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MULTISTAGE DECISION MAKING AND CONTROL UNDER FUZZINESS INVOLVING OBJECTIVE AND SUBJECTIVE ASPECTS

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We extend the basic Bellman and Zadeh's (1970) model of multistage decision making (control) in a fuzzy environment to include both objective and subjective evaluations of how well fuzzy constraints on decisions (controls) and fuzzy goals on states (outputs) are satisfied. We discuss the solution by an extended fuzzy dynamic programming model. We show an example of regional development planning.

Keywords. multistage decision making (control) under fuzziness, fuzzy dynamic programming, socio-economic regional development planning.

1 Introduction

The idea of Bellman and Zadeh's (1970) general approach to decision making in a fuzzy environment is that:

- a set of options (alternatives, variants, decisions, ...) is given,
- fuzzy constraints and fuzzy goals are specified as fuzzy sets in the set of options,
- a fuzzy decision, which serves the purpose of an objective function intoducing an order in the set of options, is defined as a confluence (aggregation) of the fuzzy constraints and fuzzy goals,
- an optimal option (alternative, decision, ...) is sought which maximizes the membership function of the fuzzy decision.

Particularly relevant is an extension of Bellman and Zadeh's (1970) model to multistage decision making (control) under fuzziness [cf. Kacprzyk's (1997) book].

Basically, the multistage decision making (control) process under fuzziness proceeds as sketched in Figure 1. We start from an initial state at control stage (time) $t = 0$, x_0, make a decision (apply a control) at $t = 0$, u_0, attain a state at time $t = 1$, x_1, apply u_1, ... Finally, at stage $t = N-1$ in state x_{N-1} we apply u_{N-1} and attain the final state x_N.

Dynamics of the system under control, S, reflects the state transitions from x_t to x_{t+1} under u_t, the consecutive u_t's are subjected to fuzzy constraints, $\mu_{C^t}(u_t)$, and on the x_{t+1}'s, fuzzy goals, $\mu_{G^{t+1}}(u_{t+1})$, are imposed, $t = 0, 1, \ldots, N-1$.

The performance of the control process is expressed by how well, at all the consecutive stages, the fuzzy constraints on controls and fuzzy goals on states are satisfied. And an optimal sequence of decisions (controls) at the consecutive control stages, $u_0^*, \ldots, u_{N-1}^*$, is sought.

An extension of Bellman and Zadeh's (1970) model outlined in this paper is motivated by results of Kacprzyk and Straszak (1982a, b, 1984) where a model of regional development planning has been proposed [cf. also of Kacprzyk, Romero and Gomide (1998, 1999)]. This will be discussed in Section 4.

The basic solution technique for fuzzy multistage decision making (control) problems is dynamic programming [cf. Kacprzyk (1997) or Kacprzyk and Esogbue (1996)], and this will be dealt with here. Other approaches include: branch and bound (Kacprzyk, 1978, 1979, 1983, 1997), a neural network emulating dynamic programming (Francelin and Gomide, 1993, Kacprzyk, Romero and Gomide, 1999a, b), and genetic algorithms (Kacprzyk, 1995a–c). Details may be found in Kacprzyk's book (1997).

2 Bellman and Zadeh's general approach to decision making and control under fuzziness – the source and extended model

If $X = \{x\}$ is some set of possible *options* (alternatives, variants, choices, decisions, ...), then:

- the *fuzzy goal* is defined as a fuzzy set G in X, characterized by its membership function $\mu_G : X \longrightarrow [0,1]$ such that $\mu_G(x) \in [0,1]$ specifies the grade of membership of a particular option $x \in X$ in the fuzzy goal G;
- the *fuzzy constraint* is similarly defined as a fuzzy set C in the set of options X, characterized by $\mu_C : X \longrightarrow [0,1]$ such that $\mu_C(x) \in [0,1]$

specifies the grade of membership of a particular option $x \in X$ in the fuzzy constraint C.

The general problem formulation is:

$$\text{"Attain } G \text{ \underline{and} satisfy } C\text{"} \tag{1}$$

which leads to a fuzzy decision

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x), \qquad \text{for each } x \in X \tag{2}$$

where "$\wedge$" is the minimum operation, i.e. $a \wedge b = \min(a, b)$; "$\wedge$" may be replaced by another appropriate operation as, e.g., a t-norm [cf. Kacprzyk's (1997) book for detail].

The *maximizing (decision)* is defined as an $x^* \in X$ such that

$$\mu_D(x^*) = \max_{x \in X} \mu_D(x) \tag{3}$$

In virtually all real life decision making problems an important role is played by the human factor. One of relevant implications in our context is that the satisfaction of constraints and attainment of goals have both an objective and subjective aspect. They can be accomodated in the source Bellman and Zadeh's (1970) framework outlined above by introducing:

- an *objective fuzzy goal* $\mu_{G_o}(x)$;
- a *subjective fuzzy goal* $\mu_{G_s}(x)$;
- an *objective fuzzy constraint* $\mu_{C_o}(x)$; and
- a *subjective fuzzy constraint* $\mu_{C_s}(x)$.

