

# **New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications**

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**Systems Research Institute  
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Dedicated to Professor Beloslav Riečan on his 75th anniversary

# On intuitionistic fuzzy approach to generalized net prognostics

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## Abstract

A procedure for the aggregation of statistical data derived from the simulation of a GN model has been described. The method for utilization of that information has been described in details for the case when the same GN model is run for prognostic purposes. The concept of intuitionistic fuzziness is used to construct evaluations of the tokens’ transfer during the simulation runs, which are then interpreted as probabilities for tokens’ transfer during the prognostic ones.

**Keywords:** generalized nets, intuitionistic fuzziness, modelling, prognostics, simulation.

## 1 Introduction

The present paper introduces a new way of performing prognostics of processes, modelled by generalized nets, using intuitionistic fuzzy evaluations of the tokens’ records of transfer via the net transitions. This approach enables taking future decisions with respect to the plausibility of tokens’ movement inside the net, on the basis of a certain number of model simulations in the past.

## 2 Description of the extended abstract GN model

Let us have an *abstract model* of a real process constructed with the apparatus of generalized nets (GNs, see [1]). The model consists of  $n$  transitions,  $T_1, \dots, T_n$  and a set of places, both input and output, collectively denoted by  $p_{i,j}$  and  $q_{i,k}$

where  $i \in \{1, \dots, n\}, j \in \{1, \dots, f_i\}, k \in \{1, \dots, g_i\}$  where  $f_i$  and  $g_i$  are, respectively, the maximal number of input and output places of the  $i$ -th transition. In Figure 1, the abstract GN model is schematically illustrated in the grey zone, and further extended with additional infrastructural elements that reflect the novelty of the presently proposed approach.

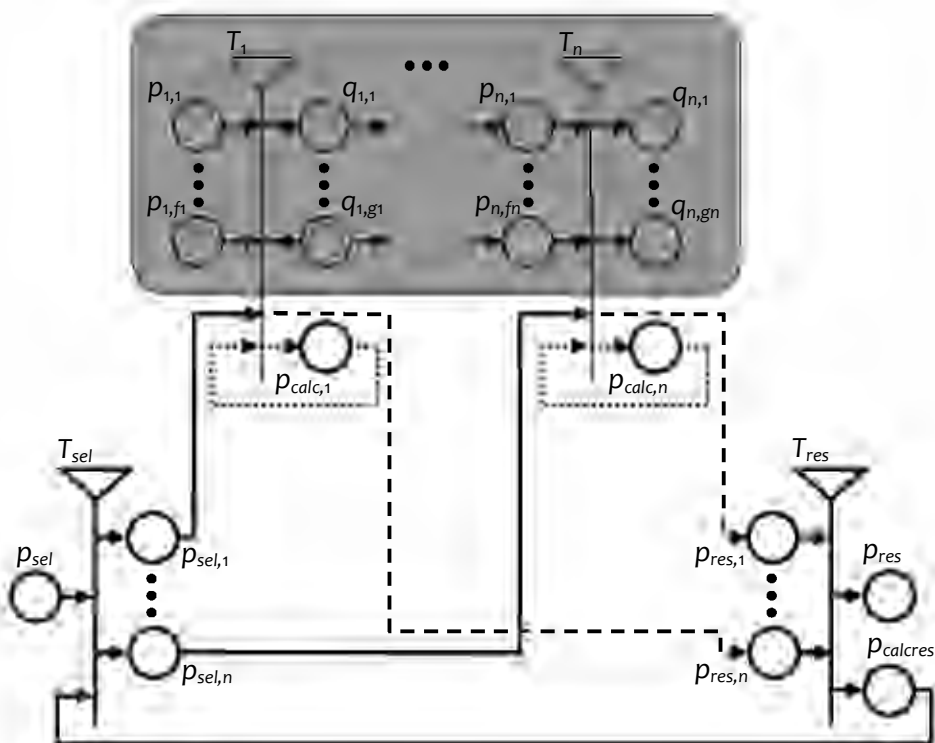


Figure 1: Proposed extension of an abstract GN model

These extra infrastructural elements are:

