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SOCIAL SECURITY REFORM

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Chapter 2:

A Way to Formalization

Individual utilities and the choice of security system

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The general acceptance and success of the present social security reform in Poland depends much on the rational behaviour and rational decisions taken by people being insured.

The present paper describes methodological support for such decisions.

1. Return and risk

When one decides to be insured in a security system, such as a pension fund, he pays to the fund a part of present income (P_0) in order to get (in the future) a monetary return $P_1 > P_0$.

The rate of return

$$R = \frac{P_1 - P_0}{P_0}, \tag{1}$$

is obviously a random variable. Assume R to be normally distributed with the expected value $\overline{R}$ and standard deviation σ. Assume also that the person insured is concerned with:

a. The expected monetary return

$$Z = P_0 \overline{R}, \tag{2}$$

b. The „worse case" monetary return:

$$Y = P_0\left[\overline{R} - \kappa(p)\sigma\right], \tag{3}$$

where $\kappa(p)$ is the „cost of risk", attached to σ. It depends on the subjectively accepted probability of worse case (p) (see Fig. 1). For example, when one assumes $p = 1/6$, one gets $\kappa(1/6) \approx 1$, and the worse case return R is not more than $\overline{R} - \sigma$, i.e. $R \leq \overline{R} - \sigma$.

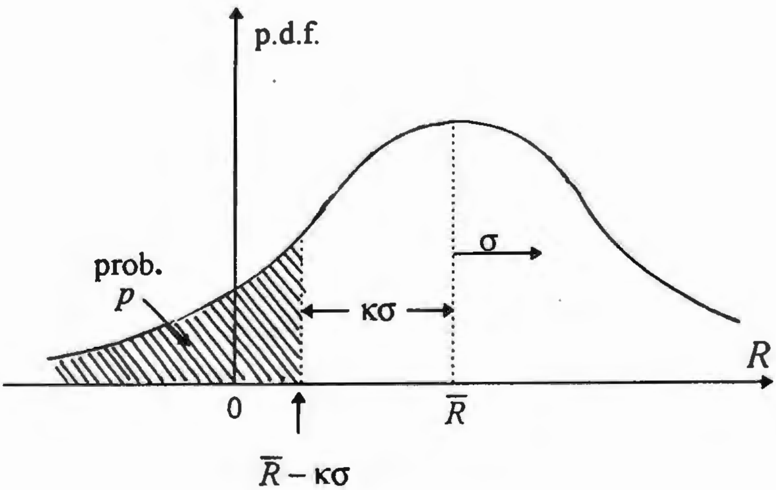

Fig. 1. Illustration for the "worse case" return.

2. Utility of risky insurance

Assume the two-factors, constant return to scale, utility function

$$U = \phi[Z, Y] = YF\left(\frac{Z}{Y}\right), \tag{4}$$

with $F(\cdot) > 0$, $F'(\cdot) > 0$, $F''(\cdot) < 0$, to be given.

The function $F(\cdot) > 0$ is strictly concave (Fig. 2).

Introduce the notion of security index (S):

$$S = \frac{Y}{Z} = 1 - \kappa\frac{\sigma}{R}. \tag{5}$$

The smaller the risk (σ) is the higher the security index S. For $\sigma = 0$ one gets the maximum of $S = 1$.

When one approximates $F(x)$ by the function:

$$F(x) = \alpha x^{\beta}, \quad \alpha > 0, \quad 0 \le \beta \le 1,$$

one gets

$$U = \alpha P_0 \overline{R} S^{1-\beta}. \tag{6}$$

The utility (6) increases along with expected return $\overline{R}$ and the security S. The subjective parameter $1-\beta$ can be regarded as sensitivity of U with respect to small variations of security (dS/S), Indeed, $\frac{dU}{U} : \frac{dS}{S} = 1 - \beta$.