The general problem formulation (1) is now extended to:

$$\text{"Attain } [G_o \text{ and } G_s] \text{ \underline{and} satisfy } [C_o \text{ and } C_s]\text{"} \tag{4}$$

which leads to the fuzzy decision

$$\mu_D(x) = [\mu_{G_o}(x) \wedge \mu_{G_s}(x)] \wedge [\mu_{C_o}(x) \wedge \mu_{C_s}(x)], \qquad \text{for each } x \in X \tag{5}$$

where "$\wedge$", i.e. $a \wedge b = \min(a, b)$ may be replaced by, e.g., a t-norm; notice that we can use different operations, not just one, for each aggregation.

The *maximizing*, or *optimal* decision is defined as in (3).

If we have $n > 1$ fuzzy goals – $G^1, \ldots, G^n$ defined in Y, $m > 1$ fuzzy constraints – $C^1, \ldots, C^m$ defined in X, and a function $f : X \longrightarrow Y$, $y = f(x)$, then

$$\begin{aligned} \mu_D(x) = {} & \mu_{G^1}[f(x)] \wedge \cdots \wedge \mu_{G^n}[f(x)] \wedge \\ & \wedge \mu_{C^1}(x) \wedge \cdots \wedge \mu_{C^n}(x), \qquad \text{for each } x \in X \end{aligned} \tag{6}$$

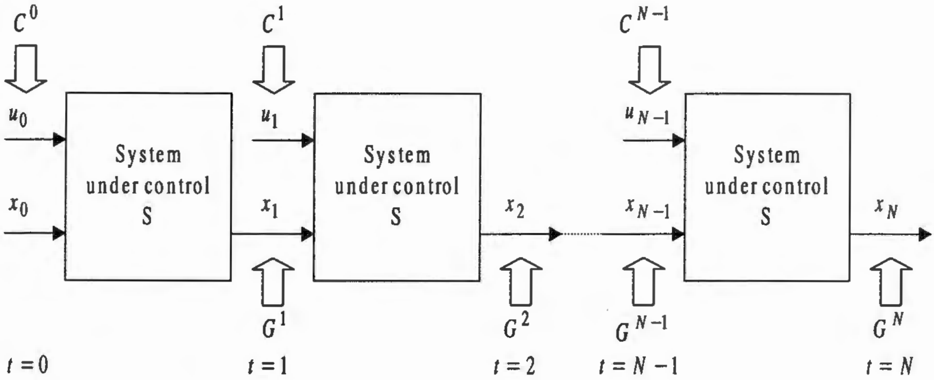

Figure 1: A general framework of multistage decision making (control) under fuzzy constraints and goals

Then, in the case of objective and subjective fuzzy constraints and goals, if we have: $n_o > 1$ objective fuzzy goals – $G_o^1, \ldots, G_o^{n_o}$ defined in Y, $n_s > 1$ subjective fuzzy goals – $G_s^1, \ldots, G_s^{n_s}$ defined in Y, $m_o > 1$ objective fuzzy constraints – $C_o^1, \ldots, C_o^{m_o}$ defined in X, $m_s > 1$ subjective fuzzy constraints – $C_s^1, \ldots, C_s^{m_s}$ defined in X, and a function $f : X \longrightarrow Y$, $y = f(x)$, then

$$\begin{aligned}
&\mu_D(x) = \\
&\quad = (\mu_{G_o^1}[f(x)] \wedge \cdots \wedge \mu_{G_o^{n_o}}[f(x)]) \wedge (\mu_{G_s^1}[f(x)] \wedge \cdots \wedge \mu_{G_s^{n_s}}[f(x)]) \wedge \\
&\quad \wedge [\mu_{C_o^1}(x) \wedge \cdots \wedge \mu_{C_o^{m_o}}(x)] \wedge [\mu_{C_s^1}(x) \wedge \cdots \wedge \mu_{C_s^{m_s}}(x)] \wedge \\
&\quad \wedge [\mu_{C_s^1}(x) \wedge \cdots \wedge \mu_{C_s^{m_s}}(x)], \qquad \text{for each } x \in X
\end{aligned} \tag{7}$$

In all the above cases the *maximizing decision* is defined as (3), i.e. $\mu_D(x^*) = \max_{x \in X} \mu_D(x)$.

3 Multistage decision making (control) under fuzziness in Bellman and Zadeh's setting – the source and extended model

The *control process* proceeds basically as mentioned in Section 1 and as is depicted in Figure 1.

The decision (control) space is $U = \{u\} = \{c_1, \ldots, c_m\}$, the state (output) space is $X = \{x\} = \{s_1, \ldots, s_n\}$, and both are finite. We start from $x_0 \in X$, apply $u_0 \in U$, which is subjected to a fuzzy constraint $\mu_{C^0}(u_0)$, and attain $x_1 \in X$ via a known state transition equation of the system under control S; a fuzzy goal $\mu_{G^1}(x_1)$ is imposed on x_1. Next, we apply u_1, subjected to $\mu_{C^1}(u_1)$, and attain x_2, subjected to $\mu_{G^2}(x_2)$, etc.

