New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

SRI PAS IBS PAN

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications

Systems Research Institute
Polish Academy of Sciences

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

IBS PAN SRI PAS

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 83-894-7541-3

Dedicated to Professor Beloslav Riečan on his 75th anniversary

Fuzzy decision making in toxoplasmosis medication

Elisabeth Rakus-Andersson[1] and Alicja Kurnatowska[2]
[1] Blekinge Institute of Technology,
Valhallavägen 1, 37179 Karlskrona, Sweden
[2] Medical University of Łódź,
Plac Hallera 1, 90-647, Łódź, Poland
Elisabeth.Andersson@bth.se, Katbiol@poczta.onet.pl

Abstract

Theoretical fuzzy decision-making models mostly developed by Zadeh, Bellman, Jain and Yager can be adopted as useful tools to estimation of the total effectiveness-utility of a drug when appreciating its positive influence on a collection of symptoms characteristic of a considered diagnosis.

The expected effectiveness of the medicine is evaluated by a physician as a verbal expression for each distinct symptom. By converting the words at first to fuzzy sets and then numbers we can regard the effectiveness structures as entries of a utility matrix that constitutes the common basic component of all methods. We involve the matrix in a number of computations due to different decision algorithms to obtain a sequence of tested medicines in conformity with their abilities to soothe the unfavorable impact of symptoms.

An adjustment of the large spectrum of applied fuzzy decision-making models to the extraction of the best medicines provides us with some deviations in obtained results but we are thus capable to select this method whose effects closest converge to the physicians' judgments and expectations.

In the current paper we apply fuzzy decision making algorithms to rank medicines in multifocal toxoplasmosis.

Keywords: fuzzy decision-making, fuzzy utility matrix, powers of symptom importance, utilities of medicines in toxoplasmosis, Bellman-Zadeh decision making, unequal objectives, minimization of regret, OWA operators.

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications *(K.T. Atanassov, W. Homenda, O. Hryniewicz, J.Kacprzyk, M. Krawczak, Z. Nahorski, E Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2012.*

1 Introduction

Theoretical fuzzy decision-making models mostly develop in [3, 4–8, 9, 10], give rise to successfully accomplished technical applications. However, there are not so many medical applications to decision-making proposals, especially they are lacking in the domain of pharmacy matters.

After visiting the homepages of some pharmacological concerns, e.g., AstraZeneca in Sweden, we realize that the most popular mathematical methods utilized in appreciation of medicine availability in the treatment of patients are statistical tests, like descriptive statistics, regression and analysis of variance, a general linear model (GLM) and others [11]. Statistical tests are very helpful in grading the curative power of medicines; nevertheless they cannot effectively handle either interactions among medicines or imprecise estimations of the medicine influence on a collection of symptoms that should retreat after the treatment.

Fuzzy set theory operating with computing with words [12–15] and with systems accommodated to imprecise or vague data provides us with a mathematical apparatus giving answers to different posed questions regarding pharmacology. We can refer to such tools as the adaptation of fuzzy control in pharmacology models [16], the recognition prime-decision model (RPD) in appreciation of drugs [17] or the process of medicine extraction by the method of midpoints [18]. Other solutions evaluating medicines, worth mentioning, are: classification of medicines by fuzzy matrices [9], rough sets in evaluation of medicines [19] and the use of neuro-fuzzy structures contra rough sets in the possible evaluation of drugs [20].

Anyway, if we formulate the task to solve as determination of a hierarchical ladder in a sample of medicines that affect the same symptoms typical of a considered diagnosis with respect to the choice of the most efficacious medicine then we cannot find any positions in literature except own previously made attempts [21–28]. We thus intend to select the most promising results of fuzzy decision-making models adapted to the selection of the most effective medicine when comparing it to others in the process of the patients' recoveries. In order to show the models' action we test medicines proved in toxoplasmosis.

To start with the discussion concerning the choice of drugs we sketch the components of fuzzy decision-making model in Section 2. In section 3 we interpret the Bellman-Zadeh model of decision making [1]. To improve the obtained results we insert weights-powers [4–8] when selecting the best medicine in accordance to the algorithm of unequal objectives. At last, we perform OWA-operations [29–30] to benefit the model of regret minimization in optimal medicine decision making. We have thus made a survey of fuzzy decision making versions from the oldest to the newest ones to show improvements of operations involved in numerical calculations.

2 The general outline of a drug-decision making model

We introduce the notions of a space of states $X = \{x_1,...,x_m\}$ and a decision space (a space of alternatives) $A = \{a_1,...,a_n\}$. We consider a decision model in which n alternatives $a_1,...,a_n \in A$ act as drugs used to treat patients who suffer from a disease. The medicines should influence m states $x_1,...,x_m \in X$, which are identified with m symptoms typical of the morbid unit considered [21–28].