- two new transitions  $T_{sel}$  and  $T_{res}$ ,
- $3n+3$  new places:
  - $p_{sel}$  (input for  $T_{sel}$  and for the whole new GN),
  - $p_{sel,1}, \dots, p_{sel,n}$ ,
  - $p_{calc,1}, \dots, p_{calc,n}$  that are  $n$  places for interim calculations attached to each of the  $n$  transitions of the original GN of the modelled process,
  - $p_{res,1}, \dots, p_{res,n}$ ,
  - $p_{calres}$ ,
  - $p_{res}$  (output for  $T_{res}$  and for the whole new GN).



Before we proceed with the dynamics of the newly constructed GN, let us comment on one important aspect of its transitions and places: their priorities.

The first consideration to be made is that the two new transitions  $T_{sel}$  and  $T_{res}$  have, respectively, the highest and the lowest priority among all. Within the set of the transitions  $T_i$  ( $i \in \{1, \dots, n\}$ ) different priorities may hold, as determined by the particular modelled process, but they are not a subject of discussion.

The second consideration is that in the whole net we have four levels of places' priority:

- In the first level, the places with the highest priority are  $p_{sel}, p_{sel,1}, \dots, p_{sel,n}$  and  $p_{calcres}$ , i.e. the input and output places of the transition with the highest priority  $T_{sel}$ .
- The second level contains the totality of places from the standard GN that describe the real process, namely places  $p_{i,j}$  and  $q_{i,k}$ , where  $i \in \{1, \dots, n\}$  is the number of transitions and  $j \in \{1, \dots, fi\}, k \in \{1, \dots, gi\}$ . In Figure 1, these are illustrated in the grey zone. Within the set of the places from the second group, places may have different priorities as determined by the particular process that is being modelled, but their priorities are generally lower than those on the first level and generally higher than those on the third level.
- The third group of places consists of the places  $p_{calc,i}, i \in \{1, \dots, n\}$  that are attached as output/input places of  $T_i$ .
- The last group of places that exhibit the lowest priority in the new net are  $p_{res,1}, \dots, p_{res,n}$  and  $p_{res}$ , i.e. the input and output places of the transition with the lowest priority  $T_{res}$ .

The so described additional elements of the GN static nature form a new contour for the dynamic elements (tokens) that carry meta information about the original modelled process. Let us denote these tokens by  $\alpha, \beta$  and  $\gamma_1, \dots, \gamma_n$ .

An  $\alpha$ -token enters the global input place of the new GN  $p_{sel}$  with the characteristic current user's selection (hence, the index  $sel$ ) of what his/her intention is when running the model: whether he/she runs it for simulation purposes, or for prognostic ones. Therefore, the  $\alpha$ -token enters the place with exactly one of the two following characteristics:

*“simulation” or “prognostics”.*

This first choice is crucial for the execution of the original model, because it determines:

- whether information about the tokens' transfer is only going to be retrieved, statistically processed and stored in the additional places  $p_{calc,1}, \dots, p_{calc,n}, p_{calcres}$  if the user chooses to perform simulation,
- or collected information from previous simulations is actually going to be used (without changes) to run the model with prognostic purposes.

So, the different behaviour of the new GN, as stipulated by the choice made in the first input place, dictates that we consider these two scenarios separately. Before that, however, let us give some necessary working definitions.

## 2.1 Working definitions

Let us remind that the index matrix (see [3]) of predicates of a given transition contains information of the possible routes of tokens' transfer from the input to the output places of the transition. The cells of the index matrix may contain either the unconditional explicit values "true" and "false", or some predicates that are subject to calculation and may result in either "true" or "false" according to certain conditions. When a given cell contains the value "true", this means that any token in the respective input place of the transition can transfer to an output place, while if the value is "false", it may simply be considered that there is no arc, linking the input and the output place.