The deterministic system under control is described by a *state transition equation*

$$x_{t+1} = f(x_t, u_t), \qquad t = 0, 1, \ldots \tag{8}$$

where $x_t, x_{t+1} \in X = \{s_1, \ldots, s_n\}$ are the states at t and $t+1$, respectively, and $u_t \in U = \{c_1, \ldots, c_m\}$ is the decision (control) at t.

At t, $t = 0, 1, \ldots$, $u_t \in U$ is subjected to a *fuzzy constraint* $\mu_{C^t}(u_t)$, and on $x_{t+1} \in X$ a *fuzzy goal* is imposed, $\mu_{G^{t+1}}(x_{t+1})$.

The fixed and specified in advance *initial state* is $x_0 \in X$, and the *termination time* (planning horizon), $N \in \{1, 2, \ldots\}$, is finite, and fixed and specified in advance.

The *performance* of the particular stage t, $t = 0, 1, \ldots, N-1$, is given by

$$v_t = \mu_{C^t}(u_t) \wedge \mu_{G^{t+1}}(x_{t+1}) = \mu_{C^t}(u_t) \wedge \mu_{G^{t+1}}[f(x_t, u_t)] \tag{9}$$

while the *performance* of the whole multistage decision making (control) process is given by the fuzzy decision

$$\begin{aligned} \mu_D(u_0, \ldots, u_{N-1} \mid x_0) &= v_0 \wedge v_1 \wedge \ldots \wedge v_{N-1} = \\ &= [\mu_{C^0}(u_0) \wedge \mu_{G^1}(x_1)] \wedge \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(x_N)] \end{aligned} \tag{10}$$

The problem is to find an optimal sequence of decisions (controls) $u_0^*, \ldots, u_{N-1}^*$ such that

$$\mu_D(u_0^*, \ldots, u_{N-1}^* \mid x_0) = \max_{u_0, \ldots, u_{N-1} \in U} \mu_D(u_0, \ldots, u_{N-1} \mid x_0) \tag{11}$$

Kacprzyk's (1997) book provides and wide coverage of various aspects and extensions to this basic formulation.

In case of an extention proposed in this paper and outlined in Section 2 in which the objective and subjective fuzzy constraints and fuzzy goals are assumed, we have, at each $t = 0, 1, \ldots, N-1$:

- an objective fuzzy constraint $\mu_{C_t^1}(u_t)$ and a subjective fuzzy constraint $\mu_{C_s^t}(u_t)$, and
- an objective fuzzy goal $\mu_{G_o^{t+1}}(u_{t+1})$ and a subjective fuzzy constraint $\mu_{G_s^{t+1}}(u_{t+1})$.

The (extended) performance of the particular decision making (control) stage t, $t = 0, 1, \ldots, N-1$, is then given by

$$\overline{v}_t = [\mu_{C_o^t}(u_t) \wedge \mu_{C_s^t}(u_t)] \wedge [\mu_{G_o^t}(x_t) \wedge \mu_{G_s^t}(x_t)] \tag{12}$$

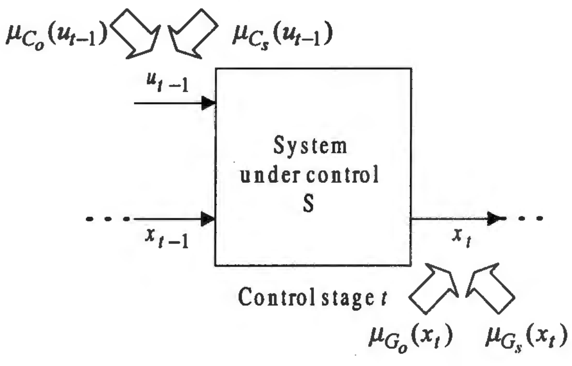

Figure 2: Evaluation of (extended) performance of decision making (control) stage t

which can be schematically shown as in Figure 2.

The (extended) performance of the whole multistage decision making (control) process is then given by the fuzzy decision

$$\begin{aligned} &\mu_{\overline{D}}(u_0,\ldots,u_{N-1}\mid x_0) = \overline{v}_0 \wedge \overline{v}_1 \wedge \ldots \wedge \overline{v}_{N-1} = \\ &= \{[\mu_{C_o^0}(u_0)\wedge \mu_{C_s^0}(u_0)] \wedge [\mu_{G_o^1}(x_1)\wedge\mu_{G_s^1}(x_1)]\}\wedge\ldots \\ &= \wedge \{[\mu_{C_o^{N-1}}(u_{N-1})\wedge \mu_{C_s^{N-1}}(u_{N-1})]\wedge[\mu_{G_o^N}(x_N)\wedge\mu_{G_s^N}(x_N)]\} \end{aligned} \quad (13)$$

and we seek again an $u_0^*,\ldots,u_{N-1}^*$ such that

$$\mu_{\overline{D}}(u_0^*,\ldots,u_{N-1}^*\mid x_0) = \max_{u_0,\ldots,u_{N-1}\in U} \mu_{\overline{D}}(u_0,\ldots,u_{N-1}\mid x_0) \quad (14)$$

It often happens that the (subjective) human satisfaction resulting from the attainment of some level of x_{t+1} depends not only on the "objectively attained" value but on how this value looks like in comparision with the past or future prospects. This relevant problem is discussed in Kacprzyk's (1997) book, but will not be considered in this paper.

