Polska Akademia Nauk • Instytut Badań Systemowych

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Książka jubileuszowa
z okazji
70-lecia urodzin

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pod redakcją
Jakuba Gutenbauma

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UNCERTAIN VARIABLES AND THEIR APPLICATIONS IN KNOWLEDGE-BASED PATTERN RECOGNITION

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Abstract: *The purpose of this work is to show how uncertain variables may be applied to a class of pattern recognition systems described by a relational knowledge representation with unknown parameters. The short description of uncertain logics and variables is given in the first part of the work. In the second part, formulations and solutions of the pattern recognition problem for different cases are presented. Two simple examples illustrate the approach considered in the paper.*

Keywords: *uncertain variables, uncertain systems, pattern recognition, knowledge-based systems.*

1. Introduction

The idea of the so called uncertain variables based on uncertain logics has been introduced and developed as a tool for the analysis and decision problems in a class of uncertain systems described by traditional mathematical models or by the relational knowledge representations (Bubnicki 1998, 2000c, 2001b, 2001c, 2002a). The uncertain variable is described by certainty distribution given by an expert and expressing his/her knowledge concerning different approximate values of the variable. The uncertain variables are related to random and fuzzy variables but there are also essential differences. The so called *soft variables* as a generalization of random, uncertain and fuzzy variables have been presented in (Bubnicki 2001d, 2001e).

The purpose of this work is to show how the uncertain variables may be applied to a class of pattern recognition systems with a relational knowledge representation containing unknown parameters which are assumed to be the values of uncertain variables described by certainty distributions. Such a description may be much simpler and nearer to traditional models of the recognition problem than the description in the form of fuzzy rules. In the first part of the work a short presentation of the uncertain logics and variables is given. Details can be found in (Bubnicki 2001b, 2001c, 2002a).

2. Uncertain logics and variables

Consider a universal set Ω, $\omega \in \Omega$, a set $\overline{X} \subseteq R^k$, a function $g: \Omega \to \overline{X}$, a crisp property (predicate) $P(\overline{x})$ and the crisp property $\Psi(\omega, P)$ generated by P and g: "For $\overline{x} = g(\omega) \triangleq \overline{x}(\omega)$ assigned to ω the property P is satisfied". Let us introduce now the property $G_\omega(x) =$ "$\overline{x}(\omega) \cong x$" for $x \in X \subseteq \overline{X}$, which means: "$\overline{x}$ is approximately equal to x" or "x is the approximate value of $\overline{x}$". The properties P and G_ω generate the soft property $\overline{\Psi}(\omega, P)$ in Ω: "the approximate value of $\overline{x}(\omega)$ satisfies P", i.e.

$$\overline{\Psi}(\omega, P) = \text{"}\overline{x}(\omega) \mathrel{\tilde{\in}} D_x\text{"}, \qquad D_x = \{\overline{x} \in \overline{X} : P(\overline{x})\}, \tag{1}$$

which means: "$\overline{x}$ approximately belongs to D_x". Denote by $h_\omega(x)$ the logical value of $G_\omega(x)$:

$$w[\overline{x}(\omega) \cong x] \triangleq h_\omega(x), \qquad \bigwedge_{x \in X} h_\omega(x) \geq 0, \qquad \max_{x \in X} h_\omega(x) = 1. \tag{2}$$

Definition 1. The *uncertain logic L* is defined by Ω, $\overline{X}$, X, crisp predicates $P(\overline{x})$, the properties $G_\omega(x)$ and the corresponding functions $h_\omega(x)$ for $\omega \in \Omega$. In this logic we consider soft properties (1) generated by P and G_ω. The logical value of $\overline{\Psi}$ is

$$w[\overline{\Psi}(\omega, P)] \triangleq v[\overline{\Psi}(\omega, P)] = \begin{cases} \max\limits_{x \in D_x} h_\omega(x) & \text{for } D_x \neq \varnothing \\ 0 & \text{for } D_x = \varnothing \end{cases}$$

and is called a *certainty index*. The operations are defined as follows:

$$v[\neg\overline{\Psi}(\omega, P)] = 1 - v[\overline{\Psi}(\omega, P)], \tag{3}$$

$$v[\Psi_1(\omega, P_1) \vee \Psi_2(\omega, P_2)] = \max\{v[\Psi_1(\omega, P_1), v[\Psi_2(\omega, P_2)]\}, \tag{4}$$

$$v[\Psi_1(\omega,P_1)\wedge\Psi_2(\omega,P_2)]=\begin{cases} 0 \text{ if for each } x \;\; w(P_1\wedge P_2)=0 \\ \min\{v[\Psi_1(\omega,P_1),v[\Psi_1(\omega,P_2)]\} \;\; \text{otherwise} \end{cases} \quad (5)$$

where Ψ_1 is $\overline{\Psi}$ or $\neg\overline{\Psi}$, and Ψ_2 is $\overline{\Psi}$ or $\neg\overline{\Psi}$ □

