

# **Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications**

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Systems Research Institute  
Polish Academy of Sciences  
Newelska 6, 01-447 Warsaw, Poland  
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# Modeling the nonlinear autoregressive network with exogenous inputs with a generalized net

**Sotir Sotirov<sup>1</sup>, Valentina Kukenska<sup>1</sup>, Maria Hristova<sup>2</sup>,  
Ivelina Vardeva<sup>1</sup>, Lilia Staneva<sup>1</sup>, Juli Barzov<sup>1</sup>,  
Stoyan Dimitrov<sup>1</sup> and Stela Stoyanova<sup>1</sup>**

<sup>1</sup> Prof. Asen Zlatarov University, Bourgas-8010, Bulgaria,  
*ssotirov@btu.bg*

<sup>2</sup> Todor Kableshkov University of Transport, Sofia, Bulgaria  
*mhristova@vtu.bg*

## Abstract

The proposed GN model presents the functioning of the Nonlinear Autoregressive Network with Exogenous Inputs (NARX). This is the one of the recurrent dynamic neural networks with feedback connection with several layers. Instead of other types neural network now we are describing the process during the time.

**Keywords:** nonlinear autoregressive network with exogenous inputs, generalized nets.

## 1 Introduction

In a series of papers the process of functioning and the results of the work of different types of neural networks are described, modelled in terms of generalized nets [3, 4, 8-11].

According to [23], neural networks can be classified into dynamic and static categories. Static (feedforward) networks have no feedback elements and carry no delays; the output is calculated directly from the input through feedforward connections. In dynamic networks, the output depends not only on the current input to the network, but also on the current or previous inputs, outputs, or the states of the network. The training of dynamic networks is very similar to the training of static feedforward networks, Dynamic networks can be divided into two categories: those that have only feedforward connections, and those that have feedback, or recurrent, connections.

The dynamic network has memory. Its response at any given time depends not only on the current input, but on the history of the input sequence. If the network does not have any feedback connections, then only a finite amount of records in the history will affect the response.

Dynamic networks are generally more powerful than static networks (although somewhat more difficult to train) [16]. Dynamic networks can be used for prediction in financial markets [18], channel equalization in communication systems [19], phase detection in power systems, sorting [20], fault detection [21], speech recognition and even the prediction of protein structure in genetics [22]. A discussion of many more dynamic network applications can be found in [23]. One principal application of dynamic neural networks is found in control systems. Dynamic networks are also well suited for filtering.

One of the dynamic kinds of neural networks is the Nonlinear Autoregressive Network with Exogenous Inputs (NARX), illustrated on Fig. 1.

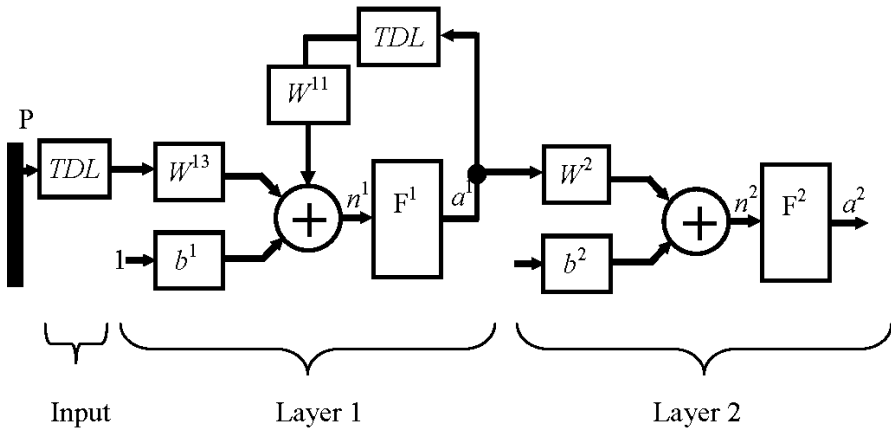


Figure 1: NARX neural network

The definition equation for the NARX is

$$y(t) = f(y(t-1), y(t-2), \dots, y(t-n_y), u(t-1), u(t-2), \dots, u(t-n_u)) \quad (1)$$

where the next value of the dependent output signal  $y(t)$  is regressed on previous values of the output signal and previous values of an independent (exogenous) input signal.

## 2 A GN-model

All definitions related to the concept *generalized net* (GN) are taken from [1,2]. The GN, describing the process of the work of the Nonlinear Autoregressive Network with Exogenous Inputs, is shown on Fig. 2.

