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TOWARDS MORE REALISTIC MODELING OF INTERNATIONAL ECONOMIC COOPERATION VIA FUZZY MATHEMATICAL PROGRAMMING AND COOPERATIVE GAMES

Janusz Kacprzyk and Jerzy Hołubiec

Systems Research Institute, Polish Academy of Sciences

ul. Newelska 6, 01 - 447 Warsaw, Poland

Abstract. A new 'soft' cooperative - game - theoretic model of international economic cooperation is proposed. Basically, the participating countries form coalitions and then choose appropriate economic strategies to maximize a benefit or payoff (their shares of some commodity). To account for the inherent 'softness' of this problem, we apply fuzzy mathematical (linear) programming to model the economic behavior of the individual countries and their coalitions. This makes it possible to introduce 'soft' (imprecise or fuzzy) aspiration levels for the objective function values and the satisfaction of the constraints. This replaces the conventional strict and rigid optimization and constraint satisfaction which are often unrealistic in practice. We derive as a solution the C - core which is coalitionally stable.

Keywords: International economic cooperation, international conflict resolution, international stability, fuzzy mathematical programming, cooperative games.

1. INTRODUCTION

International stability is not only a prerequisite for the successful development of mankind but even a "to be or nor to be" for the today's world. Issues related to it are therefore thoroughly studied by both governmental agencies, and scholarly and research institutions.

The economic aspects are here certainly of utmost importance. Among them, the ones related to *international economic cooperation* are particularly important since the cooperation, if appropriately carried out, may significantly help raise living standards, increase efficiency due to the effect of scale, make countries less vulnerable to economic perturbations, etc.

This paper is concerned with the modeling of international economic cooperation using a cooperative - game - theoretic perspective [cf. Ameljańczyk, Hołubiec and Piasecki (1978), Ameljańczyk and Hołubiec (1983, 1984), and Hołubiec (1985)]. We present a 'softening' of that approach in the spirit of Gambarelli, Hołubiec and Kacprzyk (1988). The rationale is that since the economic cooperation problems concern highly aggregated, often not clear - cut entities and relations on which data are rarely fully available and reliable, then strict requirements of the conventional models as to the precision of data are often unrealistic and should be alleviated.

Fuzzy sets theory which provides formal means for representing imprecise concepts and relations (e.g., given by linguistic terms "around 5", much more than 10", "moderate", etc.) is used. In particular, we use fuzzy mathematical programming to model the economic behavior of the individual countries and their coalitions. Basically, too strict and unrealistic requirements to seek exact optima of the objective function(s) and to exactly satisfy the constraints are replaced by the requirements to attain some (possibly imprecisely defined) aspiration levels on the values of the objective function(s) and constraints.

2. A BRIEF INTRODUCTION TO FUZZY SETS AND FUZZY MATHEMATICAL PROGRAMMING

A *fuzzy set* is basically meant to represent imprecise or vague concepts as "large numbers". Formally, a fuzzy set A in a universe of discourse X = {x}, written $A \underset{\sim}{\subset} X$, is represented by, and often informally equated with, its *membership function* $\mu_A: X \rightarrow [0, 1]$ such that $\mu_A(x) \in [0, 1]$ states to what degree x belongs to A: from 0 for full nonbelonginginess to 1 for full belongingness, through all intermediate values.

The basic operations on fuzzy sets are:

- the *complement*

$$\mu_{\neg A}(x) = 1 - \mu_A(x) \qquad \forall x \in X \tag{1}$$

- the *intersection*

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \qquad \forall x \in X \tag{2}$$

- the *union*

$$\mu_{A + B}(x) = \max(\mu_A(x), \mu_B(x)) \qquad \forall x \in X \tag{3}$$

Linguistically, the complement corresponds to "not", the intersection to "and", and the union to "or".

Virtually all works related to *decision making under fuzziness* stem from Bellman and Zadeh's (1970) framework. Its basic elements are: a *fuzzy goal* $G \underset{\sim}{\subset} X$, a *fuzzy constraints* $C \underset{\sim}{\subset} X$, and a *fuzzy decision* $D \underset{\sim}{\subset} X$; $X = \{x\}$ is a space of options (alternatives, decisions,...).