We will say that a token located in an input place  $l_i$  is *eligible* to transfer to the transition's output place  $l_j$  whenever in the  $(i, j)$ -th cell of the transition's index matrix of predicates the value is either "true" (unconditional transfer) or some predicate  $W_{i,j}$ , whose truth value is currently calculated as "true" (conditional transfer).

Whenever a token is eligible to transfer, it will transfer given enough time on the current time-step. This extra time dependence means that some tokens, which are eligible to transfer, may not successfully accomplish the procedure on the current time-moment, i.e. they will remain in the input place at least by the time of the next activation of the transition.

The same token will be *ineligible* to transfer to a given output place when the calculated truth value of the predicate linking its input to that output place is (conditionally) "false", i.e. current conditions do not allow the token's transfer, however such transfer may still be permitted under other circumstances. Thus, reflecting the logic behind the model, we will fully ignore the case of the unconditional ineligibility to transfer (i.e. explicit value "false").

As it can be directly seen, the set of "successful" tokens (let us denote its cardinality by  $\mu$ ) and the set of "unsuccessful" tokens (respectively, cardinality  $\nu$ ) are both subsets of the set of all tokens in the input places of a given transition, let denote its cardinality by  $\tau$ . What may not be so easily seen, however, is that both sets may *not* complement up to  $\tau$ . It is possible that some tokens in the input places are eligible for transfer to output places (due to the truth of the calculated predicates in the transition's predicate index matrix), but they are not able to do so due to the limited duration of the transition's active state. Hence,

$$0 \leq \frac{\tau - \mu - \nu}{\tau} \leq 1.$$

This situation bears a resemblance with the case of *intuitionistic fuzzy* (see [2]) degrees of membership ( $\mu/\tau$ ), non-membership ( $\nu/\tau$ ) and uncertainty (their complement to 1).

One more observation can be made about the *time-scale* and *elementary time-step* of the newly constructed GN. Both are practically identical to those of the originally modelled process, but the *duration* of the new GN's functioning will be exactly one (elementary) time-step longer than the duration of the original net. This observation follows from the fact that once the necessary meta information about the transitions of the simulated process is stored in the tokens in places  $p_{res,1}, \dots, p_{res,n}$ , it needs to be finally aggregated in place  $p_{calres}$  (tokens  $\gamma_i$  merge back to one token  $\gamma$ ) and stored for further simulations or prognostics.

Now, after we have given these working definitions, let us continue with the description of the model. First, we will describe it in the scenario when the model is run for simulation purposes, and second, when we run it for the purpose of prognostics. As a rule of the thumb, multiple simulation runs need to be performed before a prognostic run that claims for plausibility.

## 2.2 Scenario 1: simulation

When the user chooses to run the model for simulation purposes, information is retrieved about the behaviour of the originally modelled process. We take into account several variables related to the tokens circulating in the original model.

The first variable is the total number of tokens (let us refer to them with  $\tau$ ) that stay in the input places of each transition. It is noteworthy that during the GN's overall functioning a transition may get activated more than once, therefore, each transition's activation by the end of the net's functioning shall be taken into account.

Another variable is the number of tokens that have successfully managed to transfer to output places during the active state of the respective transition. Let us call these  $\mu$  tokens. Not only have they been eligible to transfer, but also time has been enough for them to accomplish it. The precise route of these token transfers are not being considered, being a part of the original model's internal logic.

We also maintain a variable for the number of tokens that have been ineligible to transfer, because the calculation of the predicate in the respective cell of the transition's predicate index matrix has resulted in "false". We choose to denote them by  $\nu$ .

The numbers of  $\tau$ ,  $\mu$  and  $\nu$  tokens are:

- counted in a step-wise manner in places  $p_{calc,i}$  for each transition  $T_i$  ( $i \in \{1, \dots, n\}$ ) of the originally modelled process, for each activation of the transitions in the course of a given simulation, and
- collected in place  $p_{calcres}$  at the end of each simulation. As long as the model is run for simulation purposes information will be collected from the original GN and stored in  $p_{calcres}$ .

