

SYSTEMS RESEARCH INSTITUTE
POLISH ACADEMY OF SCIENCES

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS

CONTRACTED STUDY AGREEMENT REG /POL/1

**"CONCEPTS AND TOOLS FOR STRATEGIC REGIONAL
SOCIO-ECONOMIC CHANGE POLICY"**

STUDY REPORT

PART 1

BACKGROUND METHODOLOGIES

**COORDINATOR, IIASA: A. KOCHETKOV
COORDINATOR, SRI PAS: A. STRASZAK**

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Consisting of 3 Parts

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III. TOWARDS A MORE REALISTIC DECISION SUPPORT VIA "HUMAN-CONSISTENT" COMMONSENSE-KNOWLEDGE-BASED DECISION MODELS

by Janusz Kacprzyk

III.1 INTRODUCTION

As a consequence of an unprecedented growth in the use of computers which has touched in recent years virtually all human activities: business, technology, commerce, science, etc., the analysis of man - computer systems is becoming one of the more important issues and the key factor to successful progress.

Among the man-computer systems, a particular role is played by those which are meant to support the human being in performing functions involving some inherently human capabilities as, e.g., reasoning or decision making. We will concentrate here on the latter, i.e. decision making.

Although experience accumulated by mankind throughout the centuries does make it possible to effectively and efficiently cope with a multitude of decision situations, current decisions are made in difficult, complex, competitive and ill-structured settings. These settings are full of uncertainty, subjectivity, imprecision, etc. in data, relations or value systems.

This, as well as unusually high potential gains or losses due to a proper or improper decision, suggests that human decision making processes should be assisted by some (computerized) decision support systems. Recent developments in computing technology do justify this idea.

At present, and presumably in the foreseeable future, it seems that the most efficient use of decision support systems will certainly be to assist and help the decision maker arrive at a proper decision but by no means to fully replace him or her. The system should therefore carry out some of the tasks it is better suited for, such as routine processing of relatively well defined and structured data, and then provide the user with some "good" solution guidelines. The final choice involving a delicate analysis of preferences, tradeoffs, etc. should then be left

to the user whose human skills are far superior in this respect.

To efficiently arrive at a proper and well timed decision within the above basic framework of decision support, some synergy between man and machine should exist. This includes trying to use the best capabilities and features of both parties involved, and attempting to make one party's conduct consistent with that of the other. The former aspect has already been mentioned. The latter, which is more important in our context, is somehow unidirectional. Namely, it is true that humans usually change their conduct while operating in a computerized environment. Unfortunately, the change of human nature is difficult to obtain. The other direction, to make the machine more "human-consistent", i.e., "to fit the task to the man" as mentioned before, seems more promising and will also be adopted here. Parenthetically, let us notice that the rationale behind the 5th Generation computing technology parallels this reasoning.

The human consistency of decision support systems has two aspects. The first is related to communication (interface) between the user and system and involves, among others, input of data and commands and output of results in a user-friendly way, preferably in a natural language which is the only fully natural means of human communication. The second aspect is related to algorithms, procedures, etc. employed by the system to obtain a solution. They are normally built upon some technical mathematical concept as, e.g., optimum, pure rationality, clear-cut constraints, etc. which need not necessarily reflect their human perception. This inconsistency may often inhibit human acceptance of the results provided by the system and hence make their usefulness doubtful.

The above two types of human consistency of decision support systems are of utmost practical importance, and both should be taken into account. The first, which might be called the input/output consistency, is more often dealt with. The second, which might be termed the algorithmic/procedural consistency, unfortunately, is not often considered in the field of decision support systems (a related need of "soft" models and approaches in systems analysis seems to be more strongly emphasized - see, e.g., Rapoport, 1970 or Checkland, 1972).