We wish to "satisfy C and attain G" which leads to the *fuzzy decision* given by

$$\mu_D(x) = \min(\mu_C(x), \mu_G(x)) \qquad \forall x \in X \tag{4}$$

which yields the 'goodness' of an $x \in X$ as a solution to the problem considered: from 1 for fully satisfactory to 0 for fully unacceptable, through all intermediate values.

Thus an $x^* \in X$ such that

$$\mu_D(x^*) = \sup_{x \in X} \mu_D(x) \tag{5}$$

is an *optimal* (nonfuzzy!) *solution* (decision) sought. For further details, interpretations, extensions, etc. see Kacprzyk (1983, 1986).

Linear programming (LP) is to find an $x^* \in X$ such that

$$c^T x \to \min_{x \in R^n} \tag{6}$$

subject to: $Ax \leq b$; $x \geq 0$

i.e. to find an $x^* \in X$ which maximizes an objective function subject to some constraints; A is an $m \times n$ matrix of constraint coefficients, x is an n - vector of nonnegative real variables, and b is an m - vextor of right - hand - sides.

However, in many practical problems the use of (6) is difficult due to its 'rigidity' in that 'strict' satisfaction of the constraints and 'strict' minimization of the objective function are required.

Zimmermann (1976) has therefore suggested replacing (6) by

$$c^T x \lesssim z;\ Ax \lesssim b;\ x \geq 0 \tag{7}$$

to be read as that the value of the objective function $c^T x$ should be possibly less than an *aspiration level* z, and the constraints should be possibly well satisfied.

Thus if we denote $B = (cA)^T$ and $d = (zb)^T$, then (7) becomes

$$Bx \lesssim d;\ x \geq 0 \tag{8}$$

where B is an $(m + 1) \times n$ matrix, and d is an $(m + 1)$ - vector.

Obviously, (8) may be rewritten for each element x_j of $x = [x_1, \ldots, x_n]^T$ as follows

$$(Bx)_i \lesssim d_i;\ x_j \geq 0 \qquad i = 1, \ldots, m + 1;\ j = 1, \ldots, n \tag{9}$$

To formalize " $\lesssim$ " we construct the function

$$\begin{aligned} \mu_i(x) &= 1 && \text{for } (Bx)_i \leq d_i \\ &= 1 - ((Bx)_i - d_i)/p_i) && \text{for } d_i < (Bx)_i < d_i + p_i \\ &= 0 && \text{for } (Bx)_i \geq d_i + p_i \end{aligned} \tag{10}$$

to be meant as that we are fully satisfied (= 1) if $(Bx)_i$ does not exceed a satisfaction level d_i, we are fully dissatisfied (= 0) if it exceeds $d_i + p_i$, and we are partially satisfied if it is between d_i and $d_i + p_i$.

To reflect d_i and $d_i + p_i$ we will write in the sequel (10) as

$$(Bx)_i \lesssim \{d_i, d_i + p_p\};\ x_j \geq 0 \tag{11}$$

and similarly for (7) and (8); $p_i = 0$ is equivalent to the conventional strict constraint.

Problem (11) now becomes finding an $x^* \in R^n$ such that

$$\min(\mu_1(x), \ldots, \mu_{m+1}(x)) \to \max_{x \geq 0} \tag{12}$$

which is equivalent to the following conventional linear programming problem: find $x^* \in R^n$ and $w^* \in [0, 1]$ such that

$$\begin{aligned} & w \to \max \\ & \text{subject to: } wp_i + (Bx)_i \leq d_i + p_i \qquad i = 1, \ldots, m + 1 \\ & \qquad\qquad\quad w \in [0, 1];\ x \geq 0 \end{aligned} \tag{13}$$

3. MODELING INTERNATIONAL ECONOMIC COOPERATION VIA MULTIPERSON COOPERATIVE GAMES AND FUZZY MATHEMATICAL PROGRAMMING

We will now first 'soften' the model governing the economic behavior of both the individual countries and their coalitions, and then incorporate this model into a cooperative game.

3.1. A 'soft' model of economic behavior using fuzzy mathematical programming

Denote by: $M' = \{m\} = \{1,\ldots,M\}$ the set of countries, $K' = \{k\} = \{1,\ldots,K\}$ the set of sectors (individual economic activities), $N' = \{n\} = \{1,\ldots,N\}$ the set of commodities, and $I' = \{i\} = \{1,\ldots,I\}$ the set of resource types.