Problem (11) can be solved using the following two basic traditional techniques: dynamic programming, and branch-and-bound, and also using the following two new ones: a neural network, and a genetic algorithm.

The application of dynamic programming for the solution of problem (11) was proposed in Bellman and Zadeh (1970). First, we rewrite (11) as to find $u_0^*,\ldots,u_{N-1}^*$ such that

$$\begin{aligned} &\mu_D(u_0^*,\ldots,u_{N-1}\mid x_0) = \\ &= \max_{u_0,\ldots,u_{N-1}} [\mu_{C^0}(u_0)\wedge\mu_{G^1}(x_1)\wedge\ldots \\ &\ldots\wedge\mu_{C^{N-1}}(u_{N-1})\wedge\mu_{G^N}(f(x_{N-1},u_{N-1}))] \end{aligned} \quad (15)$$

and then, since

$$\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))$$

depends only on u_{N-1}, then the maximization with respect to $u_0, \ldots, u_{N-1}$ in (15) can be split into:

- the maximization with respect to $u_0, \ldots, u_{N-2}$, and
- the maximization with respect to u_{N-1},

written as

$$\begin{aligned} &\mu_D(u_0^*, \ldots, u_{N-1}^* \mid x_0) = \\ &= \max_{u_0, \ldots, u_{N-2}} \{\mu_{C^0}(u_0) \wedge \mu_{G^1}(x_1) \wedge \ldots \\ &\quad \ldots \wedge \mu_{C^{N-2}}(u_{N-2}) \wedge \mu_{G^{N-1}}(x_{N-1}) \wedge \\ &\quad \wedge \max_{u_{N-1}} [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))]\} \end{aligned} \qquad (16)$$

which may be continued for u_{N-2}, u_{N-3}, etc.

This backward iteration leads to the following set of fuzzy dynamic programming recurrence equations:

$$\begin{cases} \mu_{\overline{G}^{N-i}}(x_{N-i}) = \\ \qquad = \max_{u_{N-i}} [\mu_{C^{N-i}}(u_{N-i}) \wedge \mu_{G^{N-i}}(x_{N-i}) \wedge \mu_{\overline{G}^{N-i+1}}(x_{N-i+1})] \\ x_{N-i+1} = f(x_{N-i}, u_{N-i}); \qquad i = 0, 1, \ldots, N \end{cases} \qquad (17)$$

where $\mu_{\overline{G}^{N-i}}(x_{N-i})$ is viewed as a fuzzy goal at control stage $t = N - i$ induced by the fuzzy goal at $t = N - i + 1$, $i = 0, 1, \ldots, N$; $\mu_{\overline{G}^N}(x_N) = \mu_{G^N}(x_N)$.

The $u_0, \ldots, u_{N-1}$ sought is given by the successive maximizing values of u_{N-i}, $i = 1, \ldots, N$ in (17) which are obtained as functions of x_{N-i}, i.e. as an *optimal policy*, $a_{N-i} : X \longrightarrow U$, such that $u_{N-i} = a_{N-i}(x_{N-i})$.

Using the objective and subjective fuzzy constraints and fuzzy goals: $\mu_{C_o^{N-i}}(u_{N-i})$ and $\mu_{C_s^{N-i}}(u_{N-i})$ on the one hand, and $\mu_{G_o^{N-i+1}}(x_{N-i+1})$ and $\mu_{G_s^{N-i+1}}(x_{N-i+1})$ on the other hand, for $i = 1, 2, \ldots, N$, we arrive at the following set of (extended) dynamic programming recurrent equations:

$$\begin{cases} \mu_{\overline{G}^{N-i}}(x_{N-i}) = \\ \qquad = \max_{u_{N-i}} \{[\mu_{C_o^{N-i}}(u_{N-i}) \wedge \mu_{C_s^{N-i}}(u_{N-i})] \wedge \\ \qquad\qquad [\mu_{G_o^{N-i}}(x_{N-i}) \wedge \mu_{G_s^{N-i}}(x_{N-i}) \wedge \mu_{\overline{G}^{N-i+1}}(x_{N-i+1})]\} \\ x_{N-i+1} = f(x_{N-i}, u_{N-i}); \qquad i = 0, 1, \ldots, N \end{cases} \qquad (18)$$

We will now show an application of the above model in socio-economic regional development planning.

