes even more crucial". It is useful to note at this point that Internet pricing is essential not only for better possible understanding of the network operation but it may also provide means to achieve rational behavior of the network users, who otherwise may 'overgraze' the existing resources. Proper pricing of the network services is also necessary for many other purposes, not only for congestion control (DaSilva 2000, Falkner 2000). Dynamic pricing may be useful, in particular, for balancing demand for access to application servers and for proper valuation of different classes and qualities of service (Armitage 2000). With Internet, the Price Method has met a large-scale system for which this method seems to be extremely well suited. It seems rather obvious that the general problem formulation presented in Section 1 may not make a framework general enough to accommodate all network pricing mechanisms and applications. Yet, this problem and its properties should be helpful, as stated above, to better understand those mechanisms and to propose their modifications. Let us concentrate on the congestion control schemes.

In several congestion control mechanisms, as recently proposed by Kelly et al. (1998), Low and Lapsley (1999), La and Anantharam (2000), Malinowski (2001), the network is represented by N traffic sources, representing particular source-destination pairs, and a grid of a set of L links. The links, together with associated routers, are the network resources of limited traffic carrying capacity c_l (c_l can be expressed e.g. in terms of packets per period). Each source is i supposed to use a set $L(i) \subseteq L$ of links. These sets define and $L \times N$ routing matrix R (fixed routing is assumed); the element R_{li} of R is equal to 1 if $l \in L(i)$ and is equal to 0 otherwise. Each source i has at time t an associated transmission rate $x_i(t)$; the set of transmission rates determines the aggregate flow $y_l(t)$ through each link, by the equation (Low et al. 2002):

$$y_l(t) = \sum_i R_{li} x_i(t - \tau_{li}^f), \tag{12}$$

where τ_{li}^f is the transmission delay (forward delay) from sources to links.

Then, the feedback mechanism communicates to sources the congestion information about the network. This congestion measure – the price $p_l(t)$ – is a positive valued quantity associated with the link l. The fundamental assumption is made that sources have access to the aggregate price of all links in their route (this information can be e.g. piggybacked on the ACK packet messages):

$$q_i(t) = \sum_l R_{li} p_l(t - \tau_{li}^b). \tag{13}$$

Here we allow for backward delays τ_{li}^b in the feedback path.

2.1. TCP congestion control and network flow optimization by price instruments.

The above model (12, 13) includes (Low et al. 2002) the mechanism being present in a number of existing congestion control protocols, with a different interpretation for price in different protocols (e.g. packet loss probability in TCP Reno and queuing delay in TCP Vegas).

To define the congestion control system, it remains to specify in what manner the sources adjust their transmission rates based on aggregate prices (13) – the TCP algorithm – and in what manner the links (the link routers) adjust their prices based on aggregate rates – the Active Queue Management algorithm. Low et al. (2002) propose the following dynamic laws:

at the source level:

$$d/dt(z_i) = F_i(z_i,q_i), \; x_i = G_i(z_i,q_i) \tag{14}$$

and, at the link level:

$$d/dt(w_l) = H_l(y_l,w_l), \; p_l = K_l(y_l,w_l) \tag{15}$$

where the key restrictions in those control laws is that they must be decentralized (i. e distributed between the large number of sources and – usually much smaller – the number of links). In (14) and (15) above z_i and w_l represent, respectively, the states of the source and link controllers.

If we consider the network with the above feedback at equilibrium, assuming the transmission rates and prices at some steady-state values x^*, y^*, p^*, q^*, then relations (12, 13) yield immediately the following relationships: $y^* = Rq^*$ and $q^* = R^T p^*$. As far as eqns. (14) are concerned we make an assumption that at equilibrium the rates satisfy the relationship

$$x_i^* = f_i(q_i^*),$$

where $f_i(\cdot)$ is a positive, strictly monotone decreasing function; this function in the static case is just given by the source static law. This assumption allows us to introduce an optimization interpretation for the equilibrium. Namely, if we introduce a source utility function $U_i(x_i)$, then we can define this function to be an integral of $f_i^{-1}(x_i)$; that is $U_i'(x_i) = f_i^{-1}(x_i)$.