More formally, the form of the first transition  $T_{sel}$  is:

$$T_{sel} = \langle \{p_{sel}, Pres\}, \{p_{sel_1}, \dots, p_{sel_n}\}, IM_{sel} \rangle$$

where:

$$IM_{sel} = \begin{array}{c|ccc} & p_{sel,1} & \dots & p_{sel,n} \\ \hline p_{sel} & true & \dots & true \\ p_{calcres} & true & \dots & true \end{array}$$

This means that at the beginning of the simulation, the  $\alpha$ -token in place  $p_{sel}$  splits into  $n$  identical copies that unconditionally transfer to all places  $p_{sel,1}, \dots, p_{sel,n}$  of transition  $T_{sel}$ , which represent the  $n$  additional input places of the  $n$  transitions of the modelled process (this unconditional transfer being coded in the index matrix of predicates for  $T_{sel}$  as the value “true”).

Simultaneously, in place  $p_{calcres}$ , which is the second input place of  $T_{sel}$ , a splittable  $\beta$ -token stays with the following characteristic:

$$x^\beta = "R, VAL_1, VAL_1^*, VAL_2, VAL_2^*, \dots, VAL_n, VAL_n^*"$$

where”

- $R$  is the subsequent number of the current running of the model.
- $VAL_1, \dots, VAL_n$  are  $n$  arrays of stored values for the current running of the model, which are in the form:

$$VAL_i = " < R, A_{R,i}, \sum_{s=1}^{A_{R,i}} \tau_{R,i,s}, \sum_{s=1}^{A_{R,i}} \mu_{R,i,s}, \sum_{s=1}^{A_{R,i}} \nu_{R,i,s} > "$$

where  $i \in \{1, \dots, n\}$  is the subsequent number of the transition and  $A_{R,i}$  is the total number of activations of transition  $T_i$  during the  $R$ -th running of the GN model. The third parameter, namely:

$$\sum_{s=1}^{A_{R,i}} \tau_{R,i,s}$$

corresponds to the total number  $\tau$  of tokens in all input places of  $T_i$ , accumulated during all the  $A_{R,i}$  activations of the transition throughout the  $R$ -th

running of the model. Similar is the meaning of the fourth and fifth parameters, where  $\mu$  corresponds to those of the tokens that were eligible and managed to successfully transfer from an input to an output place during all the  $A_{R,i}$  activations of the transition throughout the  $R$ -th running of the model. Finally,  $\nu$  corresponds to the number of tokens in the input place were ineligible to transfer to an output place.

- $VAL_1^*, \dots, VAL_n^*$  are  $n$  arrays that cumulatively store the sums of the respective values of the  $\tau$ -,  $\mu$ - and  $\nu$ - tokens during all previous runnings of the model, in the form:

$$VAL_i^* = \left\langle R, \sum_{s=1}^R A_{i,s}, \sum_{s=1}^R \tau_{i,s}, \sum_{s=1}^R \mu_{i,s}, \sum_{s=1}^R \nu_{i,s} \right\rangle,$$

$R$  here being the total number of runnings (simulations) done so far, and the rest values being the sums of the respective values per running. Initially, when  $R = 0$ ,

$$VAL_i = \langle 0, 0, 0, 0, 0 \rangle$$

and

$$VAL_i^* = \langle 0, 0, 0, 0, 0 \rangle.$$

Later, when  $R$  is a natural number larger than 0,

$$VAL_i = \langle R, 0, 0, 0, 0 \rangle$$

and the final characteristic of token  $\beta$  is:

$$x^\beta = \langle R, \langle R, 0, 0, 0, 0 \rangle, VAL_1^*, \langle R, 0, 0, 0, 0 \rangle, VAL_2^*, \dots, \langle R, 0, 0, 0, 0 \rangle, VAL_n^* \rangle.$$