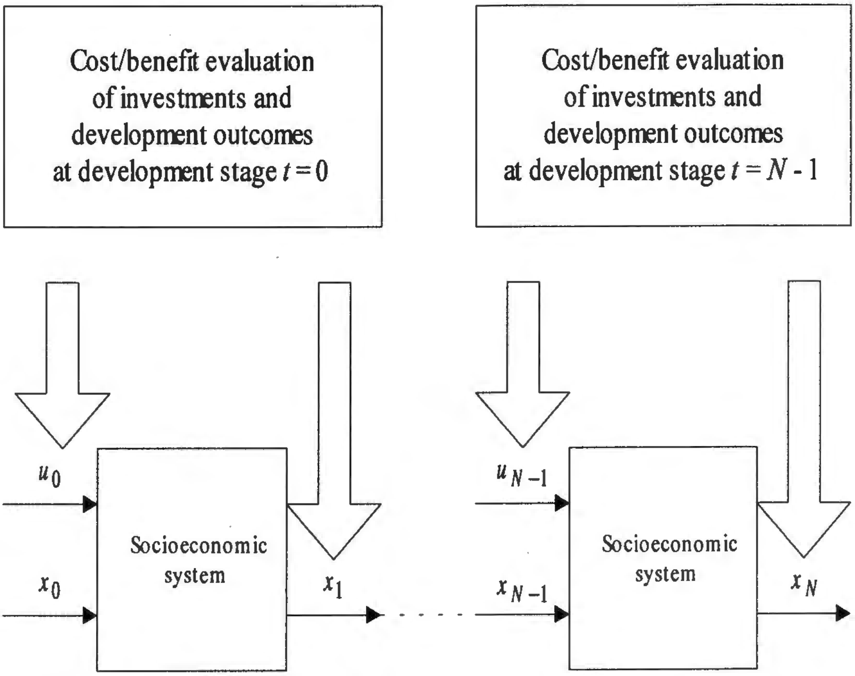

Figure 3: Essential elements of socioeconomic regional development

4 Socioeconomic regional development planning under fuzziness

Regional development planning is a difficult problem which involves various aspects (political, economic, social, environmental, technological, etc.), different parties (inhabitants, authorities of different levels, formal and informal groups, etc.), and many imprecisely defined quantities. To overcome these difficulties, the use of fuzzy sets has been proposed by Kacprzyk and Straszak (1982a, b, 1984) who consider a (rural) region plagued by severe difficulties, mainly caused by a poor *life quality* perceived. To improved it, (mostly external) funds (investments) are needed, and their amount and temporal distribution is sought.

4.1 A multistage fuzzy decision making model of regional develement planning

The essence of socioeconomic regional development may be depicted as in Figure 3. The region is represented by a socioeconomic dynamic system under

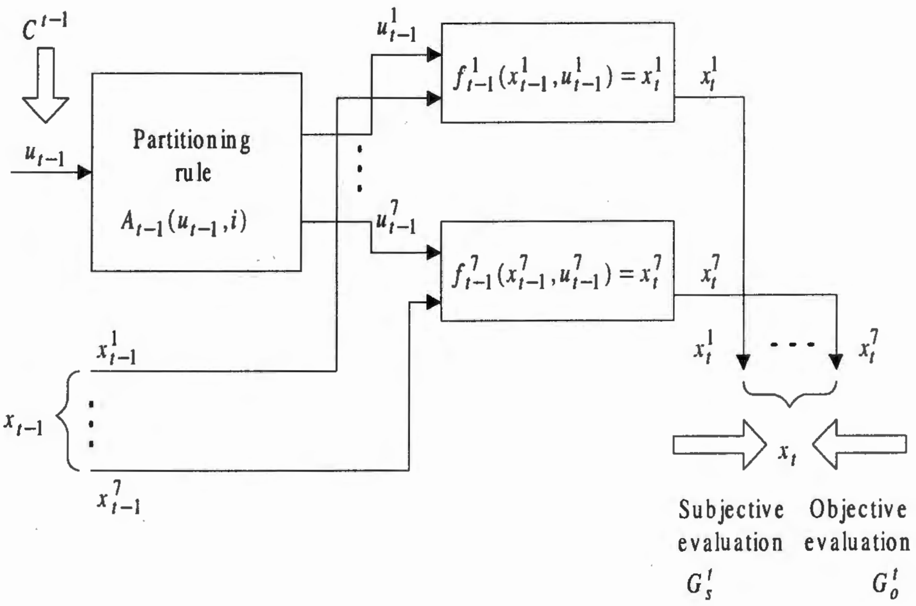

Figure 4: Basic elements of the socioeconomic system under control

control (cf. Figure 4) whose state at development (planning) stage $t-1$, X_{t-1}, is characterized by a set of relevant socioeconomic life quality indicators. Then, the decision (control), which is investment, at $t-1$, u_{t-1}, changes X_{t-1} to X_t; $t = 1, \ldots, N$, and N is a finite, fixed and specified planning horizon. The assessment of a planning stage t, $t = 1, \ldots, N$, is performed by accounting for both the "goodness" of the u_{t-1} applied (i.e. costs), and the "goodness" of the X_t attained (i.e. benefits) – the former is related to constraints, and the latter to goals.