It is easy to note that G_ω is a special case of $\overline{\Psi}$ for $D_x = \{x\}$ (a singleton) and

$$v[\bar{x}(\omega)\cong x]=h_\omega(x), \qquad v[\bar{x}(\omega)\tilde{\neq} x]=1-h_\omega(x).$$

For the logic L one can prove the following statements:

$$v[\overline{\Psi}(\omega,P_1\vee P_2)]=v[\overline{\Psi}(\omega,P_1)\vee\overline{\Psi}(\omega,P_2)], \quad (6)$$

$$v[\overline{\Psi}(\omega,P_1\wedge P_2)]\le\min\{v[\overline{\Psi}(\omega,P_1)],v[\overline{\Psi}(\omega,P_2)]\}, \quad (7)$$

$$v[\overline{\Psi}(\omega,\neg P)]\ge v[\neg\overline{\Psi}(\omega,P)]. \quad (8)$$

The interpretation (semantics) of the uncertain logic L is the following: The uncertain logic operates with crisp predicates P, but for the given ω it is not possible to state if P is true or false because the function g and consequently the value $\bar{x}$ are unknown. The function $h_\omega(x)$ is given by an expert, who by "looking at" ω obtains some information concerning $\bar{x}$ and uses it to evaluate his opinion regarding $\bar{x}\cong x$. For the same (Ω, X) we may have the different logics (the different h_ω) determined by different experts.

Definition 2 (the *uncertain logic C*). The first part is the same as in Def.1. The certainty index of $\overline{\Psi}$ and the operations are defined as follows:

$$v_c[\overline{\Psi}(\omega,P)=\frac{1}{2}\{v[\overline{\Psi}(\omega,P)]+1-v[\overline{\Psi}(\omega,\neg P)]\}$$

$$=\frac{1}{2}[\max_{x\in D_x} h_\omega(x)+1-\max_{x\in\overline{D}_x} h_\omega(x)]$$

where $\overline{D}_x$ is a complement of D_x,

$$\neg\overline{\Psi}(\omega,P)=\overline{\Psi}(\omega,\neg P), \quad (9)$$

$$\overline{\Psi}(\omega,P_1)\vee\overline{\Psi}(\omega,P_2)=\overline{\Psi}(\omega,P_1\vee P_2), \quad (10)$$

$$\overline{\Psi}(\omega,P_1)\wedge\overline{\Psi}(\omega,P_2)=\overline{\Psi}(\omega,P_1\wedge P_2) \quad □ \quad (11)$$

One can note that G_ω is a special case of $\overline{\Psi}$ and

$$v_c[\bar{x}(\omega) \cong x] = \frac{1}{2}[h_\omega(x) + 1 - \max_{\breve{x} \in X - \{x\}} h_\omega(\breve{x})]. \tag{12}$$

For the logic *C* one can prove the following statements:

$$v_c[\overline{\Psi}(\omega, P_1 \vee P_2)] \geq \max\{v_c[\overline{\Psi}(\omega, P_1)], v_c[\overline{\Psi}(\omega, P_2)]\}, \tag{13}$$

$$v_c[\overline{\Psi}(\omega, P_1 \wedge P_2)] \leq \min\{v_c[\overline{\Psi}(\omega, P_1)], v_c[\overline{\Psi}(\omega, P_2)]\}, \tag{14}$$

$$v_c[\neg\overline{\Psi}(\omega, P)] = 1 - v_c[\overline{\Psi}(\omega, P)]. \tag{15}$$

The variable $\bar{x}$ for a fixed ω will be called an uncertain variable. Two versions of uncertain variables will be defined by: $h(x)$ given by an expert and the definitions of certainty indexes $w(\bar{x} \tilde{\in} D_x)$, $w(\bar{x} \tilde{\notin} D_x)$, $w(\bar{x} \tilde{\in} D_1 \vee \bar{x} \tilde{\in} D_2)$, $w(\bar{x} \tilde{\in} D_1 \wedge \bar{x} \tilde{\in} D_2)$.