Then, by this construction the equilibrium rate x_i^* will solve the following local source problem:

$$\max_{x_i \geq 0} [U_i(x_i) - q_i^* x_i]. \tag{16}$$

The above equation has an obvious economic interpretation – at the equilibrium each source i maximizes its profit equal to utility minus payment charged by the network. It is important to note that this interpretation can be made for any reasonable congestion control protocol.

The role of prices p (and $q = R^T p$) at steady-state is to coordinate the actions of the individual sources; in fact to ensure that the solutions of (16) together solve the network flow optimization problem

$$\max_{x \geq 0} \sum_i U_i(x_i) \tag{17}$$

$$\text{subject to } Rx \leq c, \text{ where } c^T = (c_1, \ldots, c_L), \tag{18}$$

in other words – maximize aggregate utility across all sources, subject to link capacity constraints.

It can be immediately seen that the optimization problem defined by (17, 18) is a particular instance of the optimization problem (1-4), with none coupling constraints existing and with linear global resource constraints (18) in place of general nonlinear constraints (4). Problem (16) represents in this case the local problem associated with the i-th source and the coordination strategy (11) can be employed to find the coordinating prices.

It has to be understood, however, that the above optimization problem formulation is just an "optimization" interpretation of the network equilibrium which can be reached in steady-state conditions by using a stable control protocol. It does not mean that the local network users (the sources) *are conscious of their utilities or are willing to maximize them when paying "price" q for a unit transmission rate*. The source transmission rates x_i are decided by the control protocol, like TCP Reno or TCP Vegas, or other. Yet, this "optimization" interpretation demonstrates that an optimization framework together with price coordination may allow for better understanding of the network control mechanisms. In particular, Low et al. (2002) provide the utility functions which – if used by sources to control their transmission rates – would yield the equilibria which are attainable under several different network control protocols.

2.2. Utility sensitive sources; network coordination by price instruments

Assume now that the traffic sources (or source-destination pairs) *are indeed utility oriented*, i.e. they have utilities $U_i(x_i)$ and *are willing* to maximize profits defined as $U_i(x_i) - q_i x_i$ (as in eqn. (16)). In such case it would seem possible – at least in theory – to propose the following congestion control scheme – CCS-1:

For given link prices $p_l(k), l = 1,\dots,L$, at time k, the sources solve local problems

$$\max_{x_i \in I_i} [U_i(x_i) - q_i(k)x_i], \text{ where } q_i(k) = \sum_{l \in L(i)} p_l(k) \tag{19}$$

and where $L(i)$ is the set of those links which are used for transmission by the i-th source (path for source-destination pair); $I_i = [x_{i,\min}, x_{i,\max}]$. The solutions $x_i^*(k) = x_i(q_i(k))$ are signaled to all concerned links, which then adjust their link prices – for the next iteration $k+1$ – according to the following rule

$$p_l^{(k+1)} = \{p_l^{(k)} + \gamma[\sum_{i \in S(l)} x_i^*(k) - c_l]\}_+ \tag{20}$$

where $S(l)$ is the set of all sources transmitting through the link l and γ is a positive step – chosen to allow the scheme to converge. Then the new link prices are signaled to links, etc., until the convergence is obtained.

Law and Lapsley (1999) studied the above scheme in detail. It should be observed that this scheme – a simple application of the basic Price Method – is not directly concerned with an actual network operation; at least until it converges. If the traffic sources transmit during period k at the rate $x_i^*(k)$, then the routers associated with the congested links buffer and backlog or, finally, drop packets which cannot be sent over those links.

It was proposed by Malinowski (2001) to modify the above scheme in the following way (scheme CCS-2). Once the sources determine their rates $x_i^*(k)$ by solving local problems (19) at the beginning of time period k, the resulting traffic is routed through the network and each link experiences and observes the actual flow rate $y_l^r(k)$. Then the link prices are changed, for the next period $k+1$, according to the distributed pricing (coordination) rule

$$p_l^{(k+1)} = \{ p_l^{(k)} + \gamma [y_i^r (k) - c_l^*] \}_+ , \tag{21}$$

where c_l^* is chosen to be smaller than the full link capacity c_l.