In this paper we deal with the algorithmic/procedural consistency. Our basic philosophy is that, from a pragmatic point of view, the "quality criterion" of a decision support system is its usefulness, i.e. ability to provide the user with implementable solution guidelines. And only those guidelines which do not depart too much from the user's experience, perception or commonsense may fulfill their purpose. The algorithms and procedures to be employed should therefore somehow parallel the way the human user perceives their essence and intention.

Among attempts to attain that, an important one is to use models which might be called "knowledge-based" as opposed to the conventional "data-based" ones. Human reasoning is certainly much more "knowledge-based", in the sense that it uses many non-numeric data and production-rule-like, dependences, than data-based, i.e. based on numeric data, mathematical equations, etc. Let us also notice that expert systems, which will hopefully be the most powerful means for dealing with diverse real world problems, are knowledge-based too.

One of the most important types of knowledge is commonsense knowledge. It is extensively used by humans making it possible to find a solution even in situations with almost no information. Clearly, such commonsense solutions may not be ideal but they are rarely really bad, and never absurd. Commonsense is a formidable human feature which is unfortunately not possessed by the computer - with all of the negative implications as, e.g., a danger of absurd results in case of incomplete or unreliable data. Introduction of commonsense knowledge into decision support systems would therefore greatly improve their human consistency, and hence facilitate their practical use. Unfortunately, a formal representation and manipulation of commonsense is conceptually difficult and far from being solved.

For practical purposes, Zadeh's (1984) approach to commonsense knowledge is presumably the most promising. It views commonsense knowledge as a collection of dispositions, i.e. propositions involving implicit linguistic quantifiers. For instance, a disposition "winter days are cold" is in fact meant as, say, "most winter days are cold", where "most" is a linguistic quantifier. Manipulation of dispositions is done by some fuzzy-logic-based calculus.

The approach is simple and elegant.

We will show in this paper how the use of Zadeh's approach to commonsense knowledge leads to a new class of more human-consistent multicriteria and multistage (control) decision making models. These models are chosen because virtually all decisions made in reality involve multiple aspects and some dynamics. Basically, in the multicriteria case the models allow one to find an optimal solution which best satisfies, say, most (almost all, much more than 50%, etc.) of the important criteria. Notice that in conventional models we seek an optimal solution to best satisfy all of the criteria. In the multistage (control) case, the models allow one to find an optimal sequence of controls to best satisfy the goals and constraints at, say, most (almost all, etc.) of the earlier control stages. Let us also notice that a similar approach leads to a new class of group decision making and consensus formation models (see, e.g., Kacprzyk, 1984a, 1985a, 1985b, 1985d) which will not be presented here.

First, we sketch the idea of Zadeh's approach to commonsense knowledge. Then, we consecutively apply it to derive new multicriteria and multistage (control) decision making models. Mathematics will be kept to a minimum and technicalities will be avoided to assure readability. Finally, we give some concluding remarks and bibliography.

For convenience to the reader let us briefly review some of the basic fuzzy-sets-related elements and notation which will be employed.

A fuzzy set A in X , written $A \subseteq X$, say $A = \text{"large"} \subseteq \{0, 1, \dots, 10\}$ to be meant as a fuzzy set A labelled "large (number)", is represented by μ_A and often equated with μ_A - its membership function $f_A : X \rightarrow [0, 1]$ which states to what degree x belongs to A : from 0 to full belongingness, through all intermediate values. For a finite $X = \{x_1, \dots, x_n\}$, we write $A = f_A(x_1)/x_1 + \dots + f_A(x_n)/x_n$ where "+" is set-theoretic and " $f_A(x_i)/x_i$ " means the pair $(x_i, f_A(x_i))$.

Very important for our purposes is a general framework for decision making under fuzziness according to Bellman and Zadeh (1970). Its basic elements are: a fuzzy goal $G \subseteq X$, a fuzzy constraint $C \subseteq X$, and a fuzzy decision $D \subseteq X$. To show the essence of this approach, let us use Fig.1.