The matrices of technological coefficients are: $A^m = [a^m_{nk}]$ for the unit consumption (use) of commodity n in sector k in country m, $B^m = [b^m_{nk}]$ for the unit production of commodity n in sector k in country m, and $D^m = [d^m_{ik}]$ for the unit use of resource i in sector k in country m; m, n, k, i range over M', N', K', I'.

The *economic strategy* of country m, which is the decision variable, is the vector $U^m = [u^m_1,\ldots,u^m_K]^T$ where u^m_k is the production volume of sector k in country m.

The function

$$F^m(U^m) = \sum_{k=1}^{K} \sum_{n=1}^{N} (b^m_{nk} - a^m_{nk})u^m_k \tag{14}$$

is called the *production system operator* of country m and yields an economic effect resulting from strategy U^m.

We now introduce the *constraints* to which the economy is subjected. They are given as the following arrays.

$X^m = [(\underline{x}^m_1, \bar{x}^m_1),\ldots,(\underline{x}^m_N, \bar{x}^m_N)]$ is the volume of commodity n in country m which is available for consumption. This should be understood as follows: we wish to attain the consumption level of commodity n in country m at least $\bar{x}^m_n$ which is our satisfaction level (satisfaction = 1); the consumption level may be lower than $\bar{x}^m_n$ although the lower the less desirable, and $\underline{x}^m_n$ is the lowest possible consumption level.

$\bar{\underline{U}}^m = [(\underline{u}^m_1, \bar{u}^m_1),\ldots,(\underline{u}^m_K, \bar{u}^m_K)]$ is the array representing the limitations of production capacities in the particular sectors of country m. Likewise, $Z^m = [(\underline{z}^m_1, \bar{z}^m_1),\ldots,(\underline{z}^m_I, \bar{z}^m_I)]$ gives the limitations on the resource types available to country m.

Assume for simplicity that the above constraints on the consumption level, production capacity, and resource availability are the only ones to be dealt with, i.e. we neglect those on labor force, feasible technologies, etc.

A *feasible economic strategy* is therefore one which satisfies:

- the *production capacity constraints*

$$0 \le U^m \le \underline{U}^m \qquad (15a)$$

or

$$0 \le u_k^m \underset{\sim}{\le} \{\underline{u}_k^m, \bar{u}_k^m\} \qquad (15b)$$

where " $\underset{\sim}{\le}$ " is understood as in (11), i.e. it is satisfied to some degree, between 0 and 1, given by [cf. (10)]

$$\mu_{km}^1(u_k^m) = 1 \qquad \text{for } u_k^m \le \underline{u}_k^m$$

$$= 1 - (u_k^m - \underline{u}_k^m)/(\bar{u}_k^m - \underline{u}_k^m) \qquad \text{for } \underline{u}_k^m < u_k^m < \bar{u}_k^m \qquad (16)$$

$$= 0 \qquad \text{for } u_k^m \ge \bar{u}_k^m$$

- the *resource availability constraint*

$$D^m U^m \underset{\sim}{\le} Z^m \qquad (17a)$$

or

$$w_i^m = \sum_{k=1}^{K} d_{ik}^m u_k^m \underset{\sim}{\le} \{\underline{z}_i^m, \bar{z}_i^m\} \qquad (17b)$$

which is satisfied to the degree

$$\mu_{im}^2(w_i^m) = 1 \qquad \text{for } w_i^m \le \underline{z}_i^m$$

$$= 1 - (w_i^m - \underline{z}_i^m)/(\bar{z}_i^m - \underline{z}_i^m) \qquad \text{for } \underline{z}_i^m < w_i^m < \bar{z}_i^m \qquad (18)$$

$$= 0 \qquad \text{for } w_i^m \ge \bar{z}_i^m$$

- the *consumption level constraint*

$$(B^m - A^m)U^m \underset{\sim}{\ge} X^m \qquad (19a)$$

or

$$g_n^m = \sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)u_k^m \underset{\sim}{\ge} \{\underline{x}_n^m, \bar{x}_n^m\} \qquad (19b)$$