Definition 3. *L-uncertain variable* $\bar{x}$ is defined by *X*, the function $h(x) = v(\bar{x} \cong x)$ given by an expert and the following definitions:

$$v(\bar{x} \tilde{\in} D_x) = \max_{x \in D_x} h(x) \text{ for } D_x \neq \emptyset \text{ and } 0 \text{ for } D_x = \emptyset, \tag{16}$$

$$v(\bar{x} \tilde{\notin} D_x) = 1 - v(\bar{x} \tilde{\in} D_x), \tag{17}$$

$$v(\bar{x} \tilde{\in} D_1 \vee \bar{x} \tilde{\in} D_2) = \max\{v(\bar{x} \tilde{\in} D_1), v(\bar{x} \tilde{\in} D_2)\}, \tag{18}$$

$$v(\bar{x} \tilde{\in} D_1 \wedge \bar{x} \tilde{\in} D_2) = \begin{cases} \min\{v(\bar{x} \tilde{\in} D_1), v(\bar{x} \tilde{\in} D_2)\} & \text{for } D_1 \cap D_2 \neq \emptyset \\ 0 & \text{for } D_1 \cap D_2 = \emptyset. \end{cases} \tag{19}$$

The function $h(x)$ will be called *L-certainty distribution* □

The definition of *L-uncertain* variable is based on logic *L*. Then, for (1) the properties (6), (7), (8) are satisfied. In particular, (8) becomes:

$$v(\bar{x} \tilde{\in} \overline{D}_x) \geq v(\bar{x} \tilde{\notin} D_x) = 1 - v(\bar{x} \tilde{\in} D_x).$$

Definition 4. *C-uncertain variable* is defined by *X*, $h(x) = v(\bar{x} \cong x)$ given by an expert and the following definitions:

$$v_c(\bar{x} \tilde{\in} D_x) = \frac{1}{2}[\max_{x \in D_x} h(x) + 1 - \max_{x \in \overline{D}_x} h(x)], \tag{20}$$

$$v_c(\bar{x} \,\tilde{\notin}\, D_x) = 1 - v_c(\bar{x} \,\tilde{\in}\, D_x), \tag{21}$$

$$v_c(\bar{x} \,\tilde{\in}\, D_1 \vee \bar{x} \,\tilde{\in}\, D_2) = v_c(\bar{x} \,\tilde{\in}\, D_1 \cup D_2), \tag{22}$$

$$v_c(\bar{x} \,\tilde{\in}\, D_1 \wedge \bar{x} \,\tilde{\in}\, D_2) = v_c(\bar{x} \,\tilde{\in}\, D_1 \cap D_2) \quad \square \tag{23}$$

The definition of *C*-uncertain variable is based on logic *C*. Then for (1) the properties (13), (14), (15) are satisfied. According to (9) and (15) $v_c(\bar{x} \,\tilde{\notin}\, D_x) = v_c(\bar{x} \,\tilde{\in}\, \overline{D}_x)$. The function $v_c(\bar{x} \cong x) \triangleq h_c(x)$ may be called *C-certainty distribution.*

Let us consider a plant with the input vector $u \in U$ and the output vector $y \in Y$, described by a relation $R(u, y; x) \subset U \times Y$ where x is an unknown parameter which is assumed to be a value of an uncertain variable $\bar{x}$ described by the certainty distribution $h_x(x)$.

Analysis problem: For the given R, $h_x(x)$, $D_u \subset U$ (obtained as the result of the observation) and $D_y \subset Y$ (given by a user), find $v[D_y \,\tilde{\subseteq}\, D_y(\bar{x})]$ where

$$D_y(x) = \{ y \in Y : \bigvee_{u \in D_u} (u, y) \in R(u, y; x) \}$$

is the set of all possible outputs. To solve the problem one should determine $D_x(D_u) = \{ x \in X : D_y \subseteq D_y(x) \}$. Then

$$v[D_y \,\tilde{\subseteq}\, D_y(\bar{x})] = v[\bar{x} \,\tilde{\in}\, D_x(D_u)] = \max_{x \in D_x(D_u)} h_x(x). \tag{24}$$

The value (24) denotes the certainty index of the property: for the approximate value of $\bar{x}$ the set of all possible outputs contains the set D_y given by a user.