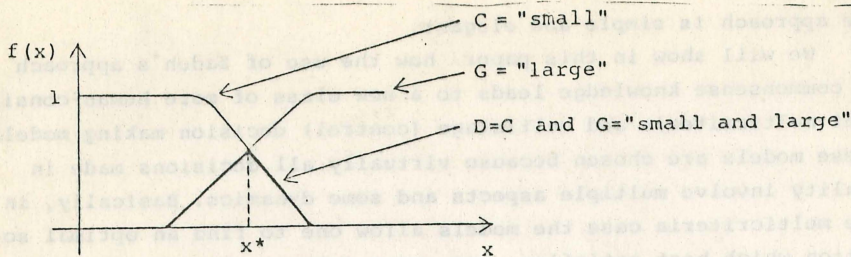


Fig.1. Basic elements of Bellman and Zadeh's approach to decision making under fuzziness.

Let us assume our fuzzy constraint is $C = \text{"small (number)"}$ and our goal is $G = \text{"large (number)"}$ whose membership functions, $f_C(x)$ and $f_G(x)$, are as given.

We wish to

"satisfy C and attain G "

which corresponds to the fuzzy decision $D \subseteq X$ whose membership function is

$$f_D(x) = f_C(x) \wedge f_G(x) \quad \text{for each } x \in X$$

where " \wedge " is "minimum", i.e. $a \wedge b = \min(a, b)$, and represents the connective "and".

The fuzzy decision gives the "goodness" of each x as a solution of the considered problem. Thus, x^* is the best (optimal) solution because $f_D(x)$ takes on its maximum value for x^* ; we will write $x^* = \arg \max_{x \in X} f_D(x)$, which means that x^* is an x which maximizes $f_D(x)$.

This general framework may easily be extended to cover the cases of multiple fuzzy constraints and goals, and of fuzzy goals and constraints defined in different spaces which are relevant for our purposes.

For more information on fuzzy sets, see, e.g., Kacprzyk (1983b).

III.2 BASIC ELEMENTS OF ZADEH'S APPROACH TO THE REPRESENTATION AND MANIPULATION OF COMMENSENSE KNOWLEDGE

In Zadeh's (1984) approach, commonsense knowledge is viewed as a collection of dispositions, i.e. propositions with implicit fuzzy linguistic quantifiers. For instance, a disposition "winter days are cold" does implicitly involve some fuzzy quantifier,

say "most", "almost all", etc., i.e. should in fact be read as "most winter days are cold". Since the traditional logical systems provide no means for handling propositions with fuzzy quantifiers, Zadeh (1983, 1984) develops the following fuzzy-logic-based calculus of linguistically quantified propositions.

A linguistically quantified proposition, exemplified by "most experts are convinced", may be generically written as

$$QY\text{'s are } F \tag{1}$$

where Q is a linguistic quantifier (most), $Y = \{y\}$ is a set of objects (experts), and F is a property (convinced).

We can also add importance, assumed to be a fuzzy set $B \subseteq Y$, to (1) obtaining

$$QBY\text{'s are } F \tag{2}$$

i.e. "most of the important experts are convinced".

The main problem now is to determine truth ($QY\text{'s are } F$), or truth ($QBY\text{'s are } F$), knowing truth (y_i is F) for each $y_i \in Y$. In the classic approach proposed by Zadeh (1983, 1984), a linguistic quantifier Q is assumed to be a fuzzy set in $[0, 1]$, $Q \subseteq [0, 1]$, characterized by its membership function $f_Q(r)$, $r \in [0, 1]$. Notice that this is the case for a proportional quantifier, say "most", while for an absolute quantifier, say "about 5", $Q \subseteq R$, i.e. is a fuzzy set in the real line. Throughout this paper we will use the proportional quantifiers which seem to be better suited for our purposes. Analogous properties also hold for the absolute quantifiers.