If a rational decision maker makes a decision $a_i \in A$, $i = 1, 2, \ldots, n$, concerning states-results $x_j \in X$, $j = 1, 2, ..., m$, then the problem is reduced to the consideration of the ordered triplet (X, A, U), where X is a set of states-results, A – a set of decisions and U – the utility matrix [1, 2, 4–8, 10, 21–28]

$$U = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_m \end{array} \\ \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{bmatrix} \end{array} \tag{1}$$

in which each element u_{ij}, $i = 1,\ldots,n$, $j = 1,\ldots,m$, is a representative value belonging to [0, 1] for the fuzzy utility following from the decision a_i with the result x_j.

The theoretical model with the triplet (X, A, U) can find its practical application in the processes of choosing an optimal drug from a sample of tested medicines [23–28].

To solve the decision problem under circumstances given above means to find the best decision a_i influenced by all constraints.

If a given disease is recognized by the symptoms accompanying it, then we, by giving a medicine, try to liquidate these symptoms or at least try to reduce their unfavorable influence upon the patient's health. Not all symptoms retreat after the cure has been carried out. Sometimes, one can only soothe their negative effects by, for example, the lowering of an excessive level of the indicator, the relief of pain and the like, but cannot ascertain that the patient is fully free from them. The problem of choosing an optimal drug (decision), which soothes the symptoms or has some power to eliminate them in full, corresponds to the theoretical assumptions presented above.

In order to show the algorithm for finding such a decision let us consider a model with *n* drugs $a_1, a_2, ..., a_n \in A$. On the basis of the physician's decision, the drugs are prescribed to the patient (thus may be treated as decisions $a_1, a_2, ..., a_n$) with a view to have an effect on *m* symptoms $x_1, x_2, ..., x_m \in X$ representing certain states characteristic of the given disease. To simplify the symbols let us further assume that each symptom $x_j \in X$, where *X* is a space of symptoms (states), is understood as the result of the treatment of the symptom after the cure with the drugs $a_1, a_2, ..., a_n$ has been carried out.

The relationship between a medicine and a symptom is determined in the term of utility. Let us discuss the formalized technique of stating the representatives of utilities without using intuitive or perceptional estimations.

On the basis of earlier experiments the physician knows how to define in words the curative drug efficiency in the case of considered symptoms. We intend to replace his verbal judgments by numerical expressions to be able to insert them in the mathematical model [12–15, 31–32]. In accordance with the physician's advice, we suggest a list of terms that introduces a linguistic variable [13–14] named "*the curative drug effectiveness regarding a symptom*" = {R_1 ="*none*", R_2 = "*almost none*", R_3 = "*very little*", R_4 = "*little*", R_5 = "*rather little"*, R_6 = "*medium*", R_7 = "*rather large*", R_8 = "*large*", R_9 = "*very large*", R_{10} = "*almost complete*", R_{11} = "*complete*"}.

We will now show how to find the numerical expressions replacing the terms from the list. Each notion, contained in the list, is the name of a fuzzy set [33].

Assume that all sets from the list "*the curative drug effectiveness regarding a symptom*" are defined in the space $Z = [0,100]$ that is suitable as a reference set to measure a number of patients who have been affected by a medicine in the grade corresponding to each name.

We propose constrains for the fuzzy sets R_1-R_{11} by applying linear functions [26-28]

$$L(z,\alpha,\beta) = \begin{cases} 0 & \text{for} \quad z \le \alpha, \\ \dfrac{z-\alpha}{\beta-\alpha} & \text{for} \quad \alpha < z \le \beta, \\ 1 & \text{for} \quad z > \beta, \end{cases} \tag{2}$$

and

$$\pi(z,\alpha,\gamma,\beta)=\begin{cases}0 & \text{for} \quad z\le\alpha,\\ L(z,\alpha,\gamma) & \text{for} \quad \alpha<z\le\gamma,\\ 1-L(z,\gamma,\beta) & \text{for} \quad \gamma<z\le\beta,\\ 0 & \text{for} \quad z>\beta,\end{cases} \tag{3}$$

where z is an independent variable from [0,100], whereas α, β, γ are parameters.

Let us now define

$$\mu_{R_t}(z)=\begin{cases}1-L(z,\alpha_t,\beta_t) & \text{for} \quad t=1,2,3,4,5,\\ L(z,\alpha_t,\beta_t) & \text{for} \quad t=7,8,9,10,11,\end{cases} \tag{4}$$

and

$$\mu_{R_6}(z)=\pi(z,\alpha_6,\gamma,\beta_6), \tag{5}$$

in which α_t, β_t, γ are the borders for supports of the fuzzy sets R_1-R_{11}.

We decide the values of the boundary parameters α_t, β_t, γ in Ex. 1 below.

Example 1

Figure 1 collects the graphs of restrictions made for fuzzy sets R_1-R_{11} that are approved as the terms composing the contents of the effectiveness list.