Decision problem: For the given R, $h_x(x)$ and D_y find the decision u^* maximizing the certainty index of the property: for the approximate value of $\bar{x}$ the set of all possible outputs belongs to D_y given by a user. Then

$$u^* = \arg\max_u v[D_y(\bar{x}) \,\tilde{\subseteq}\, D_y] = \arg\max_u \max_{x \in \hat{D}_x(u)} h_x(x)$$

where

$$\hat{D}_x(u) = \{x \in X : D_y(x) \subseteq D_y\} = \{x \in X : u \in D_u(x)\}$$

where $D_u(x) \subset U$ is the largest set such that the implication $u \in D_u(x) \rightarrow y \in D_y$ is satisfied, i.e. $D_u(x) = \{u \in U : D_y(x) \subseteq D_y\}$.
In the above formulations $\bar{x}$ has been treated as *L*-uncertain variable. The considerations for the *C*-uncertain variable have analogous forms.

3. Pattern recognition

Let an object to be recognized or classified be characterized by a vector of features $u \in U$ which may be observed, and the index of a class j to which the object belongs; $j \in \{1, 2, ..., M\} \triangleq J$, where M is the number of classes. The set of objects may be described by a relational knowledge representation $R(u, j) \in U \times J$ which is reduced to the sequence of sets

$$D_u(j) \subset U, \quad j = 1, 2, ..., M,$$

i.e.

$$D_u(j) = \{u \in U : (u, j) \in R(u, j)\}.$$

Assume that as a result of the observation it is known that $u \in D_u \subset U$. The recognition problem may consist in finding the set of all possible indices j, i.e. the set of all possible classes to which the object may belong (Bubnicki 1993, 2001a, Szala 2002).

Recognition problem: For a given sequence $D_u(j)$, $j \in \overline{1, M}$ and the result of observation D_u find the smallest set $D_j \subset J$ for which the implication

$$u \in D_u \rightarrow j \in D_j$$

is satisfied. This is the specific analysis problem for the relational plant and

$$D_j = \{j \in J : \ D_u \cap D_u(j) \neq \varnothing\}$$

where $\varnothing$ denotes an empty set. In particular, if $D_u = \{u\}$, i.e. we obtain the exact result of the measurement, then

$$D_j = \{j \in J : u \in D_u(j)\}.$$

Now let us assume that the knowledge representation contains a vector of unknown parameters $x \in X$ and x is assumed to be a value of an uncertain variable $\bar{x}$ described by a certainty distribution $h_x(x)$ given by an expert.

Recognition problem for uncertain parameters: For the given sequence $D_u(j;x)$, $h_x(x)$, D_u and the set $\hat{D}_j \subset J$ given by a user one should find the certainty index that the set $\hat{D}_j$ belongs to the set of all possible classes

$$D_j(x) = \{ j \in J : D_u \cap D_u(j;x) \neq \varnothing \}. \tag{25}$$

It is easy to see that

$$v[\hat{D}_j \,\tilde{\subseteq}\, D_j(\bar{x})] = v[\bar{x} \,\tilde{\in}\, D_x(\hat{D}_j)] \tag{26}$$

where

$$D_x(\hat{D}_j) = \{ x \in X : \hat{D}_j \subseteq D_j(x) \}. \tag{27}$$

Then

$$v[\hat{D}_j \,\tilde{\subseteq}\, D_j(\bar{x})] = \max_{x \in D_x(\hat{D}_j)} h_x(x). \tag{28}$$

In particular, for $\hat{D}_j = \{j\}$ one can formulate the optimization problem consisting in the determination of a class j maximizing the certainty index that j belongs to the set of all possible classes.