As already mentioned, both tokens  $\alpha$  and  $\beta$  split to  $n$  tokens ( $\alpha_1, \dots, \alpha_n$  and  $\beta_1, \dots, \beta_n$ ) during their transfer from the respective input places of transition  $T_{sel}$  to the output places  $p_{sel,1}, \dots, p_{sel,n}$ . Tokens  $\alpha_i$  have identical characteristics as token  $\alpha$ , namely “simulation”. Tokens  $\beta_i$  have the characteristic “ $VAL_i, VAL_i^*$ ” which are the respective parts of the characteristic of token  $\beta$ . In the output places, each  $\alpha_i$  merges with the respective  $\beta_i$  token into  $n$  new tokens:  $\gamma_i$ . The initial characteristics of  $\gamma_i$  is:

$$\langle x^\alpha, x^{\beta_i} \rangle$$

where:

- $x^\alpha$  is the characteristic of token  $\alpha$  (“simulation”), and

- $x^{\beta_i}$  is the characteristic of token  $\beta_i$ , namely “ $VAL_i, VAL_i^*$ ”. As mentioned, in the initial moment these arrays consist only of zeros.

So, the  $n$   $\gamma$  tokens are unconditionally transferred to the  $n$  input places  $p_{sel,1}, p_{sel,2}, \dots, p_{sel,n}$  of the  $n$  transitions which comprise the original modelled process. Let us describe formally only the  $i$ -th of these  $n$  transitions after the modifications, introduced by the present novel approach. For the sake of easy comprehension, the segment of the transition’s index matrix of predicates that corresponds to the original model is also coloured in grey, as in Figure 1.

$$T_i \Leftarrow \{p_{i,1}, \dots, p_{i,fi}, p_{sel,i}, p_{calc,i}\}, \{q_{i,1}, \dots, q_{i,gi}, p_{calc,i}, \dots, p_{res,i}\}, IM_i >$$

where:

$$IM_i = \begin{array}{c|ccc|cc} & q_{i,1} & \dots & q_{i,gi} & p_{calc,i} & p_{res,i} \\ \hline p_{i,1} & & & & false & false \\ \vdots & \vdots & W_{i,j,k} & \vdots & \vdots & \vdots \\ p_{i,fi} & & & & false & false \\ \hline p_{sel,i} & false & \dots & false & true & false \\ p_{calc,i} & false & \dots & false & W_{calc,i} & \neg W_{calc,i} \end{array},$$

where:

- $W_{i,j,k}$  are the respective predicates of the original modelled process (whose form is not to be discussed in the present research).
- $W_{calc,i}$  = “the time of functioning of the original GN has not yet finished”.
- $\neg W_{calc,i}$  is the negation of predicate  $W_{calc,i}$ .

The  $\gamma_i$  token that stays in place  $p_{sel,i}$  unconditionally enters place  $p_{calc,i}$  and is ineligible to direct transfer to place  $p_{res,i}$ . It may only reach place  $p_{res,i}$  if it has looped for at least one iteration through place  $p_{calc,i}$  (the place for interim calculation of the values of the arrays “ $VAL_i, VAL_i^*$ ” that reflect the tokens’ transfer during the simulation). Hence, as predicate  $W_{calc,i}$  stipulates, the token will be looping in place  $p_{calc,i}$  as long as the original generalized net model is functioning, thus recording all the information of the tokens’ transfer during all the possible activations of transition  $T_i$ . As soon as the original model stops functioning, the  $\gamma_i$  token that has been looping in place  $p_{calc,i}$  recording the data of the “successful”, “unsuccessful” tokens’ transfers through transition  $T_i$ , moves to place  $p_{res,i}$  with the characteristic “ $VAL_i, VAL_i^*$ ” where the respective values of the arrays represent the accumulation of this information across all transition’s firings throughout the whole simulation of the original GN. This is possible, since the priority of places  $p_{calc,i}$  ( $i \in \{1, \dots, n\}$ ) is lower than the priorities of the places  $p_{i,1}, \dots, p_{i,fi}, q_{i,1}, \dots, q_{i,gi}$ .