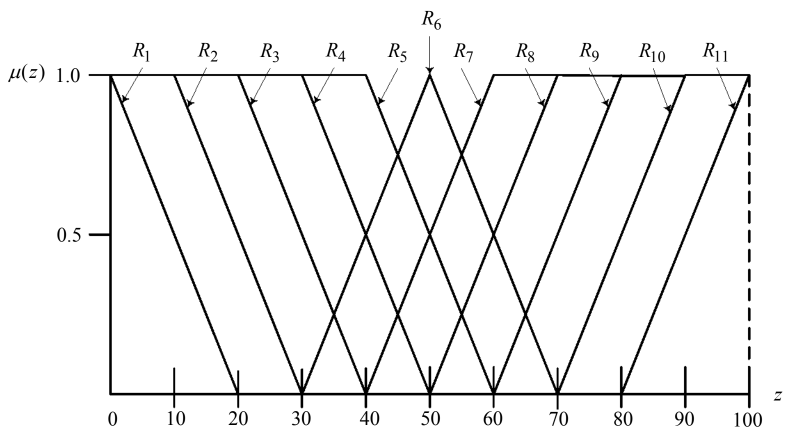

Figure 1: The fuzzy sets R_1-R_{11}

To each name of effectiveness, expanded as a continuous fuzzy set, we would like to assign only one value. To find the adequate $z\in[0,100]$ representing the effectiveness terms R_1-R_{11} we adopt as z the values α_t for t =1, 2, 3, 4, 5, and β_t for $t = 7, 8, 9, 10, 11$ in compliance with (4), respectively γ due to (5). We simply read off the values of α_t, β_t and γ from Fig. 1 in order not to introduce evident calculations. These z-values are elements of a new fuzzy set "*effectiveness*" whose membership function is expressed over the interval [0,100] by $\mu_{\text{"effectiveness"}}(z) = L(z,0,100)$. For the z-representatives of R_1-R_{11} we compute membership values $\mu_{\text{"effectiveness"}}(z)$, which replace the terms of effectiveness-utility as quantities u_{ij}. Hence, we determine the relation between terms of utilities and their quantified replacements. The relationship is stated in Table 1.

Table 1: The relationship between verbal terms of utilities and their numerical substitutes

Effectiveness	z-value for effectiveness	$\mu_{effectiveness}(z) = u_{ij}$
R_1=*"none"*	0	0
R_2=*"almost none"*	10	0.1
R_3=*"very little"*	20	0.2
R_4=*"little"*	30	0.3
R_5=*"rather little"*	40	0.4
R_6=*"medium"*	50	0.5
R_7=*"rather large"*	60	0.6
R_8=*"large"*	70	0.7
R_9=*"very large"*	80	0.8
R_{10}=*"almost complete"*	90	0.9
R_{11}=*"complete"*	100	1

Instead of linear restrictions of sets R_1–R_{11}, which constitute the simplest designs, we can implement other types of membership functions, e.g., s-class functions [27].

To state a connection between a_i (medicine) and the effectiveness of the retreat of x_j (symptom) the physician uses the word from the list "*the curative drug effectiveness regarding a symptom*" and this word is "translated" into the quantity u_{ij}, $i = 1,\ldots,m$, $j = 1,\ldots,n$.

Example 2

Let us sample the clinical data coming from the investigation carried out among patients who suffer from D_1 = "*Multifocal toxoplasmosis*" [34–37]. We consider the most typical symptoms accompanying the illness, i.e., x_1 = "*febrility*", x_2 = "*headache and dizziness*", x_3 = "*apathy and deconcetration*", x_4 = "*weakness*", x_5 = "*muscle pain*", x_6 = "*pain of enlarged occipital lymph nodules*", x_7 = "*pain of enlarged submandibularis lymph nodules*", x_8 = "*pain of enlarged periauricular lymph nodules*", x_9 = "*pain of enlarged axillary lymph nodules*", x_{10} = "*pain of enlarged inguinal lymph nodules*" and x_{11} = "*chilling*".

In the treatment of D_1 a physician has recommended a_1 = *Pyrimethamine*, a_2 = *Trimetoprime + sulfametoxazol*, a_3 = *Pyrimethamine + sulfametoxazol*, a_4 = *Pyrimethamine + sulfadoxine + folinic acid*, a_5 = *Spiramycine* and a_6 = *Pyrimethamine + sulfadoxine + Spiramycine* as the medicines expected to improve the patient's state. The physician has also decided that the matrix U should have entries consisting of effectiveness terms as a tableau

$$U = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} R_3 & R_6 & R_3 & R_3 & R_3 & R_3 & R_4 & R_4 & R_4 & R_5 & R_3 \\ R_6 & R_6 & R_4 & R_4 & R_4 & R_4 & R_3 & R_3 & R_4 & R_6 & R_6 \\ R_6 & R_6 & R_6 & R_4 & R_4 & R_4 & R_4 & R_4 & R_6 & R_6 & R_5 \\ R_8 & R_7 & R_6 & R_6 & R_5 & R_5 & R_5 & R_6 & R_6 & R_7 & R_6 \\ R_8 & R_8 & R_6 & R_6 & R_5 & R_6 & R_6 & R_6 & R_7 & R_7 & R_6 \\ R_8 & R_5 & R_5 & R_5 & R_6 & R_6 & R_6 & R_6 & R_5 & R_5 & R_6 \end{bmatrix} \end{array}$$