Optimal recognition problem: For the given sequence $D_u(j;x)$, $h_x(x)$ and D_u one should find j^* maximizing

$$v[j \,\tilde{\in}\, D_j(\bar{x})] \triangleq v(j).$$

Using (26), (27) and (28) for $\hat{D}_j = \{j\}$ we obtain

$$v(j) = v[\bar{x} \,\tilde{\in}\, D_x(j)] = \max_{x \in D_x(j)} h_x(x) \tag{29}$$

where

$$D_x(j) = \{ x \in X : j \in D_j(x) \} \tag{30}$$

and $D_j(x)$ is determined by (25). Then

$$j^* = \arg\max_j v(j) = \arg\max_j \max_{x \in D_x(j)} h_x(x). \tag{31}$$

Assume that the different unknown parameters are separated in the different sets, i.e. the knowledge representation is described by the sets $D_u(j;x_j)$

where $x_j \in X_j$ are subvectors of x, different for the different j. Assume also that $\overline{x}_j$ and $\overline{x}_i$ are independent uncertain variables for $i \neq j$ and $\overline{x}_j$ is described by the certainty distribution $h_{xj}(x_j)$. In this case, according to (25)

$$j \in D_j(x) \Leftrightarrow D_u \cap D_u(j;x_j) \neq \varnothing .$$

Then

$$v(j) = v[\bigvee_{u \in D_u} u \,\tilde{\in}\, D_u(j;\overline{x}_j)] = v[\overline{x}_j \,\tilde{\in}\, D_{xj}(j)] \tag{32}$$

where

$$D_{xj}(j) = \{x_j \in X_j : \bigvee_{u \in D_u} u \in D_u(j;x_j)\} . \tag{33}$$

Finally

$$j^* = \arg\max_j \max_{x_j \in D_{xj}(j)} h_{xj}(x_j) . \tag{34}$$

In particular, for $D_u = \{u\}$, (25), (32) and (33) become

$$D_j(x) = \{j \in J : u \in D_u(j;x)\}$$

$$v(j) = v[u \,\tilde{\in}\, D_u(j;\overline{x}_j)] = v[\overline{x}_j \,\tilde{\in}\, D_{xj}(j)] = \max_{x_j \in D_{xj}(j)} h_{xj}(x_j) , \tag{35}$$

$$D_{xj}(j) = \{x_j \in X_j : u \in D_u(j;x_j)\} . \tag{36}$$

The procedure of finding j^* based on the knowledge representation $< D_u(j;x),\ j \in \overline{1,M};\ h_x(x) >$ or the block scheme of the corresponding recognition system is illustrated in Fig. 1. The solution may be not unique, i.e. $v(j)$ may take the maximum value for the different j^*. The result $v(j) = 0$ for each $j \in J$ means that the result of the observation $u \in D_u$ is not possible or there is a contradiction between the result of the observation and the knowledge representation given by an expert.

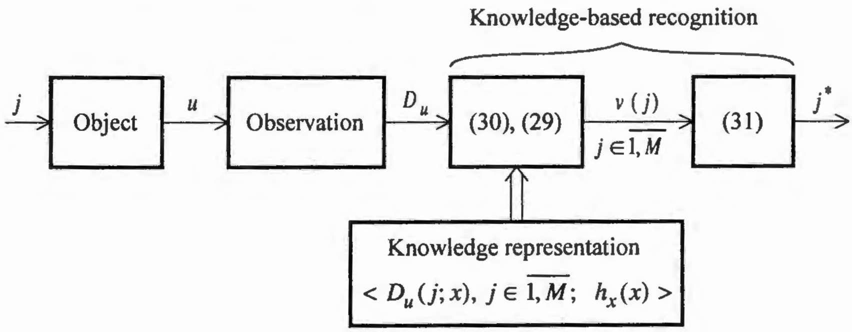

Fig. 1.

If $\bar{x}$ is considered as a C-uncertain variable then

$$j_c^* = \arg\max_j v_c(j)$$

where

$$v_c(j) = \frac{1}{2}\{v[\bar{x} \tilde{\in} D_x(j)] + 1 - v[\bar{x} \tilde{\in} \overline{D}_x(j)]\},$$

$\overline{D}_x(j) = X - D_x(j)$. Finally

$$v_c(j) = \frac{1}{2}[\max_{x \in D_x(j)} h_x(x) + 1 - \max_{x \in \overline{D}_x(j)} h_x(x)]. \tag{37}$$

The certainty indices $v_c(j)$ corresponding to (32) and (35) have the analogous form.