Property F is defined as a fuzzy set in Y , $F \subseteq Y$, whose membership function $f_F(y_i)$ gives to what degree $y_i \in Y$ possesses property F . If $Y = \{y_1, \dots, y_p\}$, then it is assumed that truth (y_i is F) = $f_F(y_i)$, $i = 1, \dots, p$.

The calculation of truth ($QY\text{'s are } F$) is based on the non-fuzzy cardinalities (the so-called Σ Counts, see Zadeh, 1983) of the respective fuzzy sets and proceeds as follows:

1. Calculate
$$r = \Sigma \text{ Count } (F) / \Sigma \text{ Count } (Y) = \frac{1}{P} \sum_{i=1}^P f_F(y_i) \tag{3}$$

2. Calculate
$$\text{truth } (QY\text{'s are } F) = f_Q(r) \tag{4}$$

Importance may be introduced into the above as follows.

$B =$ "important" is defined as a fuzzy set in $Y, B \subseteq Y$, such that $f_B(y_i) \in [0,1]$ is a degree of importance of y_i : the higher its value, the more important y_i .

We first rewrite "QBY's are F" (e.g., "most of the important experts are convinced") as "Q (B and F) Y's are B" (e.g., "most of the (important and convinced) experts are important") which leads to the following counterparts of (3) and (4):

1. Calculate

$$r^- = \frac{\prod_{i=1}^P (f_B(y_i) \wedge f_F(y_i))}{\sum_{i=1}^P f_B(y_i)} \quad (5)$$

2. Calculate

$$\text{truth (QBY's are F)} = f_Q(r^-) \quad (6)$$

Example 1. Let us have 3 experts, X, V and Z, i.e. $Y =$ "experts" = $\{X, V, Z\}$. Let $F =$ "convinced" = $0.1/X + 0.6/V + 0.8/Z$, that is X is convinced (as to an issue in question) to degree 0.1, i.e. practically not at all, V - to degree 0.6, i.e. moderately, and Z - to degree 0.8, i.e. quite strongly. Let $B =$ "important" = $0.2/X + 0.5/V + 0.6/Z$, that is the importance of X is 0.2, that of V is 0.5 and that of Z is 0.6; notice that none of the experts is considered very important (e.g., competent). Let $Q =$ "most" be given as

$$f_{\text{"most"}}(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x-0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases} \quad (7)$$

Then, on the one hand, $r = 0.5$ and

$$\text{truth ("most experts are convinced")} = 2 \cdot 0.5 - 0.6 = 0.4$$

On the other hand, $r = 0.9$ and

$$\text{truth ("most of the important experts are convinced")} = 1$$

For our particular purposes the above basic means for the representation and manipulation of commonsense knowledge in Zadeh's setting are sufficient, although in the sequel we introduce some other, more specific notions. Evidently, it is easy to imagine that the basic problem associated with the use of that knowledge in most applications is inference based on dispositions.

Some details may be found in Zadeh (1984) but the issue is far from a real solution.

Let us remark that in the procedure outlined above, we can also use fuzzy cardinalities of the respective fuzzy sets, the so-called FCounts, instead of the nonfuzzy ones (Σ Counts). This does not change the essence of the approach but may complicate the calculations to a considerable extent. Some details may be found in Kacprzyk (1985c) and Zadeh (1983).

Let us also note that the presented method may be viewed as yielding a consensory-like aggregation of the pieces of evidence " y_i is F", $i=1, \dots, p$. For details on this important issue, refer to Kacprzyk (1983a), Kacprzyk and Yager (1984a, 1984b) or Yager (1983).

The presented approach to dealing with linguistically quantified propositions is not the only one. An alternative procedure, yielding a competitive-like aggregation, has been proposed by Yager (1983) and used in Kacprzyk (1983a), Kacprzyk and Yager (1984a, 1984b), Yager (1983), etc. We will not discuss it here.

Finally, let us notice that Zadeh's approach to commonsense knowledge is not the only one. Among some other approaches, which may be useful for our purposes as they are constructive and relatively simple conceptually and computationally, hence operational, some default-logic-based approaches (Reiter, 1980; Reiter and Criscuolo, 1983) should be mentioned. For some details, see also Ferreny and Prade (1984).