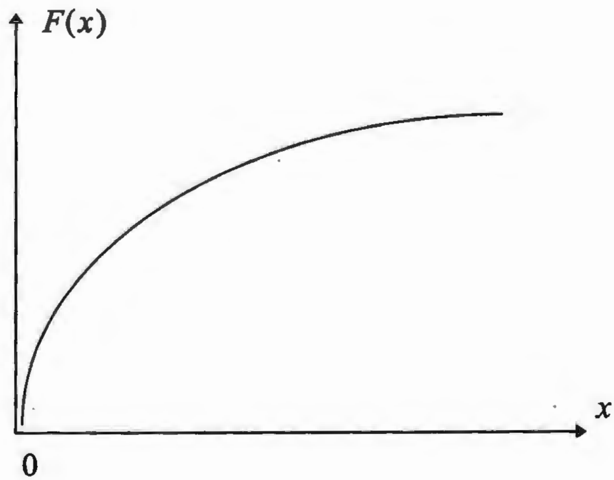

Fig. 2. Assumptions on *F(x)*.

3. The choice (acceptance) of insurance

Consider two insurances characterized by security indices:

a. $S = 1$ (risk-free security) and $U_f = \alpha P_0 R_f$,

b. $S < 1$ (a risky security) $U = \alpha P_0 \overline{R} S^{1-\beta}$.

The insurance (b) is preferred to (a) (can be accepted) if $U \ge U_f$, i.e. when

$$\bar{R} \geq \frac{R_f}{S^{1-\beta}} . \tag{7}$$

According to (7) insurance (b) can be accepted if the expected return $\bar{R}$ is not less than the risk-free-return R_f (which is offered by the long-term government bonds) divided by $S^{1-\beta}$. If S decreases, this must be compensated for (in order to be accepted) by a large enough increase of expected return $\bar{R}$.

4. Scenario analysis

In order to apply (7) as a criterion of insurance choice one should determine the parameters $\bar{R}$, σ, characterizing the existing security systems (insurance funds), which is not easy.

One possible way is to apply the historically observed, say $\bar{R}_h$, σ_h; data. However, for valuation purposes the ex ante values, denoted $\bar{R}_a$, σ_a, are preferred.

In order to derive $\bar{R}_a$, σ_a, the scenario analysis can be used.

In the simple, binomial scenario model, one can write

$$R_a = \begin{cases} \bar{R}_h + \sigma_h & \text{with probability} \quad p_1, \\ \bar{R}_h - \sigma_h & \text{with probability} \quad p_2, \end{cases}$$

where $p_1 + p_2 = 1$.

The expected value for R_a becomes

$$\bar{R}_a = p_1\left(\bar{R}_h + \sigma_h\right) + p_2\left(\bar{R}_h - \sigma_h\right) = \bar{R}_h + \left(p_1 - p_2\right)\sigma_h , \tag{8}$$

and

$$\sigma_a = \left\{p_1\left[\bar{R}_a - \bar{R}_h - \sigma_h\right]^2 + p_2\left[\bar{R}_a - \bar{R}_h + \sigma_h\right]^2\right\}^{1/2} = 2\sqrt{p_1 p_2}\,\sigma_a . \tag{9}$$

Then one can derive the ex ante security index

$$S_a = 1 - \kappa\frac{\sigma_a}{\bar{R}_a} . \tag{10}$$

In the present case the acceptance criterion becomes

$$\bar{R}_a \geq \frac{R_f}{S_a^{1-\beta}} . \tag{11}$$

In order to use the present approach the subjective parameters (κ, β) and risk free return R_f should be also specified.

5. Example

Let the pension insurance fund be characterized by the historical parameters $\overline{R}_h = 0.15$, $\sigma_h = 0.10$. A risk averse person, described by $\kappa = 1$, $\beta = 0$, considers joining the fund. He, or she, believes that $\overline{R}_h$ will go up with probability $p_1 = 0.6$, and will go down with $p_2 = 0.4$. Employing the binomial model he, or she, obtains by (8), (9), (10):

$$\overline{R}_a = 0.15 + (0.6 - 0.4) \cdot 0.1 = 0.17,$$

$$\sigma_a = 2\sqrt{0.24} \cdot 0.1 = 0.099,$$

$$S_a = 1 - \frac{0.099}{0.17} = 0.423.$$

Assuming the risk-free rate of return $R_f = 0.07$ one gets, by (11):

$$0.17 > \frac{0.07}{0.423} = 0.165.$$

Then, the pension fund can be accepted. However, the acceptance depends on the subjective parameters κ, β. Another, more risk averse person, e.g. with $\kappa = 1.5$; $\beta = 0$, will reject the pension fund. That person would rather prefer to buy the risk free government bonds.

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