4. Examples

Example 1: Let $u, x_j \in R^1$, the sets $D_u(j;x_j)$ be described by the inequalities

$$x_j \le u \le 2x_j, \quad j = 1, 2, ..., M$$

and the certainty distributions $h_{xj}(x_j)$ have a parabolic form for each j (Fig. 2):

$$h_{xj}(x_j)=\begin{cases}-(x_j-d_j)^2+1 & \text{for } d_j-1\le x_j\le d_j+1\\ 0 & \text{otherwise}\end{cases}$$

where $d_j > 1$.

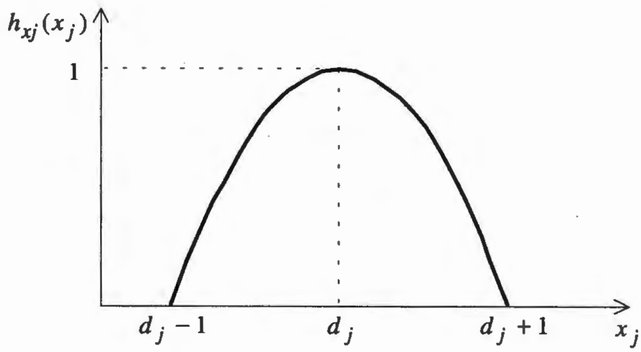

Fig. 2.

In this case the sets (36) for the given u are described by the inequality

$$\frac{u}{2}\le x_j\le u\,.$$

By applying (35) one obtains $v(j)$ as a function of d_j, illustrated in Fig. 3:

$$v(j)=\begin{cases}0 & \text{for} & d_j\le\frac{u}{2}-1\\ -(\frac{u}{2}-d_j)^2+1 & \text{for} & \frac{u}{2}-1\le d_j\le\frac{u}{2}\\ 1 & \text{for} & \frac{u}{2}\le d_j\le u\\ -(u-d_j)^2+1 & \text{for} & u\le d_j\le u+1\\ 0 & \text{for} & d_j\ge u+1\,.\end{cases}$$

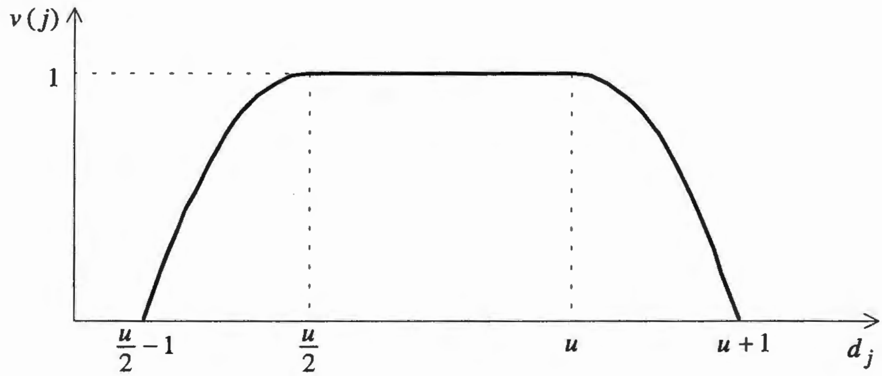

Fig. 3.

For example, for $M = 3$, $u = 5$, $d_1 = 2$, $d_2 = 5.2$, $d_3 = 6$ we obtain $v(1) = 0.75$, $v(2) = 0.96$ and $v(3) = 0$. Then $j^* = 2$, which means that for $u = 5$ the certainty index that $j = 2$ belongs to the set of the possible classes has the maximum value equal to 0.96. For $d_1, d_2, d_3 \in [\frac{u}{2}, u]$ one obtains $j^* = 1$ or 2 or 3 and $v(j^*) = 1$.

Let us consider $\bar{x}$ as a *C*-uncertain variable for the same numerical data. To obtain $v_c(j)$ according to (37) it is necessary to determine

$$v[\bar{x}_j \tilde{\in} \overline{D}_{xj}(j)] = \max_{x_j \in \overline{D}_{xj}(j)} h_{xj}(x_j) \stackrel{\Delta}{=} v_n(j). \tag{38}$$

In our case the set $\overline{D}_{xj}(j) = X_j - D_{xj}(j)$ is determined by the inequalities

$$x_j < \frac{u}{2} \qquad \text{or} \qquad x_j > u.$$

Using (38) we obtain $v_n(1) = v_n(2) = v_n(3) = 1$. Then

$$v_c(j) = \frac{1}{2}[v(j) + 1 - v_n(j)] = \frac{1}{2}v(j), \tag{39}$$

i.e. $v_c(1) = 0.375$, $v_c(2) = 0.48$, $v_c(3) = 0$ and $j_c^* = 2$ with the certainty index $v_c(j^*) = 0.48$.

