PROCEEDINGS OF THE
INTERNATIONAL WORKSHOP ON

# SOCIAL SECURITY REFORM

Roman Kulikowski
Gordon J. MacDonald
*Editors*

Systems Research Institute
Polish Academy of Sciences

IIASA

Systems Research Institute • Polish Academy of Sciences
International Institute for Applied Systems Analysis

# SOCIAL SECURITY REFORM

Proceedings of the International Workshop
held in Warsaw on 23-24 September 1997

**Roman Kulikowski**
**Gordon MacDonald**
Editors

Warsaw 1998

Publication of this volume was made possible due to financial assistance of:

- International Institute for Applied Systems Analysis
- Polish Committee for Cooperation with IIASA
- State Committee for Scientific Research of the Republic of Poland



ISBN 83-85847-14-6

# Chapter 2:

# A Way to Formalization

# A new model of asset allocation in risky and risk-free assets*

**Leszek S. Zaremba**

*Systems Research Institute, Polish Academy of Sciences*

Our model concerns finding an optimal fraction $f^*$ of an initial stake $S$ to be invested in risky assets on each single market system, such as Warsaw Stock Exchange, NYSE, etc., which is characterized by a gain of $A$ dollars and a loss of $B$ dollars per each dollar invested during one period (a week, a month, etc.). If the probability of winning equals $\frac{1}{2}$ then each concrete sequence of $n$ trades (each scenario „*i*" with $i$ gains and ($n$-$i$) losses) occurs with probability $p_i = \binom{n}{i} \cdot 2^{-n}$.
Depending on a scenario and fraction $f$ to be invested in risky assets, the initial stake $S$ will increase or decrease to the amount $S(1+fA)^i(1-fB)^{n-i}$ during $n$ trading periods. Knowing the probabilities of occurence of all scenarios, we assume in this model that during $n \cdot 2^n$ periods each scenario „*i*" will occur proportionally to its probability.

Based on this, one can compute the expected final wealth, *EFW(f)*, after $n \cdot 2^n$ sessions.

By a performance index to be maximized the money manager understands the average speed of fund growth per $n$ periods during $n \cdot 2^n$ periods, called by Vince (1990, 1995) the terminal wealth relative, *TWR(f)*. It can be shown that

* Intervention at Panel Discussion.

$TWR(f)=\left[\sqrt{(1+fA)(1-fB)}\right]^{n}$, and the optimal fraction $f^{*}=\frac{1}{2}\left[\frac{1}{B}-\frac{1}{A}\right]$. Further, $TWR(f^{*})$ depends solely on the ratio $\frac{A}{B}$ rather than on $A$ and $B$ separately. After having computed optimal fractions for each market system, we determine the optimal fraction to be traded on two uncorrelated systems simultaneously, characterized by the same parameters $A$, $B$. Generalizing previous formulas from a single System I to the composite system I-II we obtain a formula for $TWR^{I-II}(f)$, to conclude that the optimal fraction $f^{I-II}$ to be invested on the composite system I-II is the larger root of the equation

$$-f^{2}(A-B)AB+f\left[\frac{3}{4}(A-B)^{2}-AB\right]+(A-B)=0.$$

It can be shown that $f^{I-II}>f^{*}$ which suggests the composite system I-II is more advantageous than any single one it comprises. Summarizing, investing simultaneously on two similar and uncorrelated market systems is more advantageous than on any of the two single ones.

## References

Vince R. (1990) *Portfolio Management Formulas*, Wiley.
Vince R. (1995) *The New Money Management*, Wiley.

ISBN 83-85847-14-6

Systems Research Institute
Polish Academy of Sciences

International Institute for
Applied Systems Analysis