Now, let us discuss the continuation of the tokens  $\gamma_i$  in the second newly added transition  $T_{res}$ . It is noteworthy that due to the lowest priority of this transition, it only becomes active when the overall functioning of the original GN has completely finished, i.e. when the simulation is over. Its formal definition is the following:

$$T_{res} = \langle \{p_{res,1}, \dots, p_{res,n}\}, \{p_{res}, p_{calcres}\}, IM_{res} \rangle$$

where:

$$IM_{res} = \frac{\quad}{\begin{array}{c} p_{res,1} \\ \dots \\ p_{res,n} \end{array}} \left| \begin{array}{cc} p_{res} & p_{calcres} \\ W_{sel} & true \\ \dots & \dots \\ W_{sel} & true \end{array} \right.$$

where  $W_{sel} = "x^\alpha \text{ is prognostics}"$ . Therefore, during the first scenario, this predicate has truth value "false".

The tokens  $\gamma_i$  from the input places  $p_{res,i}$  unconditionally transfer to the output place  $p_{calcres}$ , where they merge into one token,  $\beta$ , with the characteristic:

$$x^\beta = \overline{\overline{R}}, \overline{\overline{VAL_1}}, \overline{\overline{VAL_1^*}}, \overline{\overline{VAL_2}}, \overline{\overline{VAL_2^*}}, \dots, \overline{\overline{VAL_n}}, \overline{\overline{VAL_n^*}}$$

where:

- $\overline{\overline{R}} = R + 1$ ,
- $\overline{\overline{VAL_i}} = \langle \overline{\overline{R}}, 0, 0, 0, 0 \rangle$ ,
- $\overline{\overline{VAL_i^*}} = \overline{\overline{VAL_i^*}} + \overline{\overline{VAL_i}}$ .

Thus, the final data about the tokens' transfers from the latest simulation of the model is not kept but is aggregated with the data obtained from all previous runnings of the model.

In the first scenario, when the purpose of the running is "simulation" (as coded in the characteristic of the initial  $\alpha$  token), no tokens  $\gamma_i$  transfer to place  $p_{res}$  of transition  $T_{res}$ . Reaching this output place is only possible when the purpose of the model running has been "prognostics". So, this is the second scenario that we are going to discuss now.

### 2.3 Scenario 2: prognostics

During the second scenario, the functioning of the first transition  $T_{sel}$  is identical to the one in the first scenario.

The major difference between the two scenarios is in the transitions  $T_i$  ( $i \in \{1, \dots, n\}$ ). Before we describe them formally, let us denote by  $r_{i,j,k}$  a random variable in the  $[0;1]$  interval that is related to transition  $T_i$ . This variable, corresponding to the  $j$ -th input place and  $k$ -th output place of the  $i$ -th transition,

will be used to determine the type of the new predicates  $V_{i,j,k}$  that will replace the original predicates as seen in the simulation scenario.

Depending on the goal of the prognostic run, there are two different variants for the development of the processes in the originally modelled GN. They can be conventionally called “fuzzy” and “intuitionistic fuzzy” cases. In both cases the formal representation of the transition is as follows:

$$T_i \Leftarrow \{p_{i,1}, \dots, p_{i,fi}, p_{sel,i}, p_{calc,i}\}, \{q_{i,1}, \dots, q_{i,gi}, p_{calc,i}, \dots, p_{res,i}\}, IM_i >$$

where:

$$IM_i = \begin{array}{c|ccc|cc} & q_{i,1} & \dots & q_{i,gi} & p_{calc,i} & p_{res,i} \\ \hline p_{i,1} & & & & false & false \\ \vdots & \vdots & V_{i,j,k} & \vdots & \vdots & \vdots \\ p_{i,fi} & & & & false & false \\ \hline p_{sel,i} & false & \dots & false & true & false \\ p_{calc,i} & false & \dots & false & W_{calc,i} & \neg W_{calc,i} \end{array},$$

where:

- $W_{calc,i}$  = “the time of functioning of the original GN has not yet finished”.
- $V_{i,j,k}$  are new predicates replacing those of the originally modelled process.
  - *Fuzzy case*:

$$V_{i,j,k} = "r_{i,j,k} \in \left[ 0; \frac{\sum_{s=1}^R \mu_{i,s}}{R} \right] ",$$

(the used notations are as described above). In this version, the duration of the functioning of  $T_i$  coincides with the duration of the functioning of  $T_i$  of the original net. This means that during the prognostic run there may transfer the same number of tokens as in the original net.

- *Intuitionistic fuzzy case*: In this case the  $T_i$  transition will be active for as many time-steps as the total number of the tokens in its input places. The predicate  $V_{i,j,k}$  will obtain the form of the following rule:



$$\left\{ \begin{array}{l}
\text{if } r_{i,j,k} \in \left[ 0; \frac{\sum_{s=1}^R \mu_{i,s}}{\sum_{s=1}^R \tau_{i,s}} \right] \\
\text{if } r_{i,j,k} \in \left( \frac{\sum_{s=1}^R \mu_{i,s}}{\sum_{s=1}^R \tau_{i,s}}; \frac{\sum_{s=1}^R \mu_{i,s} + \sum_{s=1}^R V_{i,s}}{\sum_{s=1}^R \tau_{i,s}} \right) \\
\text{if } r_{i,j,k} \in \left( \frac{\sum_{s=1}^R \mu_{i,s} + \sum_{s=1}^R V_{i,s}}{\sum_{s=1}^R \tau_{i,s}}; 1 \right)
\end{array} \right.
\begin{array}{l}
\text{then } V_{i,j,k} = \text{"true"} \text{ and the token} \\
\text{transfersto an output (truth of predicate)} \\
\\
\text{then } V_{i,j,k} = \text{"false"} \text{ and the token} \\
\text{does not transfer (falsity of predicate)} \\
\\
\text{then } V_{i,j,k} = \text{"false"} \text{ and the token} \\
\text{does not transfer (limited time).}
\end{array}$$

By means of the rule we can imitate the real-life functioning of the Algorithm for tokens' transfer in a transition [1, 4, 5].

Finally, the third transition  $T_{res}$  keeps its form, as described in the first scenario, but now the predicate  $W_{sel} = "x^\alpha \text{ is prognostics}"$  is evaluated as "true". This changes the behaviour and characteristics of the  $\gamma_i$  tokens. Each of them splits to two identical tokens  $\gamma'_i$  and  $\gamma''_i$ .

The  $\gamma'_i$  tokens with characteristics " $VAL_i, VAL_i^*$ " transfer to place  $p_{res}$  and merge there into token  $\delta$  with the characteristic:

$$"VAL_1^*, VAL_2^*, \dots, VAL_n^*"$$

which allows the user to trace back and understand the conditions that determined the development of the prognostic run of the model.

The  $\gamma''_i$  tokens, again with characteristics " $VAL_i, VAL_i^*$ ", transfer to place  $p_{caleres}$  and merge there into token  $\beta$  without obtaining new characteristic. Thus, in future runnings of the model, no matter whether for simulation or for prognostic purposes, this token  $\beta$  can be used again as an input of the top priority transition  $T_{sel}$ .

### 3 Conclusion

The so constructed model describes the process of collection and aggregation of statistical data about tokens' transfer from the simulations of an arbitrary GN model. Using these data, together with their intuitionistic fuzzy evaluations, we can utilize this construction for prognostic purposes.

The ideas presented here suggest the construction of an even more detailed approach towards the collection of the statistical data, when information is ga-

thered not on the level of transitions, but on the level of input/output places of a transition. This further detailization will be an object of future research.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

[Http://www.ibspan.waw.pl/ifs2011](http://www.ibspan.waw.pl/ifs2011)

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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