For $d_1 = 3$, $d_2 = 3.2$, $d_3 = 4$ we obtain $v(1) = v(2) = v(3) = 1$ and

$$v_n(1) = -(2.5-3)^2 + 1 = 0.75\,,$$

$$v_n(2) = -(2.5-3.2)^2 + 1 = 0.51\,, \qquad v_n(3) = 0\,.$$

Then

$$v_c(1) = \frac{1}{2}(1+1-0.75) = 0.625\,,$$

$$v_c(2) = \frac{1}{2}(1+1-0.51) = 0.745\,, \qquad v_c(3) = 1$$

and $j_c^* = 3$ with the certainty index $v_c(j_c^*) = 1$.

Example 2: To indicate a role of $h_x(x)$ assume that in Example 1 the certainty distributions have the exponential form

$$h_{xj}(x_j) = e^{-(x_j - d_j)^2}\,.$$

By applying (35) one obtains $v(j)$ as a function of d_j

$$v(j) = \begin{cases} e^{-(\frac{u}{2}-d_j)^2} & \text{for} \quad d_j \le \frac{u}{2} \\ 1 & \text{for} \quad \frac{u}{2} \le d_j \le u \\ e^{-(u-d_j)^2} & \text{for} \quad d_j \ge u\,. \end{cases}$$

For $M = 3$, $u = 5$, $d_1 = 2$, $d_2 = 5.2$, $d_3 = 6$ we obtain

$$v(1) = e^{-0.25}\,, \qquad v(2) = e^{-0.4}\,, \qquad v(3) = e^{-1}\,.$$

Then $j^* = 2$ with the certainty index $v(j^*) = e^{-0.4} = 0.67$. For d_1, d_2, $d_3 \in [\frac{u}{2}, u]$ one obtains $j^* = 1$ or 2 or 3 and $v(j^*) = 1$. It may be shown that for a *C*-uncertain variable one obtains

$$v_c(1) = \frac{1}{2}e^{-0.25}\,, \qquad v_c(2) = \frac{1}{2}e^{-0.04}\,, \qquad v_c(3) = \frac{1}{2}e^{-1}\,.$$

Then, $j_c^* = j^* = 2$ with the certainty index $v_c(j_c^*) = \frac{1}{2}e^{-0.4} = 0.335$. The results for $d_1 = 3$, $d_2 = 3.2$ and $d_3 = 4$ are as follows:

$$v_c(1) = 1 - \frac{1}{2}e^{-0.25}, \quad v_c(2) = 1 - \frac{1}{2}e^{-0.49}, \quad v_c(3) = 1 - \frac{1}{2}e^{-1}$$

and $j_c^* = 3$ with the certainty index $v_c(j_c^*) = 1 - \frac{1}{2}e^{-1} = 0.816$. In this particular case the results j^* and j_c^* are the same for different forms of certainty distribution (see Example 1).

5. Conclusions and related problems

The approach presented in the work may be considered to be an extension of a pattern recognition problem based on relational and logical knowledge representations (Bubnicki 1990, 1993) to an uncertain recognition system with unknown parameters. The uncertain variables are shown to be a convenient tool for decision making in a class of knowledge-based pattern recognition systems described by relational knowledge representations with unknown parameters characterized by an expert. In the case of a *C*-uncertain variable the expert's knowledge is used in a better way but the calculations are more complicated.

The numerical examples and simulations showed that the parameters in the certainty distributions have a significant influence on quality of recognition. Then, it may be reasonable to apply an adaptation consisting in self-adjustment of the parameters of the recognition algorithm or – more generally – to combine the application of uncertain variables and of the learning process described in (Bubnicki 2000a, 2000b, 2001a). The learning process consists in *step by step* knowledge validation and updating and may be treated as an extension of the known idea of adaptation via identification (Bubnicki 1980). In the convergence analysis of the learning process the stability conditions for uncertain systems presented in (Bubnicki 2000d) may be useful.

It may be interesting and promising to apply uncertain variables in two-level knowledge-based pattern recognition systems (Szala 2002) and to apply a generalization based on soft variables (Bubnicki 2001d, 2001e).

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