X_t is equated with a *life quality index* that consists of the following seven *life quality indicators* (i.e. $X_t = [x^1_t, \ldots, x^7_t]$):

- x^1_t – economic quality (e.g., wages, salaries, income, ...),
- x^2_t – environmental quality,
- x^3_t – housing quality,
- x^4_t – health service quality,
- x^5_t – infrastructure quality,
- x^6_t – work opportunity,

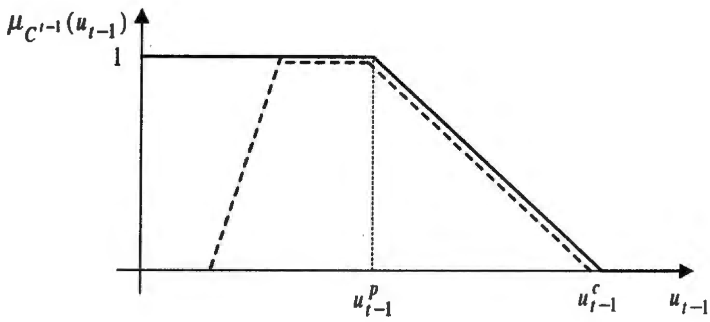

Figure 5: Fuzzy constraints on investment u_{t-1}

- x_t^7 – leisure time opportunity,

The decision at $t-1$, u_{t-1} is investment subjected to some limitation. We impose on u_{t-1} a fuzzy constraint $\mu_{C^{t-1}}(u_{t-1})$ in a piecewise linear form as shown in Figure 5 to be read as follows. The investment may be fully utilized up to u_{t-1}^p, hence $\mu_{C^{t-1}}(u_{t-1}) = 1$ for $0 < u_{t-1} < u_{t-1}^p$. However, this is usually insufficient and some additional contingency investment is needed, maximally up to u_{t-1}^c (the more the worse, of course). The fuzzy constraints are often as shown in the dotted line, i.e. too low a use of available investments should also be avoided. We will not discuss the partitioning of u_{t-1} into $u_{t-1}^1, \ldots, u_{t-1}^7$, devoted to improve the respective life quality indicators, and assume a fixed rule.

The temporal evolution of the life quality indicators is governed by the state transition equation

$$x_t^i = f_{t-1}^i(x_{t-1}^i, u_{t-1}^i), \qquad i = 1, \ldots, 7; t = 1, \ldots, N \tag{19}$$

which may be derived by, e.g., using experts' opinions, past experience, mathematical models, etc.

The development is a goal oriented task aimed at the satisfaction of some needs. Measures are needed of how well some predetermined goals are fulfilled, i.e. of how *effective* the development is, which should then be related to the investment spent, to find how *efficient* it is – cf. Kacprzyk's (1997) book for for a thorough analysis.

The effectiveness of regional development involves two aspects:

- the effectiveness of a particular development stage, and
- the effectiveness of the whole development,

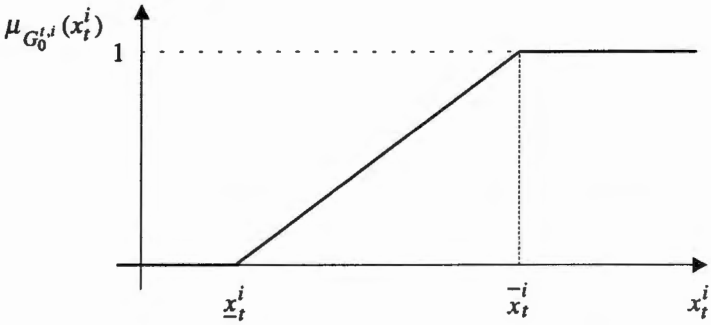

Figure 6: Objective fuzzy subgoal

The effectiveness of a stage has both an objective and subjective aspect. The objective evaluation is the determination of how well the fuzzy constraints are fulfilled, and fuzzy goals are attained.

For each life quality indicator at $t = 1, \ldots, N$, x_t^i, we define an *objective fuzzy subgoal* $G_o^{t,i}$ characterized by $\mu_{G_o^{t,i}}(x_t^i)$ as shown in Figure 6 to be read as follows: $G_o^{t,i}$ is fully satisfied for $x_t^i \geq \overline{x}_t^u$, where $\overline{x}_o^i$ is some *aspiration level* for the indicator x_t^i; therefore, $\mu_{G_o^{t,i}}(x_t^i) = 1$, for $x_t^i \geq \overline{x}_t^i$. Less preferable are $\underline{x}_t^i < x_t^i < \overline{x}_t^i$ for which $0 < \mu_{G_o^{t,i}}(x_t^i) < 1$, and $x_t^i \leq \underline{x}_t^i$ are assumed to be impossible, hence $\mu_{G_o^{t,i}}(x_t^i) = 0$.