This means that the objective is to satisfy at steady-state the modified link capacity constraints

$$\sum_{i \in S(l)} x_i^*(k) \leq c_l^* \tag{22}$$

The *headroom* $h_r = c_i - c_i^*$ is left to provide for traffic bursts and for early congestion notification. In this modified scheme there is no need for the sources to signal their desired transmission rates $x_i^*(k)$ to the links. This is an important feature, since there can be numerous sources transmitting through a given link. The new link prices are, as before, signaled to the sources before next values of the source rates are established. This scheme is now under a detailed simulation study.

The sources may, instead of choosing their transmission rates, choose payments per unit time that they would be willing to make when they are being charged by the network the price p_i for a unit of data transmitted to the destination. In particular La and Anantharam (2000) propose the following scheme (CCS-3).

During time period k the source is transmitting at rate $x_i(k) = x_i^0(v_i(k), v_i^-(k))$, where $v_i(k)$ is the payment to the network chosen by source i for this period (of unit length), and where $v_i^-(k)$ is the collection of payments proposed by all other sources. The rates $x_i(k)$ are established – at steady state – by special (proportionally fair) control protocol at the TCP layer (Mo and Walrand 1998). This protocol ensures that $x_i^0(\cdot, v_i^-(k))$ is an increasing function of $v_i(k)$ for fixed $v_i^-(k)$. Knowing both the actual (measured) rate $x_i(k)$ and the actual chosen payment per period $v_i(k)$, the source estimates the price it is paying per unit of data sent (e.g. price per standard packet) as $p_i(k) = v_i(k) / x_i(k)$. Then the source solves the following problem to choose payment $v_i(k+1)$ for the next period

$$\max_{0 \le v_i \le v_{i,\max}} [U_i(v_i / p_i(k)) - v_i] \tag{23}$$

The above local source problem is, in fact, the same as the problem (19), if we take into account that actually the source is wishing to maximize its transmission rate $x_i = v_i / p_i(k)$.

This short survey of price-based congestion control schemes is by far not exhaustive; in particular Kelly proposed a number of interesting mechanisms (e.g. Kelly et al. 1998). Low and his co-workers proposed also a number of modifications of the basic pricing scheme CCS-1; in particular by allowing for asynchronous operation of distributed decision rules. Yet most of these schemes, differing in many aspects, assume the same local problem (19) or (23) as solved by a source to choose its transmission rate or the payment per period. The last section of this paper is concerned with possible modifications and extensions of this problem and with impact of such modifications on the congestion control.

3. Local Problems - possible formulations and open questions

A number of objections or criticisms can be raised with respect to the local problems formulated as (19) or (23). Since these problems are, in fact, equivalent, the discussion will be focused on the problem (19). Let us consider the following issues:

1) If a given traffic source is utility concerned, this does not necessarily mean that the source total profit can be modeled as utility minus transmission rate times price per data unit. Actually, it may appear that the cost terms, expressed in utility units – which can be non-monetary units – exhibit constant elasticity to payments, rather than constant sensitivity; in other words the source decision making could be modeled as

$$\max_{x_i \in I_i} [U_i(x_i) - \alpha_i (q_i(k) x_i)^{\beta_i}], \tag{24}$$

where, for example $\beta_i > 1$, instead of $\max_{x_i \in I_i} [U_i(x_i) - q_i(k) x_i]$. In general then, the source problem (19) may take the form:

$$\max_{x_i \in I_i} [U_i(x_i) - C(q_i(k), x_i)], \tag{25}$$

with the utility decrement function C dependent in a nonlinear manner on $q_i(k)$ and x_i It is not clear in this case, what would be the meaning of a system equilibrium achieved, if possible at all, with such prices that provide for the link capacity constraints to be satisfied.

2) In problem (19) it is explicit that deterministic behavior of the sources is assumed; source transmission rate is supposed to depend only on the aggregate price $q_i(k)$. This may well be too much of a simplification. The internal source demand for bandwidth to transmit data, expressed through the source utility, may fluctuate – usually it will – and change also due to other factors than q_i. The source may, in particular, go off for short periods of time. One can model such behavior by assuming that the source desired *average* rate $x_i^*(k)$ within period is given as

$$x_i^*(k) = \arg \max_{x_i \in I_i} [U_i(x_i, \theta_i(k)) - q_i(k)x_i], \tag{26}$$

