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ON THE ENTRY PROBLEM.

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ABSTRACT. The problem of the entry of a new candidate (alternative) into a race is considered in order to find the necessary condition guaranteeing victory for the new one. The considerations are made under differing assumptions about the dimensionality of space, number of candidates and type of voting.

Key Words: entry problem, voting schemas.

INTRODUCTION.

One of the most important problems in applied political science is connected with the entry of a new candidate into a political race. On solving this problem one expects answers to, among others, the following questions:

- what chances has any new enterer for winning,
- what positions among other candidates should s/he take in order to maximize his or her chances of winning.

It is obvious that answers to the above questions depend substantially on: (1) the voting system used in a given situation (election), (2) the way of evaluating the candidates' positions. The first dependence has an objective character (it is given from outside), the second one is subjective in nature; one may use one or more ideological dimensions (attribute scales) in a modeland the choosen distance metrics may differ for different problems. The situation may also be more complicated bacause usually the number of candidates is considerably large.

In this paper we analyse the entry problem under plurality voting (because it is the most popular way of voting) and under approval voting (probably the best way of group decision making - Nurmi(1983)).

In literature one may find the results of investigations upon the chances of the new enterer in the following situations:

(1) n=2+1 candidates and plurality voting,

(2) n > 3 candidates and plurality voting,

(3) n=2+1 candidates and approval voting.

The situations (1) and (3) are considered under the assumption that candidates are distributed along a single attribute scale. Situation (2) is also considered in more than one ideological dimension.

Some considerations about the entry problem one may also find in the papers of Holler (1984, 1988a, 1988b) where the language and methodology of game theory were used.

In our paper we are going to present the "state of art" of the entry problem.

THE CASE OF n = 2 + 1 CANDIDATES.

Brams and Straffin (1982) have considered the new candidate's entry problem under the following assumptions:

1.1 There is a single left-right ideological dimension along which candidates take positions.

1.2 Each voter has a most-preferred position on this dimension.

1.3 Each voter has one vote and always casts it for the candidate whose position is closest to his or her most-preferred position (the used voting is plurality non-transferable).

1.4 The candidate with the most votes wins.

In the search for a new enterer's chances to win Brams and Straffin found so called "2/3 separation opportunity" for the median candidate, given the symmetry and unimodality assumptions. The following theorem contains this result:

Theorem 1 (Brams, Straffin).

If x=L and x=C are the positions of the candidates, and f(x)>0 is a continuous, unimodal density function of electorate distribution on ideological dimension [a,b], symmetric about its median M, and

$$\int_L^M f(x)dx = \int_M^C f(x)dx \geq 1/3$$

then a third candidate X can win at his or her optimal position $X^{*}=M$.

The assumption, given in the above theorem, that electorate is distributed unimodally and symmetrically around its median on the dimension [a,b] (or equivalently on [0,1]) is rather unrealistic. Taking this assumption apart we use consequently in all our futher considerations the quantity of votes, which can be obtained by a given candidate. This also means that we will use the electorate distribution function only indirectly.

Let us consider the new candidate's chance when we change plurality voting into approval voting. Consequently, we substitute assumption 1.3 by assumption 1.3.1.

1.3.1 Each voter may cast from 0 to n votes (n is the number of candidates) giving one vote to one candidate. The voting is sincere and votes are given according to the distance from his or her position along the dimension (approval voting).

We assume that the number of votes given by any voter is uniformly choosen from the set {0,1,...,n}.

Theorem 2. If X is winning under single plurality voting then X is also expected to be a winner under approval voting.

The proof one may find in Mercik (1988b).

Let $b_i^{(n)}$ denote the number of votes received by the i-th candidate (i=1,2,...,n) when there are n candidates in a race. For the sake of simplicity we assume that $\sum_{i=1}^{n} b_i^{(n)} = 1$.

Let $c_{n+1}^{(j)}$ denote a part of the votes which candidate (n+1)-st gets from the j-th candidate.

From Theorem 2 one may also obtain

$$c_3^{(L)}+c_3^{(C)}>1/2;$$

this is so called "1/2 separation opportunity", i.e. (in the sense of number of votes) the minimal distance (between L and C) giving X the opportunity to be the expected winner under approval voting and is equal to 1/2.

We see that Brams' and Straffin's "2/3 separation opportunity" for plurality voting has turned into "1/2 separation opportunity" for approval voting.

In the above theorem we talk only about the expected winner because no one can say what a voter's behaviour will be, i.e. how many votes will any voter decide to use in specific voting. We use the assumption about uniform probability, however there are some practical investigations (see: Brams 1988, Brams, Fishburn 1987, Fishburn, Little 1988, Mercik 1988c) telling us that this assumption is, in general, probably not true.

If we reject the assumption about uniform distribution over the set $\{1,2,...,n\}$ we may also expect that the centrist candidate X: $L < X < C$ will win under approval voting. This is a consequence of the central tendency of approval voting in one dimension modelling (Merçik 1986,1988d).

THE CASE OF $n \geq 3$ CANDIDATES.

We still analyse one dimensional model. Assumptions 1.1-1.4 stand but there are more than three candidates in the challenge. Definition. For a given (n+1)-st new enterer, the *i-th candidate* is called *neighbour to the j-th candidate* if this new enterer takes off some part of the i-th and j-th candidates' votes by the way of taking a fixed position on the dimension.