The objective evaluation of the life quality index at t, $X_t = [x_t^1, \ldots, x_t^7]$, is obtained by the aggregation of partial assessments of the particular life quality indicators, i.e.

$$\mu_{G_o^t}(X_t) = \mu_{G_o^{t,1}}(x_t^1) \wedge \ldots \wedge \mu_{G_o^{t,7}}(x_t^7) \tag{20}$$

and "$a \wedge b = \min(a, b)$" may be replaced here and later on by another suitable operation as, e.g., a t-norm [cf. Kacprzyk (1997)].

The use of "$\wedge$" (minimum) reflects a pessimistic, safety-first attitude, and a lack of substitutability (i.e. that a low value of one life quality indicator cannot be compensated by a higher value of another), which is often adequate.

The attained value of a particular x_t^i implies its corresponding partial social satisfaction s_t^i derived as in Figure 7, and its interpretation is basically as for Figure 6.

It should be noted that in general both $\underline{z}_t^i$ and $\overline{z}_t^i$ may be functions of the trajectory (history) of development or the reduced trajectory, and for more detail on this relevant topic we refer the reader to Kacprzyk's (1997) book.

The social satisfaction at t is now

$$s_t = s_t^1 \wedge \ldots \wedge s_t^7 \tag{21}$$

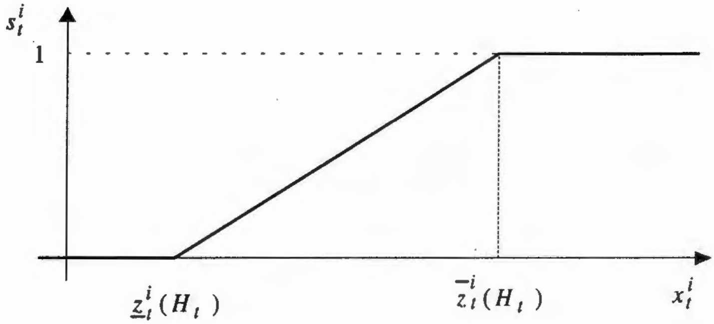

Figure 7: Partial social satisfaction

The s_t is subjected to a subjective fuzzy goal $\mu_{G_s^t}(s_t)$ which is meant similarly as its objective counterpart shown in Figure 6.

The effectiveness of t is meant as a relation of what has been attained (the life quality indices and their respective social satisfactions) to what has been "paid for" (the respective investments), i.e. is a benefit–cost relationship.

Formally, the (fuzzy) effectiveness of stage t is expressed as

$$\mu_{E^t}(u_{t-1}, X_t, s_t) = \mu_{C^{t-1}}(u_{t-1}) \wedge \mu_{G_o^t}(X_t) \wedge \mu_{G_s^t}(s_t) \tag{22}$$

and the fuzzy effectiveness measure for the whole development is

$$\mu_E(H_N) = \mu_{E^1}(u_0, X_1, s_1) \wedge \ldots \wedge \mu_{E^N}(u_{N-1}, X_N, s_n) \tag{23}$$

The fuzzy decision is

$$\begin{aligned} &\mu_D(u_0, \ldots, u_{N-1} \mid X_0, B_N) = \\ &\quad = [\mu_{C^0}(u_0) \wedge \mu_{G_o^1}(X_1) \wedge \mu_{G_s^1}(s_1)] \wedge \ldots \\ &\quad\quad \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G_o^N}(X_N) \wedge \mu_{G_s^N}(s_N)] \end{aligned} \tag{24}$$

and we seek an optimal sequence of controls (investments) $u_0^*, \ldots, u_{N-1}^*$ such that (under a given policy B_N):

$$\begin{aligned} &\mu_D(u_0^*, \ldots, u_{N-1}^* \mid X_0, B_N) = \\ &\quad = \max_{u_0, \ldots, u_{u_{N-1}}} \{[\mu_{C^0}(u_0) \wedge \mu_{G_o^1}(X_1) \wedge \mu_{G_s^1}(s_1)] \wedge \ldots \\ &\quad\quad \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G_o^N}(X_N) \wedge \mu_{G_s^N}(s_N)]\} \end{aligned} \tag{25}$$

For illustration, we will solve a simple example [cf. Kacprzyk (1983, 1997)].

Example 1 The region has a population of ca. 120,000 inhabitants, its arable land is ca. 450,000 acres, and the planning horizon is 3 years. The life quality index consists of the four life quality indicators:

- x_t^{I} – average subsidies in US$ per acre (per year),
- x_t^{II} – sanitation expenditures (water and sewage) in US$ per capita (per year),
- x_t^{III} – health care expenditures in US$ per capita (per year), and
- x_t^{IV} – expenditures for paved roads (new roads and maintenance of the existing ones) in US$ (per year).

Suppose now that the investments are partitioned into parts devoted to the improvement of the above life quality indicators due to the fixed partitioning rule $A_{t-1}(u_{t-1}, i)$: 5% for subsidies, 25% for sanitation, 45% for health care, and 25% for infrastructure.