We replace U's verbal expressions by the numbers stated in the last column of Table 1. The numerical version of U is thus determined as

$$U = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.2 & 0.2 & 0.2 & 0.2 & 0.3 & 0.3 & 0.3 & 0.4 & 0.2 \\ 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 & 0.2 & 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.5 & 0.4 & 0.4 & 0.4 & 0.5 & 0.5 & 0.6 & 0.5 \\ 0.7 & 0.7 & 0.5 & 0.5 & 0.4 & 0.5 & 0.5 & 0.5 & 0.6 & 0.6 & 0.5 \\ 0.7 & 0.4 & 0.4 & 0.4 & 0.5 & 0.5 & 0.5 & 0.5 & 0.4 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

3 The design of the Bellman-Zadeh decision model

We will now prove the earliest model of fuzzy decision making suggested by Bellman and Zadeh [1]. Due to their suggestions we form the decision set A^* with the members consisting of drugs $a_1, \ldots, a_n$ as

$$A^* = \sum_{i=1}^{n} \mu_{A^*}(a_i) / a_i, \qquad (6)$$

in which the membership degree of each a_i is shaped by an operation

$$\mu_{A^*}(a_i) = \min_{1 \le j \le m}(u_{ij}) \,. \tag{7}$$

We accept as an optimal medicine a^* this a_i whose membership degree satisfies the equation

$$\mu_{A^*}(a^*) = \max_{1 \le i \le n}(\mu_{A^*}(a_i)) \,. \tag{8}$$

Example 3

We consider the contents of matrix U constructed in Ex. 2. In accord with (6)–(8) the decision set A^* is found as

$$A^* = {}^{\min(0.2,0.3,0.2,\ldots,0.2)}\!/\!_{a_1} + {}^{\min(0.5,0.5,0.3,\ldots,0.5)}\!/\!_{a_2} + {}^{\min(0.5,0.5,0.5,\ldots,0.4)}\!/\!_{a_3} + {}^{\min(0.7,0.6,0.5,\ldots,0.5)}\!/\!_{a_4}$$
$$+ {}^{\min(0.7,0.7,0.5,\ldots,0.5)}\!/\!_{a_5} + {}^{\min(0.7,0.4,0.4,\ldots,0.5)}\!/\!_{a_6} = {}^{0.2}\!/\!_{a_1} + {}^{0.2}\!/\!_{a_2} + {}^{0.3}\!/\!_{a_3} + {}^{0.4}\!/\!_{a_4} + {}^{0.4}\!/\!_{a_5} + {}^{0.4}\!/\!_{a_6} \,.$$

After reconsidering the membership degrees of drugs in the decision set A^* in the descending order we conclude that a_i, i =1,…,6, are arranged in the sequence $a_4 = a_5 = a_6 \succ a_3 \succ a_1 = a_2$, where $a_i \succ a_j$ means that a_i acts better than a_j, i, j = 1,...,6.

The decision made seems to be very poor and cautious because of the unfavorable affection of the minimum operator when deciding the degrees of a_i. The use of minimum deprives many data values of their decisive power. We intend to improve the obtained results by attaching the importance values to the symptoms-states considered.

4 Unequal states-results in the choice of medicines

The purpose of this section is to add other factors inserted in the solution of fuzzy decision-making model. We want the model to be furnished with extraction of the most efficacious medicine provided that the particular emphasis is also impacted on assigning differing degrees of importance to states-symptoms [4–5, 26].

Let us associate with each symptom x_j, j = 1,…,m, a non negative number that indicates its power or importance in the decision according to the rule: the higher the number is the more important role of the x_j's retreat will be regarded. We assign $w_1,\ldots,w_m$ as powers-weights to $x_1,\ldots,x_m$ to modify (6) as a richer and more extended decision

$$A^*_{weighted} = \sum_{i=1}^{n} \mu_{A^*_{weighted}}(a_i) \Big/ a_i \qquad (9)$$

with membership degrees of each $a_i \in A^*_{weighted}$ computed as [5]

$$\mu_{A^*_{weighted}}(a_i) = \min_{1 \le j \le m}(u_{ij}^{\,w_j}). \qquad (10)$$

We note that each a_i always takes the value of a membership degree from [0, 1]. If w_j gets bigger then $u_{ij}^{\,w_j}$, $j = 1,\ldots,m$, $i = 1,\ldots,n$, will get smaller, closer to zero. On the contrary, $w_j \to 0$ implies $u_{ij}^{\,w_j} \to 1$. The behaviour of minimum warrants that the minimal value in the sequence of quantities belonging to [0, 1] must be a value coming from the same interval. This time the application of the minimum operator is better motivated as before since we neglect large values of $u_{ij}^{\,w_j}$ corresponding to less important symptoms.