which is satisfied to the degree

$$\mu_{nm}^3(g_n^m) = 1 \qquad \text{for } g_n^m \ge \bar{x}_n^m$$

$$= 1 - (\bar{x}_n^m - g_n^m)/(\bar{x}_n^m - \underline{x}_n^m) \qquad \text{for } \underline{x}_n^m < g_n^m < \bar{x}_n^m \qquad (20)$$

$$= 0 \qquad \text{for } g_n^m \le \underline{x}_i^m$$

We can also distinguish some representative commodity, say x_1^m, and express the consumption of $x_2^m, \ldots, x_N^m$ with respect to x_1^m using the vector $h^m = [h_1^m, \ldots, h_N^m]$; $h_n^m = x_n^m / x_1^m$ is called the *consumption structure*. In this case (19a) becomes

$$\sum_{k=1}^{K} (b_{nk}^m - a_{nk}^m)u_k^m = h_n^m \sum_{k=1}^{K} (b_{1k}^m - a_{1k}^m)u_k^m \underset{\sim}{\ge} \{\underline{x}_n^m, \bar{x}_n^m\} \qquad (21)$$

An economic strategy $U^m = [u_1^m, \ldots, u_K^m]^T$ is clearly feasible to some degree which is here assumed to be

$$\mu^S_m(U^m) = \min(\min(\mu^1_1(u^m_1),\dots,\mu^1_{Km}(u^m_K)),\ \min(\mu^2_{1m}(w^m_i,\dots$$
$$\dots,\mu^2_{Im}(w^m_I)),\ \min(\mu^3_{1m}(g^m_1),\dots,\mu^3_{Nm}(g^m_N))) \qquad (22)$$

Notice that (22) may be viewed as reflecting some "safety - first" attitude which seems to be appropriate in our context.

An economic strategy is λ - *feasible* if $\mu^S_m(U^m) = \lambda$, i.e. if it satisfies each constraint at least to degree $\lambda \in [0, 1]$.

The objective function is the production system operator (14) to be maximized. This is replaced by a more realistic requirement

$$F^m(U^m) \underset{\sim}{\geq} \underline{F}^m \qquad (23a)$$

or

$$y^m_k = \sum_{k=1}^{K} \sum_{n=1}^{N} (b^m_{ik} - a^m_{ik})u^m_k \underset{\sim}{\geq} \{\underline{f}^m, \bar{f}^m\} \qquad (23b)$$

which is satisfied to the degree

$$\mu^0_m(U^m) = \begin{cases} 1 & \text{for } y^m \geq \bar{f}^m \\ 1 - (\bar{f}^m - y^m)/(\bar{f}^m - \underline{f}^m) & \text{for } \underline{f}^m < y^m < \bar{f}^m \\ 0 & \text{for } y^m \leq \underline{f}^m \end{cases} \qquad (24)$$

An economic strategy is λ - *optimal* if $\mu^0_m(U^m) = \lambda$.

For country m the problem is now to find an optimal strategy U^{m*} and a maximal λ, λ^*, such that it yields the highest feasibility and optimality degree of U^m.

According to Section 2, this can be represented by a fuzzy linear programming problem with the fuzzy objective function (23a, b) and the fuzzy constaints (15), (17) and (19), which is equivalent to the following nonfuzzy linear programming problem: find $U^{m*} = (u^{m*}_1,\dots,u^{m*}_K)^T$ and λ^* such that

$$\lambda \to \max$$

subject to: (25)

$$\lambda(\bar{u}^m_k - \underline{u}^m_k) + u^m_k \leq \bar{u}^m_k$$

$$\lambda(\bar{z}^m_i - \underline{z}^m_i) + \sum_{k=1}^{K} d^m_{ik}u^m_k \leq \bar{z}^m_i$$

$$\lambda(\bar{x}^m_n - \underline{x}^m_n) + \sum_{k=1}^{K} (b^m_{nk} - a^m_{nk})u^m_k \geq \bar{x}^m_n$$

$$\lambda \in [0, 1];\ u^m_k \geq 0$$

3.2. <u>A multiperson cooperative game model of international economic cooperation involving fuzzy mathematical programming</u>

The economic cooperation between the countries whose economies are described by the above model is now dealt with in terms of a *multiperson cooperative game*. This game is assumed to be an ordered pair $G = (M, v)$. M is the set of *players* (individual countries) and $2^M = \{S: S \subseteq M\}$ is the set of *coalitions*, i.e. subsets of countries.