Let A^j denote the set of the j-th candidate's neighbours for the fixed position of the (n+1)-st candidate.

Let B^j denote the set of voters' positions which are not the actual winner ($\{j\}$) or the j-th candidate's neighbours.

Hence $N = \{ 1,2,...,n \} = \{ j \} \cup A^j \cup B^j$.

It is obvious that card$(A^j) \geq 0$ and card$(B^j) \geq 0$ but they may equal zero only simultaneously.

In the sense of the above, the number of votes of the new (n+1)-st enterer is:

$$b_{n+1}^{(n+1)} = c_{n+1}^{(j)} + \sum_{i \in A^j} c_{n+1}^{(i)} + \sum_{i \in B^j} c_{n+1}^{(i)} \qquad (1)$$

We distinguish candidate j as the up-to-now winner. It is obvious that in the first turn the new enterer should beat the current winner (this is necessary condition), afterwards, s/he should beat every other candidate from A^j and B^j, i.e.

$$b_{n+1}^{(n+1)} + c_{n+1}^{(j)} > b_j^{(n)} \qquad (2)$$

$$b_{n+1}^{(n+1)} + c_{n+1}^{(i)} > b_i^{(n)} \quad \text{for } i \in A^j \qquad (3)$$

$$b_{n+1}^{(n+1)} + c_{n+1}^{(k)} > b_k^{(n)} \quad \text{for } k \in B^j \qquad (4)$$

We may notice that in the one dimension model, the entry of the(n+1)-st candidate is a very local phenomenon, i.e. for $i \in B^j$: $c_{n+1}^{(i)} = 0$; the number of votes of candidates not being neighbours to the j-th candidate doesn't change itself after the (n+1)-st candidate enters.

Let a_j denote cardinality of A^j.

If the (n+1)-st candidate wins, s/he gets (Mercik,1988a) more than

$$1/(a_j + 2) \; . \; (b_j^{(n)} + \sum_{k \in A^j} b_k^{(n)}) \qquad (5)$$

and because card$(A^j)=1$ we obtain

$$b_{n+1}^{(n+1)} > 2/3 \; b_k^{(n)} \; ; \quad k \in A^j \qquad (6)$$

The above inequality one may call the "modyfied 2/3 separation opportunity"; the new enterer has to have more votes than 2/3 of the votes of any neighbours of any of the winners to date.

The case of n ≥ 3 candidates under approval voting still hasnot been analysed and needs more research both for the one dimension as well as the more than one dimension cases. Positive results have been obtained only for the entry problem of $n \geq 3$ candidates in plurality voting.

THE CASE OF MULTIDIMENSIONAL ANALYSIS.

At the very beginning of the entry problem analysis using multidimensional models we have to say that the dimensional extention is not only a simple extention of one dimensional models, say, for example, as the extention of definition of the parameters of random variable to random vector. The phenomena under investigation are usually not so local as in one dimension; see, for example, at $c_{n+1}^{(i)}$ for $i \in B^j$. It is possible to beat the current winner via collecting votes from non-neighbour candidates. It is also not so clear where the position of the (n+1)-st candidates should be. It is no longer true that the new (n+1)-st candidate should beat the current winner in the first turn. Generally, there is no prediction who (meaning: which candidate) should first be beaten.

Having in mind the above remarks we may find that restrictions (1)-(4), as formulated in the language of quantity of votes in place of the distribution function along dimensions, are still valid, and they are necessary and sufficient conditions for the (n+1)-st candidate to be the winner in a chalenge with n other candidates.

The problem of how to find a position for the ne᷊ enterer guaranting him or her victory is then a linear programming (LP) problem with restrictions (1)-(4) and the goal function: $\max b_{n+1}^{(n+1)}$. The only problem which we may find in solving this LP problem is that we don't know the exact values of the distribution of votes for the candidates for every point of multidimensional space. This is, in fact, a discrete LP problem.

In this paper we want to show only an approximation of the (n+1)-st candidate's position via some type of "separation opportunity".

For more than one dimension analysis from (1)-(4) we may find that if the (n+1)-st candidate wins, s/he gets

$$b_{n+1}^{(n+1)} > 1/(a_j + 2) \cdot (b_j^{(n)} + \sum_{k \in A^j} b_k^{(n)} + \sum_{k \in B^j} c_{n+1}^{(k)}) \qquad (7)$$

The inequality (7) one may call "multidimensional opportunity condition" for the new enterer's win - it is evidently not a sufficient condition. We should notice that the number of votes of the new enterer ($b_{n+1}^{(n+1)}$) is approximated by the quantity of the votes of all the other candidates before the new enterer has come - this kind of approximation lets us locate the new enterer's position as a point in the multidimensional space of attributes.

CONCLUSIONS.

In the paper we have presented the results of analysis of the so called entry problem finding some opportunity conditions the fulfillment of which gives the new enterer chances to win. The obtained results presented here are rather positive apart from thenegative results indicating what positions are prohibited for a new candidate if s/he wants to win.

We also think that every real life model should operate using the quantity of votes in place of the distribution of the electorate along the dimension(s).

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