We will now present the application of Zadeh's approach to derive some new, more human-consistent multicriteria and multi-stage (control) decision making models.

III.3 COMMONSENSE-KNOWLEDGE-BASED MULTICRITERIA DECISION MAKING MODELS

Virtually all decision to be made in non-trivial real situations must take into account the existence of multiple, often conflicting, objectives or criteria. This has triggered much research on multicriteria decision making.

For our purposes multicriteria decision making under fuzziness may be meant as follows. $A = \{a\} = \{a_1, \dots, a_d\}$ is a set of possible options (alternatives, decisions...) and $0 = \{0_1, \dots, 0_p\}$ is a set

of fuzzy objectives (fuzzy constraints and/or fuzzy goals). The degree to which option $a \in A$ satisfies objective $0_i \in O$ is given by

$$\text{truth}(0_i \text{ is satisfied (by } a)) = f_{0_i}(a), i=1, \dots, p. \quad (8)$$

Traditionally, it is postulated (e.g., Bellman and Zadeh, 1970; Kacprzyk, 1983b) that $a \in A$ satisfy " 0_1 and ... and 0_p " i.e. all the fuzzy objectives, and hence the degree of that satisfaction is given by the fuzzy decision

$$\begin{aligned} f_D(a \text{ "all"}) &= \text{truth}(0_1 \text{ and ... and } 0_p \text{ are satisfied}) = \\ &= \text{truth}(\text{"all" } 0_i\text{'s are satisfied}) = \text{truth}(0_1 \text{ is satisfied}) \wedge \\ &\dots \wedge \text{truth}(0_p \text{ is satisfied}) = f_{0_1}(a) \wedge \dots \wedge f_{0_p}(a) \quad (9) \end{aligned}$$

The problem is to find an optimal decision $a^* \in A$, such that

$$a^* = \arg \max_{a \in A} f_D(a \mid \text{"all"}) \quad (10)$$

The requirement to satisfy "all" the fuzzy objectives may be viewed too rigid and restrictive for practical purposes. An idea for replacing "all" by some milder requirement specified by a linguistic quantifier Q , say "most", appeared in Yager (1983), Kacprzyk and Yager (1984a, 1984b), etc. Basically, it consists of seeking an optimal decision that best satisfies Q (e.g., "most") fuzzy objectives. We will now discuss this type of problem.

Following Section 2, we introduce first the fuzzy set $S =$ "satisfied" $\subseteq O$, such that $f_S(0_i) = \text{truth}(0_i \text{ is satisfied (by } a)) = f_{0_i}(a)$, $i=1, \dots, p$, is to what degree objective 0_i is satisfied (by option a), and a fuzzy set $B =$ "important" $\subseteq O$, such that $f_B(0_i) \in [0, 1]$ is the degree of importance of objective 0_i : from 0 denoting "unimportant at all" to 1 denoting "definitely important", through all intermediate values. $Q \subseteq [0, 1]$ is a fuzzy linguistic quantifier.

The fuzzy decision is now written as

$$\begin{aligned} f_D(a \mid Q, B) &= f_D(a)Q \text{ "important"} = \text{truth}(QB \text{ } 0_i\text{'s are} \\ &\text{satisfied (by } a)) = \text{truth}(Q \text{ "important" } 0_i\text{'s are} \\ &\text{satisfied}) \quad (11) \end{aligned}$$

and gives the degree to which Q of the important (B) fuzzy objectives are satisfied (by a).

Employing (6), we obtain

$$f_D(a | Q, B) = f_Q \left(\frac{\sum_{i=1}^P (f_{B_i}(0_i) \wedge f_{0_i}(a))}{\sum_{i=1}^P f_{B_i}(0_i)} \right) \quad (12)$$

and the problem is to find an optimal option $a^* \in A$, such that

$$a^* = \arg \max_{a \in A} f_D(a | Q, B) \quad (13)$$

i.e. an optimal option which best satisfies Q of the important (B) fuzzy objectives.