A procedure for obtaining a ratio scale of importance for a group of m elements (symptoms) was developed by Saaty [38].

Assume that we have m objects (symptoms) and we want to construct a scale, rating these objects as to their importance with respect to the decision – in our case we wish to prioritize symptoms as to our expectations to get rid of them. We ask a decision-maker to compare the objects in paired comparison. If we compare object j with object k, then we will assign the values b_{jk} and b_{kj} as follows

(1) $b_{kj} = \dfrac{1}{b_{jk}}$.

(2) If object j is more important than object k then b_{jk} gets assigned a number according to the following scheme:

Intensity of importance expressed by the value of b_{jk}	*Definition*
1	Equal importance of x_j and x_k
3	Weak importance of x_j over x_k
5	Strong importance of x_j over x_k
7	Demonstrated importance of x_j over x_k
9	Absolute importance of x_j over x_k
…	
2, 4, 6, 8	Intermediate values

If object k is more important than object j, we assign the value of b_{kj} to pair (x_j, x_k).

Having obtained the above judgments an $m\times m$ importance matrix $B=\left(b_{jk}\right)_{j,k=1}^{m}$ is constructed in the drug decision problem sketched above. Matrix B constitutes a crucial part in the procedure of determining the degrees of importance $w_1, \ldots, w_m$ that affect the decision set $A^*_{weighted}$ in a substantial way. The weights are decided as components of this eigen vector that corresponds to the largest in magnitude eigen value of the matrix B.

Example 4

By involving the computation technique suggested in the description of matrix B we try to find the weights for objects x_j, $j = 1,\ldots,11$, already introduced in Ex. 2 and Ex. 3.

We want to release the patient from unfavorable symptoms in the order: x_1, x_2, x_3, $x_4 - x_5$ (equal priorities), $x_6 - x_{10}$ (equal priorities) and x_{11}. This hierarchy of the retreat of symptoms, desired by physicians and patients, is helpful when constructing a content of the matrix B as

$$B=\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{10} \\ x_{11} \end{array}\begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} 1 & 3 & 5 & 7 & 7 & 9 & 9 & 9 & 9 & 9 & 11 \\ \frac{1}{3} & 1 & 3 & 5 & 5 & 7 & 7 & 7 & 7 & 7 & 9 \\ \frac{1}{5} & \frac{1}{3} & 1 & 3 & 3 & 5 & 5 & 5 & 5 & 5 & 7 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{9} & \frac{1}{7} & \frac{1}{5} & \frac{1}{3} & \frac{1}{3} & 1 & 1 & 1 & 1 & 1 & 3 \\ \frac{1}{11} & \frac{1}{9} & \frac{1}{7} & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix} \end{array}$$

The largest eigen value $\lambda_1 = 11.43$ of B has the associated eigen vector V_1, whose coordinates are interpreted as weights $w_1,\ldots,w_{11}$ of $x_1,\ldots,x_{11}$, respectively. We establish $w_1 = 0.78$, $w_2 = 0.49$, $w_3 = 0.29$, $w_4 = 0.15$, $w_5 = 0.15$, $w_6 = 0.06$, $w_7 = 0.06$, $w_8 = 0.06$, $w_9 = 0.06$, $w_{10} = 0.06$, $w_{11} = 0.03$.

In compliance with the recommended Eqs (9) and (10) the final decision $A^*_{weighted}$ is obtained as a fuzzy set

$$\begin{aligned} A^* &= \min(0.2^{0.78},0.5^{0.49},0.2^{0.29},\ldots,0.2^{0.03})/a_1 + \min(0.5^{0.78},0.5^{0.49},0.3^{0.29},\ldots,0.5^{0.03})/a_2 \\ &+ \min(0.5^{0.78},0.5^{0.49},0.5^{0.29},\ldots,0.4^{0.03})/a_3 + \min(0.7^{0.78},0.6^{0.49},0.5^{0.29},\ldots,0.5^{0.03})/a_4 + \min(0.7^{0.78},0.7^{0.49},0.5^{0.29},\ldots,0.5^{0.03})/a_5 \\ &+ \min(0.7^{0.78},0.4^{0.49},0.4^{0.29},\ldots,0.5^{0.03})/a_6 = 0.285/a_1 + 0.582/a_2 + 0.582/a_3 + 0.757/a_4 + 0.757/a_5 + 0.638/a_6. \end{aligned}$$

After returning to (8) we conclude that the soothing effect of considered medicines is ranked in the order $a_4 = a_5 \succ a_6 \succ a_2 = a_3 \succ a_1$. We have considered not only the effectiveness of drugs regarding their action on symptoms, but also the priority of symptoms to be wished to disappear. The importance order among the symptoms points out that the ones that should disappear first, for the reason of their harm, mostly influence the patient's mental and physical condition.