where $\theta_i(k)$ is a sample of random variable θ_i. Each time the *i*-th source establishes its rate it is based on current $q_i(k)$ and on current sample $\theta_i(k)$ of θ_i; i.e. $x_i^*(k) = x_i(q_i(k), \theta_i)$ for $\theta_i = \theta_i(k)$. What can be then the objective of the price adjustment mechanism at the link level? Well, the only reasonable objective could be to seek such link prices p_l^* and the associated source prices $q_i^* = \sum_{l \in L(i)} p_l$, that the following constraints are satisfied

$$E_\theta\{\sum_{i \in S(l)} x_i(q_i(k), \theta_i)\} \le c_l^*, \tag{27}$$

where $\theta = (\theta_1, \ldots, \theta_N)$. Here again, as in the case of scheme CCS-2 described in Section 2, the headroom $h_r = c_i - c_i^*$ must be left to provide for enough space to accommodate random traffic variations. Price coordination for large problems with stochastic inputs was discussed in Malinowski (1992); it was shown there that the existence of random elements in a large-scale optimization problem formulation could make this problem very difficult, if not impossible, to solve with decomposition-coordination techniques. Such is also the case of the network flow optimization, as considered in this paper.

The price adjustment algorithm should have the form

$$p_l^{(k+1)} = \{p_l^{(k)} + \gamma\, E_\theta\{\textstyle\sum_{i\in S(l)} x_i(q_i(k),\theta_i) - c_l^*]\}_+ \tag{28}$$

but then it would not be possible to implement this procedure since the expected value in (28) cannot be computed or even estimated in an existing computer network. One may then propose to use the following approximation strategy

$$p_l^{(k+1)} = \{p_l^{(k)} + \gamma(k)\{\textstyle\sum_{i\in S(l)} x_i(q_i(k),\theta_i(k)) - c_l^*]\}_+, \tag{29}$$

where the sequence of positive step coefficients $\gamma(k)$ should satisfy the following conditions: $\gamma(k)\to 0^+$ and $\sum_k \gamma(k) = +\infty$. Under mild conditions the above scheme should converge. It might be recommended to use the following step values $\gamma(k) = a_1/(a_2+k)$, where a_1 and a_2 are natural numbers, and $a_2 >> a_1$. The approximation strategies of type (29) are for a long time well known to be very useful for identification (e.g. Mańczak 1971) of statistical relationships and for optimization of decision rules; for a recent application in network control see e.g. (Marbach et al. 2000). The presented price adjustment scheme (29) requires further studies.

3) The next issue is that the original source utility function, whether dependent on θ_i or not, may not be a strictly concave function of x_i. In fact, it can be argued that a source may exhibit S-shaped utility: for small values of transmission rates the utility goes up sharply (is locally convex) with the increasing transmission rate, then – for larger rates – the shape changes into the concave one. The basic results concerning the Price Method, summarized in Section 1, make it clear that with non-concave source utilities one may expect difficulties due to, possibly, non-unique solutions of the source problems. On the other hand, it is known that for a very big number of market players the price mechanism can work well enough even if some of those players cannot make unique decisions. In case of Internet the number of network users is also big and so the congestion control with dynamic pricing may work well in case when the source utility functions fail to be strictly concave. This issue also requires further studies; some preliminary results are given in Kozakiewicz et al. (2002).

4. Final remarks

In this paper the basic properties of the Price Method were reminded and then possible applications of the price-based mechanisms to Internet congestion control were discussed. In fact the size and the properties of the Internet motivate the renewed interest in these mechanisms. The pricing "technology" is very much needed because there do not exist other optimization based technologies that could cope with the challenges offered by the network. Similarly, control of the network results in the renewed interest in using the stochastic approximation algorithms, like that of eqn. (29), for congestion control as well as for call admission control (Marbach et al. 2000).

It was further claimed in this paper that the basic formulation of the local problems of the Price Method might need significant modifications. Those may then require new price adjustment algorithms. Thus the further study of pricing mechanisms for large-scale engineering-economic systems is indeed required.

It is finally worth to observe that in Internet control applications the price instruments can meet the postulate formulated in Malinowski (1987) in regard to those instruments. Namely, they could be made both the coordinating variables and the variables with respect to which the experimental search for a satisfactory equilibrium of this very large-scale system is performed.

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