Notice that if we do not wish to account for importance, we set $f_{B_i}(0_i) = 1$ for each $0_i \in 0$, i.e. we assume that all 0_i 's are equally important, and (12) and (13) become, respectively:

$$f_D(a | Q) = f_Q \left(\frac{1}{P} \sum_{i=1}^P f_{0_i}(a) \right) \quad (14)$$

and

$$a^* = \arg \max_{a \in A} f_D(a | Q) \quad (15)$$

It is easily seen that it is difficult to say something about the solution of (15) for an arbitrary linguistic quantifier Q , i.e. for an arbitrary function $f_Q(r)$. Fortunately enough, there are some particular relevant quantifiers in our context, the so-called nondecreasing quantifiers whose essence may be subsumed as "the more objectives that are satisfied the better". For such quantifiers, the solution of (15) is relatively easy; for details, see, e.g., Yager, 1983 or Kacprzyk and Yager, 1984a, 1984b).

Example 2. Let us have 3 options, i.e. $A = \{a_1, a_2, a_3\}$, and 3 fuzzy objectives 0_1 , 0_2 and 0_3 . Let $0_1 = 1/a_1 + 0.7/a_2 + 0.2/a_3$ which may be read as: option a_1 is definitely the best one, option a_2 may be chosen although it is not a definitely preferred choice (only to degree 0.7), and option a_3 is a relatively bad (to degree 0.2) choice although still possible. Let $0_2 = 0.2/a_1 + 1/a_2 + 0.5/a_3$, and $0_3 = 0.2/a_1 + 0.3/a_2 + 1/a_3$ to be meant analogously. Moreover, let $Q = \text{"most"}$ be given by (7), and $B = \text{"important"} = 0.3/0_1 + 0.8/0_2 + 0.1/0_3$ which expresses importances of the particular objectives as in Example 1.

Our problem is to find "an optimal option which best satisfies most of the important objectives". For the particular options we obtain the following values of the fuzzy decision

$$f_D(a_1 | \text{"most"}, \text{"important"}) = 0$$

$$f_D(a_2 | \text{"most"}, \text{"important"}) = 1$$

$$f_D(a_3 | \text{"most"}, \text{"important"}) = 0.9$$

that is the optimal option sought is a_2 .

III.4 COMMONSENSE-KNOWLEDGE-BASED MULTISTAGE DECISION MAKING (CONTROL) MODELS

Most decision situations involve some dynamics, i.e. decisions that are currently made influence not only outcomes at the following time (stage) but also those in the more distant future. To account for that, multistage decision (control) models are developed.

For our purposes, multistage decision making (control) under fuzziness may be formalized as follows. At each time (control stage) t the control $u_t \in U = \{c_1, \dots, c_m\}$ is subject to a fuzzy constraint $f_{C^t}(u_t)$, and on the state attained $x_{t+1} \in X = \{s_1, \dots, s_n\}$ a fuzzy goal $f_{G^{t+1}}(x_{t+1})$ is imposed, while the state transitions are governed by $x_{t+1} = g(x_t, u_t)$; $x_t, x_{t+1} \in X, u_t \in U, t=0, 1, \dots, N$; N is some termination time.