5 Minimization of regret

The action of the minimum operation in the final decision formulas has provided us with a very cautious prognosis referring to the drug hierarchy. Some high values of utilities emphasizing a positive effect of medicine on considered symptoms have no chance of influencing finally computed decision degrees. We can even say that the minimum operation acts as a filter for high values by depriving them of their power.

In the next trial of evaluation of the medicine order we want to obtain clearer results when applying another fuzzy decision-making technique known as a minimization of regret [7]. We preserve a decision space (a space of medicines) $A = \{a_1, ..., a_n\}$ and a space of states-symptoms $X = \{x_1, x_2, ..., x_m\}$. We form a basic payoff matrix (the old U-utility matrix)

$$C = \begin{array}{c} \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & \dots & x_j & \cdots & x_m \end{array} \\ \begin{bmatrix} & & & & \\ & & & & \\ & & c_{ij} & & \\ & & & & \\ & & & & \end{bmatrix} \end{array} \tag{11}$$

where $c_{ij} = u_{ij}$ is the payoff (utility) to a decision-maker if he connects a_i to x_j, $i = 1,\dots,n$, $j = 1,\dots,m$.

In a continuation of the proposed approach to the choice of an optimal medicine we first obtain the regret matrix R. Its components r_{ij} indicate the decision-maker's regret in selecting alternative a_i when the state of X is x_j. We then calculate the maximal regret for each alternative.

A procedure of selecting an optimal a_i should follow some steps listed below due to Algorithm 1.

Algorithm 1

1. For each x_j calculate $C_j = \max_{1 \le i \le n} c_{ij}$.
2. For each pair a_i and x_j calculate $r_{ij} = C_j - c_{ij}$.

3. Suppose that matrix B from Section 4 consists of b_{jk}, which still describe the importance scale when comparing states-symptoms x_j and x_k, j, $k = 1,\dots,m$. The coordinates of this eigen vector that assists the largest in magnitude eigen value of B thus constitute weights $w_1,\dots,w_m$ assigned to symptoms x_1, $\dots,x_m$ stated in X. The weights are involved in the computations of estimates $RT_i = w_1 r_{i1} + \cdots + w_m r_{i,m}$ for each a_i. It can be proved that the formulas derived for calculations of RT_i satisfy the conditions of OWA operators [29–30].

4. Select a^*, such that $RT_{i^*} = \min\limits_{1\le i\le n} RT_i$.

Example 5

The matrix C remains equal to the matrix U from Ex. 2. We remind of its existence as the table

$$C = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.2 & 0.2 & 0.2 & 0.2 & 0.3 & 0.3 & 0.3 & 0.4 & 0.2 \\ 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 & 0.2 & 0.3 & 0.5 & 0.5* \\ 0.5 & 0.5 & 0.5* & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.4 \\ 0.7* & 0.6 & 0.5* & 0.5* & 0.4 & 0.4 & 0.4 & 0.5* & 0.5 & 0.6* & 0.5* \\ 0.7* & 0.7* & 0.5* & 0.5* & 0.4 & 0.5* & 0.5* & 0.5* & 0.6* & 0.6* & 0.5* \\ 0.7* & 0.4 & 0.4 & 0.4 & 0.5* & 0.5* & 0.5* & 0.5* & 0.4 & 0.4 & 0.5* \end{bmatrix} \end{array}$$

in which “*” points to the largest element in each column due to Step 1.

The regret matrix R is computed as the next table

$$R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 & 0.2 & 0.3 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.3 & 0.3 & 0.3 & 0.1 & 0 \\ 0.2 & 0.2 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0 & 0 & 0.1 & 0.1 & 0.1 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.1 & 0.1 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0 \end{bmatrix} \end{array}$$

For $w_1 = 0.78,\dots,w_{11} = 0.03$ (Ex. 4) the values of RT_i, $i = 1,\dots,6$, are appreciated as

$$RT_1 = 0.5\cdot 0.78 + 0.2\cdot 0.49 + 0.3\cdot 0.29 + 0.3\cdot 0.15 + 0.3\cdot 0.15 + 0.3\cdot 0.06 \\ + 0.2\cdot 0.06 + 0.2\cdot 0.06 + 0.3\cdot 0.06 + 0.2\cdot 0.06 + 0.3\cdot 0.03 = 0.725$$

$RT_2 = 0.444$, $RT_3 = 0.365$, $RT_4 = 0.082$, $RT_5 = 0.015$ and $RT_6 = 0.215$.

Due to Step 4 of Algorithm 1 we now decide the order of drugs with respect to their curative abilities. We state them in sequence $a_5 \succ a_4 \succ a_6 \succ a_3 \succ a_2 \succ a_1$ that almost confirms the results obtained by the technique of unequal objectives. Moreover, we notice that the last decision is very clearly interpretable and easy to understand without doubts. This emphasizes an advantage of applying the OWA weighted operations that prevent a loss of substantial information. The OWA operations have resulted in the simultaneous engagement of all effectiveness quantities in mean decision-making values involved in the regret model.