Let the initial, at $t = 0$, values of the life quality indicators be:

$$x_0^{\mathrm{I}} = 0.5 \qquad x_0^{\mathrm{II}} = 15 \qquad x_0^{\mathrm{III}} = 27 \qquad x_0^{\mathrm{IV}} = 1,700,000$$

We assume the two *scenarios* (policies):

- Policy 1: $u_0 = \$8,000,000 \quad u_1 = \$8,000,000 \quad u_2 = \$8,000,000$
- Policy 2: $u_0 = \$7,500,000 \quad u_1 = \$8,000,000 \quad u_2 = \$8,500,000$

Under Policy 1 and Policy 2, the values of the life quality indicators attained are:

Policy 1:

Year(t)	u_t	x_t^{I}	x_t^{II}	x_t^{III}	x_t^{IV}
0	$8,000,000				
1	$8,000,000	0.88	16.7	30	$2,000,000
2	$8,000,000	0.88	16.7	30	$2,000,000
3		0.88	16.7	30	$2,000,000

Policy 2:

Year(t)	u_t	x_t^{I}	x_t^{II}	x_t^{III}	x_t^{IV}
0	$7,500,000				
1	$8,000,000	0.83	15.6	28.1	$1,875,000
2	$8,500,000	0.88	16.7	30	$2,000,000
3		0.94	17.7	31.9	$2,125,000

For simplicity, we only take into account the *effectiveness* of development, and the objective evaluation. The consecutive fuzzy constraints and objective fuzzy subgoals are assumed piecewise linear, i.e. their definition requires two values only (cf. Figure 5, and Figure 6): the aspiration level (i.e. the fully acceptable value) and the lowest (or highest) possible (still acceptable) value) which are:

$$
\begin{array}{llllll}
t & & & & & \\
0 & C^0: & u_0^p = \$7,500,000 & & & \\
 & & u_0^c = \$8,500,000 & & & \\
1 & C^1: & u_1^p = \$7,750,000 & & & \\
 & & u_1^c = \$9,000,000 & G_o^{1,\mathrm{I}}: \underline{x}_1^{\mathrm{I}} = 0.6 & & \overline{x}_1^{\mathrm{I}} = 0.85 \\
 & & & G_o^{1,\mathrm{II}}: \underline{x}_1^{\mathrm{II}} = 14 & & \overline{x}_1^{\mathrm{II}} = 16 \\
 & & & G_o^{1,\mathrm{III}}: \underline{x}_1^{\mathrm{III}} = 27 & & \overline{x}_1^{\mathrm{III}} = 29 \\
 & & & G_o^{1,\mathrm{IV}}: \underline{x}_1^{\mathrm{IV}} = \$1,800,000 & & \overline{x}_1^{\mathrm{IV}} = \$1,900,000 \\
2 & C^2: & u_2^p = \$8,000,000 & & & \\
 & & u_1^c = \$10,000,000 & G_o^{2,\mathrm{I}}: \underline{x}_2^{\mathrm{I}} = 0.7 & & \overline{x}_1^{\mathrm{I}} = 0.9 \\
 & & & G_o^{2,\mathrm{II}}: \underline{x}_2^{\mathrm{II}} = 15 & & \overline{x}_1^{\mathrm{II}} = 17 \\
 & & & G_o^{2,\mathrm{III}}: \underline{x}_2^{\mathrm{III}} = 28 & & \overline{x}_1^{\mathrm{III}} = 30 \\
 & & & G_o^{2,\mathrm{IV}}: \underline{x}_2^{\mathrm{IV}} = \$1,900,000 & & \overline{x}_2^{\mathrm{IV}} = \$2,000,000 \\
3 & & & G_o^{3,\mathrm{I}}: \underline{x}_3^{\mathrm{I}} = 0.75 & & \overline{x}_3^{\mathrm{I}} = 1 \\
 & & & G_o^{3,\mathrm{II}}: \underline{x}_3^{\mathrm{II}} = 16 & & \overline{x}_1^{\mathrm{II}} = 18.5 \\
 & & & G_o^{3,\mathrm{III}}: \underline{x}_3^{\mathrm{III}} = 29 & & \overline{x}_1^{\mathrm{III}} = 31 \\
 & & & G_o^{3,\mathrm{IV}}: \underline{x}_3^{\mathrm{IV}} = \$1,950,000 & & \overline{x}_3^{\mathrm{IV}} = \$2,100,000
\end{array}
$$

The evaluation (goodness) of the two investment policies is:

- Policy 1

$$\mu_D(\$8,000,000; \$8,000,000; \$8,000,000 \mid .) = 0.28$$

- Policy 2

$$\mu_D(\$7,500,000; \$8,000,000; \$8,500,000 \mid .) = 0.55$$

hence the second policy is better. □

5 Concluding remarks

We extended the basic Bellman and Zadeh's (1970) model of multistage decision making (control) in a fuzzy environment to include both objective and subjective evaluations of how well fuzzy constraints on decisions (controls) applied and fuzzy goals on states attained are satisfied. We discussed the solution by an extended fuzzy dynamic programming model, and showed the use of a neural network implementing fuzzy dynamic programming which by its inherent parallelism, makes it possible to procced with often time consuming computations in a parallel way. We considered an application for solving a socio-economic regional planning problem.

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