It is commonly postulated (e.g., Bellman and Zadeh, 1970; or Kacprzyk, 1982, 1983b) that at each t , the control u_t satisfy the fuzzy constraint C^t and the fuzzy goal G^{t+1} (in fact the fuzzy goal is satisfied not by u_t but by the resulting new state x_{t+1}), to be written as P_{t+1} : " C^t and G^{t+1} are satisfied (by u_t)". This satisfaction is evidently equal to

$$\begin{aligned} \text{truth } P_{t+1} &= \text{truth } (C^t \text{ and } G^{t+1} \text{ are satisfied}) = \\ &= f_{C^t}(u_t) \wedge f_{G^{t+1}}(x_{t+1}) \end{aligned} \quad (17)$$

Traditionally, we require a sequence of controls to satisfy the fuzzy constraints and fuzzy goals at all the subsequent control stages, hence the fuzzy decision expressing the degree of that satisfaction is

$$\begin{aligned}
 f_D(u_0, \dots, u_{N-1} | x_0, \text{"all"}) &= \text{truth } (P_1 \text{ and } \dots \text{ and } P_N | \text{"all"}) = \\
 &= \bigwedge_{t=0}^{N-1} \text{truth } P_{t+1} = \bigwedge_{t=0}^{N-1} (f_{C^t}(u_t) \wedge f_{G^{t+1}}(x_{t+1})) \quad (18)
 \end{aligned}$$

where $\bigwedge_{t=0}^{N-1} a_t = a_0 \wedge \dots \wedge a_{N-1}$

The problem is to find an optimal sequence of controls u^*_0, \dots, u^*_{N-1} , such that

$$u^*_0, \dots, u^*_{N-1} = \arg \max_{u_0, \dots, u_{N-1}} f_D(u_0, \dots, u_{N-1} | x_0, \text{"all"}) \quad (19)$$

For details and some extensions of the above basic formulation, see Kacprzyk (1982, 1983b).

As in the case of multicriteria decision making, "all" may be viewed too restrictive, and its replacement by a milder linguistic quantifier Q, say "most", has been proposed by Kacprzyk (1983a). Therefore an optimal sequence of controls is sought, u^*_0, \dots, u^*_{N-1} , which best satisfies the fuzzy constraints and goals at Q control stages, i.e.

$$\begin{aligned}
 u^*_0, \dots, u^*_{N-1} &= \arg \max_{u_0, \dots, u_{N-1}} f_D(u_0, \dots, u_{N-1} | x_0, Q) = \\
 &= \arg \max_{u_0, \dots, u_{N-1}} \left(\bigwedge_{t=0}^{N-1} | Q \right) (f_{C^t}(u_t) \wedge f_{G^{t+1}}(x_{t+1})) \quad (20)
 \end{aligned}$$

where $(\bigwedge_{t=0}^{N-1} | Q) a_t$ means that in $a_0 \wedge \dots \wedge a_{N-1}$ only Q, say "most" a_i s are included.

For simplicity, we will assume that importance is not accounted for, i.e. the importance of each control stage is the same. Let us notice that the introduction of importance, i.e. derivation of a model which allows one to seek an optimal sequence of controls best satisfying the fuzzy goals and constraints at, say most of the earlier control stages, may be viewed as the introduction of discounting which has a long tradition in conventional approaches. This important issue is however beyond the scope of this paper.

For the solution of (20) we use the two steps of Zadeh's procedure, and consecutively obtain:

$$r(u_0, \dots, u_{N-1} | x_0) = \frac{1}{N} \sum_{t=0}^{N-1} (f_{C^t}(x_t) \wedge f_{G^{t+1}}(x_{t+1})) \quad (21)$$

$$\begin{aligned} f_D(u_0, \dots, u_{N-1} | x_0, Q) &= f_Q(r(u_0, \dots, u_{N-1} | x_0)) = \\ &= f_Q\left(\frac{1}{N} \sum_{t=0}^{N-1} (f_{C^t}(u_t) \wedge f_{G^{t+1}}(x_{t+1}))\right) \end{aligned} \quad (22)$$

Therefore we seek an optimal sequence of controls, such that

$$u_0^*, \dots, u_{N-1}^* = \arg \max_{u_0, \dots, u_{N-1}} f_D(u_0, \dots, u_{N-1} | x_0, Q) \quad (23)$$

that is, the one which best satisfies the fuzzy constraints and fuzzy goals at Q, say "most", control stages.