6 Conclusions

We have presented the adaptations of some fuzzy decision making models to the conditions attributed to the process of selecting the most efficacious medicine. The decision patterns should be particularly helpful in doubtful cases when we observe unequal remedial abilities of different medicines acting on the same symptoms.

As a primary method of fuzzy decision-making we have adjusted Bellman-Zadeh model to the process of extraction of the best medicine from the collection of proposed remedies. In the next methods of unequal objectives and minimization of regret we have employed the indices of the symptoms' importance to emphasize the essence of additional factors in the final decision.

Except from Bellman-Zadeh model, that provides us with the decision based on the strict minimum operator, we have produced the results absolutely acceptable from the medical point of view, which confirms the reliability of adaptation of tested decision cases to medical assumptions. After accomplishing of the close analysis of results we should admit that the utilization of mean values or OWA operators in numerical computations yields the most significant effects. All algorithms are based on simple calculations that allow testing large databases.

We assure that all medical adaptations constitute own original contributions in fuzzy decision-making. These have already been published one by one in several international sources [21–28]. We have decided to use these algorithms once again in the case of toxoplasmosis to help the physicians in the differentiation of medicine powers. There are no such remedies, which cure a patient totally of the disease consequences. Hence, we want to prescribe the most efficacious drugs to the patient to make him/her feel better, at least, partially.

Since the physicians claim divided prioritizations of toxoplasmosis medicines then the group decision making algorithm should be tested as a complement of a final decision of medicine hierarchies.

References

[1] Bellman R-E., Zadeh, L-A. (1970) Decision Making in a Fuzzy Environment. *Management Sci*., 17, 4, 141-164.

[2] Jain R. (1976) Decision Making in the Presence of Fuzzy Variables. *IEEE Trans. Syst. Man and Cybern.* SMC-6, 698-703.

[3] Jain R. (1977) A Procedure for Multi-aspect Decision Making Using Fuzzy Sets. *Int. J. Syst. Sci*. 8, 1-7.

[4] Yager R-R. (1977) Multiple Objective Decision-Making Using Fuzzy Sets. *Int. J. Man-Machine Studies*, 9, 375-382.

[5] Yager R-R. (1978) Fuzzy Decision Making Including Unequal Objectives. *Fuzzy Sets and Systems*, 1, 87-95.

[6] Yager R-R. (1999) Modeling Uncertainty Using Partial Information. *Inf. Sci.* 121, 271-294.

[7] Yager R-R. (2004) Decision Making Using Minimization of Regret. *Int. J. of Approximate Reasoning*, 36, 109-128.

[8] Yager R-R. (2004) Uncertainty Modeling and Decision Support. *Reliability Engineering and System Safety*, 85, 341-354.

[9] Chad-Ton Su, Chien-Hsin Yang, Kuang-Hung Hsu, Wen-Ko Chiu (2006) Data Mining for the Diagnosis of Type II Diabetes from Three-dimensional Body Surface Anthropometrical Scanning Data. *Computers & Mathematics with Applications*, 51, 6-7, 1075-1092.

[10] Ji Y., Massanari R-M., Ager J., Yen J., Miller R., Ying H. (2007) A Fuzzy Logic Based on Computational Recognition-primed Decision Model. *Inf. Sci.*, 177, 4338-4353.

[11] O'Hagan A., Stevens J-W. (2002) Bayesian Methods for Design and Analysis of Cost-effectiveness Trials in the Evaluation of Health Care Technologies. *Statistical Methods in Medical Research*, 11, 469-490.

[12] Zadeh L-A. (1975) The Concept of a Linguistic Variable and its Application to Approximate Reasoning. *Inf. Sci*. 8, 199-249.

[13] Zadeh, L-A. (1975) *Calculus of Fuzzy Restrictions. Fuzzy Sets and Their Applications to Cognitive and Decision Processes.* Academic Press, London.

[14] Zadeh L-A. (1986) Test-score Semantics as a Basis for a Computational Approach to the Representation of Meaning. *Literary and Linguistic Computing*, 1, 24-35.

[15] Zadeh L-A. (1999) From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions. *IEEE Transactions on Circuits and Systems*, 45, 105-119.

[16] Mesiar R., Calvo T., Yager R-R. (2004) Quantitative Weights and Aggregation. *IEEE Transactions on Fuzzy Systems*, 12, 62-69.

[17] Kumar K-J. (2006) Fuzzy Logic and Modern Analytical Tool Coupling in Comprehensive Evaluation of Traditional and Complementary Medicine. *Pharmacognosy Magazine*, 2, 202-203.

[18] Nieto J-J., Torres A. (2003) Midpoints for Fuzzy Sets and Their Application in Medicine. *Artificial Intelligence in Medicine*, 27, 81-101.