A function, called the *characteristic function* of the game,

$$v: 2^M \to R; \; v(\emptyset) = 0 \tag{26}$$

is defined which associates with each coalition an economic effect, $v(S)$, i.e. the value of (11), which can be assured by the coalition no matter what the conduct of other players (not belonging to this coalition) is.

The algorithm for determining $v(S)$ [see, e.g., Ameljańczyk and Hołubiec (1983, 1984)], but now accounting for the 'soft' model of the economy of each country presented in Subsection 3.1, is

$$v(S) = F^*(S) = \sum_{m \in S} \sum_{k=1}^{K} (b^m_{nk} - a^m_{nk}) u^{m*}_k; \; v(\emptyset) = 0 \tag{27}$$

where $U^{m*} = (u^{m*}_1, \ldots, u^{m*}_K)$, $m \in S$, are the optimal economic strategies of the contries belonging to coalition S which are obtained by solving

$$\begin{aligned} &\lambda \to \max \\ &\text{subject to:} \\ &\lambda(\bar{u}^m_k - \underline{u}^m_k) + u^m_k \le \bar{u}^m_k; \; m \in S \\ &\lambda\Big(\sum_{m \in S} \bar{z}^m_i - \sum_{m \in S} \underline{z}^m_i\Big) + \sum_{m \in S} d^m_{ik} u^m_k \le \sum_{m \in S} \bar{z}^m_i \\ &\lambda\Big(\sum_{m \in S} \bar{x}^m_n - \sum_{m \in S} \underline{x}^m_n\Big) + \sum_{m \in S} \sum_{k=1}^{K} (b^m_{nk} - a^m_{nk}) u^m_k \ge \bar{x}^m_n \\ &\lambda \in [0, 1]; \; u^m_k \ge 0 \end{aligned} \tag{28}$$

Notice that $v(\{m\})$, $m \in M$, is some characteristic feature of country m which can be found by solving (25).

Function $v(S)$ is superadditive, i.e. $v(S) \ge \sum_{m \in S} v(\{m\})$. Intuitively, it can be jutified as follows: the aspiration levels on lowest (highest) possible levels are obtained by summing up their individual counterparts over $m \in S$. Thus if in one country a constraint limit is only partially used, it may be used by the other countries of the coalition, alleviating their limits, hence leading to a possibly higher economic effect.

Evidently, the maximal joint economic effect is for $S = M$, i.e. when all the countries cooperate, and is equal to $v(M)$ given by solving (28) for $S = M$.

If $X' = (x'_1,\ldots,x'_N)$ is the vector of global volumes of the consumption commodities, then the problem of international exchange, assuming a known and constant consumption structure h, is to determine the payoff vector $X = (x_1,\ldots,x_M)$ which gives the share of x_1 (the basic or represenative commodity) allocated to country m, $m = 1,\ldots,M$. Obviously

$$\sum_{m=1}^{K} x_m = x'_1 \tag{29}$$

Notice that in (29) the amount of x'_1 is assumed to be strictly given as opposed to 'soft' limits ($\bar{x}_n^m$ and $\underline{x}_n^m$) in (19).

Among many solution concepts , the *C - core*, C(v), has proven very useful [cf. Ameljańczyk and Hołubiec (1983, 1984)]. In our case it is defined as

$$C(v) = \{X: \sum_{m \in S} x_m \geq v(S),\ s \in 2^M;\ \sum_{m=1}^{M} x_m = v(M)\} \tag{30}$$

If $C(v) \neq \emptyset$, it contains coalitionally rational (stable) solutions (the payoff vectors, i.e. the shares of x_1) in the sense that the players (countries) cannot do any better by leaving S.

4. CONCLUDING REMARKS

We feel that the proposed model may help describe more realistically important problems of international economic cooperation. First, it explicitly accounts for an inherent 'softness' of these problems in an intuitively appealing and computationally tractable way. Second, it proposes to use some 'softer' derivations of the tools of cooperative game theory and its solution concepts which have already proven to be useful for analysing problems from the class considered.

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