Similarly as in the case of multicriteria decision making, it is also difficult to say something about the solution of (23) for an arbitrary Q. Fortunately enough, for the so-called non-decreasing quantifiers whose essence is now "the more control stages at which the fuzzy constraints and goals are satisfied the better", the solution of (23) may be obtained by dynamic programming. For details, see Kacprzyk (1983a)

Example 3. Let us have 3 control stages, i.e. N=3. Let the fuzzy constraints and goals at the particular control stages be

$$\begin{aligned} C^0 &= 0.5/c_1 + 1/c_2 & G^1 &= 0.1/s_1 + 0.6/s_2 + 1/s_3 \\ C^1 &= 1/c_1 + 0.7/c_2 & G^2 &= 0.6/s_1 + 1/s_2 + 0.5/s_3 \\ C^2 &= 1/c_1 + 0.6/c_2 & G^3 &= 1/s_1 + 0.8/s_2 + 0.3/s_3 \end{aligned}$$

to be understood as in Example 2.

Let the state transitions be governed by a state transition equation represented by the table

		x_t			
		s_1	s_2	s_3	
$x_{t+1} =$	u_t	c_1	s_3	s_3	s_3
		c_2	s_2	s_2	s_2

i.e. if, for example, the current state is $x_t = s_2$ and the current control is $u_t = c_1$, the next state becomes $x_{t+1} = s_3$.

Using dynamic programming we solve the problem "backwards", i.e. first for the last control stage $t=N-1$, then for $t=N-2, \dots$, and finally for $t=0$, determining the so-called policy functions $a_t^*(x_t)$ which give for each state x_t an optimal control at a particular control stage t .

Thus, for $t=N-1=2$, we obtain the policy functions

$$a_2^*(s_1) = c_2, \quad a_2^*(s_2) = c_2, \quad a_2^*(s_3) = c_2;$$

i.e. if at $t=2$ we are in state s_1 then the optimal control is c_2 , for $a_2^*(s_1) = c_2$, etc. Next, for $t=1$ and $t=0$, respectively

$$a_1^*(s_1) = c_2, \quad a_1^*(s_2) = c_2, \quad a_1^*(s_3) = c_2;$$

$$a_0^*(s_1) = c_1 \text{ or } c_2, \quad a_0^*(s_2) = c_1, \quad a_0^*(s_3) = c_1 \text{ or } c_2.$$

III.5 SOME REMARKS ON COMMONSENSE KNOWLEDGE IN GROUP DECISION MAKING AND CONSENSUS FORMATION MODELS

Zadeh's approach to commonsense knowledge employed here has also proved to be an efficient means for making some other types of decision making models, more human-consistent.

More specifically, it is easy to notice that a fuzzy linguistic quantifier, say, "most" or "almost all", is a natural representation of a (fuzzy) majority as perceived by humans. This is a point of departure in Kacprzyk (1984a, 1985a) where some new solution concepts for group decision are proposed as, e.g., some fuzzy cores, consensus winners, etc., using the approach employed here. Moreover, in Kacprzyk (1985b) the approach is used to define the notion of a "soft" consensus and its degree to model and monitor real, not ideal, consensus reaching processes.

III.6 CONCLUDING REMARKS

In this paper we show the introduction of some elements of commonsense knowledge, using one of its representations which seems to be particularly promising due to its simplicity, elegance and

constructiveness, into some multicriteria and multistage (control) decision making models.

The very purpose of the above is to make the models, which are to be used in decision support systems, more human-consistent.

This should greatly facilitate interaction between the user and the system because the human user will presumably be more willing to adopt solution guidelines provided by the system if the system's "reasoning" and conduct parallels to some extent those of his or her. The implementability of the system's solutions, hence the system's usefulness, should therefore be enhanced.

This paper is an attempt towards developing a research direction, to be eventually called the algorithmic/procedural "human-consistency" of decision support systems, which we feel is extremely important and should be further pursued.

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STUDY REPORT

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