[19] Chen Shi-quan (1988) Fuzzy Classification of Acute Toxicity of Poisons. *Fuzzy Sets and Systems*, 27, 3, 255-260.

[20] Gorzałczany M-B., Piasta Z. (1999) Neuro-fuzzy Approach Versus Rough-set Inspired Methodology for Intelligent Decision Support. *Inf. Sci.*, 120, 1-4, 45-68.

[21] Rakus-Andersson E. (1999) A Fuzzy Decision Making Model Applied to the Choice of the Therapy in the Case of Symptoms not Disappearing after the Treatment. In: *Proceedings of EUROFUSE-SIC'99*, Eds: deBaets B., Fodor J., Kóczy L.-T., Budapest, 1999, 298-303.

[22] Rakus-Andersson E., Gerstenkorn T. (1999) A Comparison of Different Fuzzy Decision Making Models Supporting the Optimal Therapy. *Studies in Fuzziness and Soft Computing Series,* Eds: Szczepaniak P., Lisboa P-J., Tsumoto S., Springer-verlag, Berlin Heidelberg New York, 561-572.

[23] Rakus-Andersson, E. (1999) A Fuzzy Group-Decision Making Model Applied to the Choice of the Optimal Medicine in the Case of Symptoms not Disappearing after the Treatment. *The International Journal of Computing Anticipatory Systems*, Ed. Dubois D., Chaos, Liège, 141-152.

[24] Rakus-Andersson E. (2005) A Choice of Optimal Medicines by Fuzzy Decision Making Including Unequal Objectives. *Issues in the Representation and Processing of Uncertain and Imprecise Information,* EXIT – The publishing House of The Polish Academy of Sciences, 307-321.

[25] Rakus-Andersson E. (2006) Minimization of Regret versus Unequal Multi-objective Fuzzy Decision Process in a Choice of Optimal Medicines. In: *Proceedings of the XIth International Conference IPMU 2006 – Information Processing and Management of Uncertainty in Knowledge-based Systems*, 2, Edition EDK, Paris, 1181-1189.

[26] Rakus-Andersson E., Jogréus C. (2007) The Choquet and Sugeno Integrals as Measures of Total Effectiveness of Medicines. In: *Theoretical Advances and Applications of Fuzzy Logic and Soft Computing* (*Proceed-*

ings of IFSA 2007, Cancun, Mexico), Eds: Castillo O., Melin P., Ross O-M., Cruz R-S., Pedrycz W., Kacprzyk J., Springer-verlag, Berlin Heidelberg, 253-262.

[27] Rakus-Andersson E. (2007) *The Fuzzy and Rough Techniques in Medical Diagnosis and Medication*. Springer-verlag, Berlin Heidelberg.

[28] Rakus-Andersson E. (2008) Decision-making Techniques in Ranking of Medicine Effectiveness. *Advanced Computational Intelligence Paradigms in Healthcare 3*, Eds: Sordo M., Vaidya S., Jain L-C., Springer-verlag, Berlin Heidelberg, 51-73.

[29] Yager R-R., Kacprzyk J. (1997) *The Ordered Weighted Averaging Operators: Theory and Applications*. Kluwer, Norwell, MA.

[30] Yager R-R. (2004) Generalized OWA Aggregation Operators. *Fuzzy Optimization and Decision Making*, 3, 93-107.

[31] Adlassnig K-P. (1980) A Fuzzy Logical Model of Computer-assisted Medical Diagnosis. *Methods of Information in Medicine*, 19, 3, 141-148.

[32] Adlassnig K-P. (1986) Fuzzy Modeling and Reasoning in a Medical Diagnostic Expert System. *EVD in Medizin und Biologie*, 17, 12-20.

[33] Zadeh L-A. (1965) Fuzzy sets. *Inf. Control*, 8, 338-353.

[34] Kadłubowski R., Kurnatowska A. (1999) The Outline of Medical Parasitology. Publishing Medical House, 167-176 (in Polish).

[35] Kocięcka W., Mrozewicz B., Simon E., Pakuła M. (1990) Clinical Criteria and Evaluation of Lymphonodular Toxoplasmosis. *Parasitological News*, 36, 99-119 (in Polish).

[36] Liberski P-P., Kurnatowska A., Mossakowski M-J., (2004) Parasitological Diseases of the Neurosystem. *Diseases of the Neurosystem*, (Eds:) Kozubski W., Liberski P-P., Publishing Medical House PZWL, Warsaw, 76-87 (in Polish).

[37] Liberski P-P., Sikorska B., Kurnatowska A. (2005) Parasitological Diseases. The Mossakowski Neuropathology, (Eds:) Liberski P-P., Papierz W., The Czelej Publishing House, Lublin-Poland, 375-416 (in Polish).

[38] Saaty T-L. (1978) Exploring the Interface between Hierarchies, Multiplied Objectives and Fuzzy Sets. *Fuzzy Sets and Systems* 1, 57-68.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2011

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475411