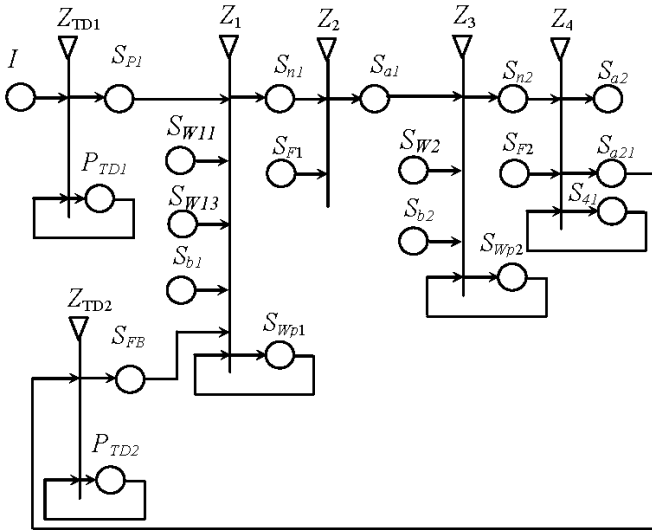


Figure 2: GN model of the NARX

Initially the following tokens enter in the GN:

- in place I -  $\alpha^1$ -token with initial characteristic  $x_0^{\alpha^1} = I(t)$ ;
- in place  $S_{W^{11}}$  -  $\beta^1$ -token with initial characteristic  $x_0^{\beta^1} = w^1$ , where  $W^{11}$  is a matrix with weight coefficient in first layer of the NARX);
- in place  $S_{W^{13}}$  -  $\beta^3$ -token with initial characteristic  $x_0^{\beta^3} = w^{13}$ , where  $W^{13}$  is a matrix with weight coefficient in feedback of the first layer of the NARX);
- in place  $S_{b^1}$  -  $\gamma^1$ -token with initial characteristic  $x_0^{\gamma^1} = b^1$ , where  $b^1$  is a matrix with biases of the first layer of the NARX);
- in place  $S_{F^1}$  - one  $\delta^1$ -token with initial characteristic  $x_0^{\delta^1} = \text{transfer function}$ ;
- in place  $S_{W^2}$  -  $\beta^2$ -token with initial characteristic



$$x_0^{\beta^2} = W^2 = \begin{vmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,R} \\ W_{2,1} & W_{2,2} & \dots & W_{2,R} \\ \dots & \dots & \dots & \dots \\ W_{S,1} & W_{S,2} & & W_{S,R} \end{vmatrix}$$

where  $W^2$  - matrix with weight coefficient for second layer of the NARX;

- in place  $S_{b^2}$  -  $\gamma^2$ -token with initial characteristic  $x_0^{\gamma^2} = b^2$ ; where  $b^2$  is a matrix with biases of the second layer of the NARX;
- in place  $S_{F^2}$  - one  $\delta^2$ -token with initial characteristic using “Transfer function”;

The GN is presented by a set of transitions:

$$A = \{ Z_{TD1}, Z_{TD2}, Z_1, Z_2, Z_3, Z_4 \},$$

where transitions describe the following processes:

- $Z_{TD1}$  – Preparing the input vector  $P^1$  from the  $I(t)$ ;
- $Z_{TD2}$  – Preparing the feedback;
- $Z_1$  – Calculating influence of the first layer of the MPL ( $n^1$ );
- $Z_2$  – Calculating the output of the first layer of the MPL ( $a^1$ );
- $Z_3$  – Calculating influence of the second layer of the MPL ( $n^2$ );
- $Z^4$  – Calculating the output of the second layer of the MPL ( $a^2$ ).

Transitions of the GN-model have the following forms.

$$Z_{TD1} = \langle \{I, P_{TD1}\}, \{S_{P^1}, P_{TD1}\}, R_{TD1}, \wedge(I, P_{TD1}) \rangle,$$

where:

$$R_{TD1} = \begin{array}{c|cc} & S_{P^1} & P_{TD1} \\ \hline I & False & True, \\ P_{TD1} & W_{P_{TD1}, P^1} & True \end{array}$$

where

$W_{P_{TD1}, P^1} =$  “In place  $P_{TD1}$  there exists  $x_k^\alpha$  “.

The  $\alpha$ -tokens obtain characteristics, as follows:

$$\begin{aligned}
 x_0^\alpha &= I(t); \\
 x_1^\alpha &= I(t), I(t-1); \\
 &\dots\dots \\
 x_{k-1}^\alpha &= I(t), I(t-1), \dots, I(t-k+1); \\
 x_k^\alpha &= I(t), I(t-1), \dots, I(t-k+1), I(t-k)
 \end{aligned} \tag{2}$$

$$Z_{TD2} = \langle \{S_{a21}, P_{TD2}\}, \{S_{FB}, P_{TD2}\}, R_{TD2}, \wedge(S_{a21}, P_{TD2}) \rangle,$$

where

$$R_{TD2} = \begin{array}{c|cc} & S_{FB} & P_{TD2} \\ S_{a21} & False & True, \\ P_{TD2} & W_{P_{TD2}, S_{FB}} & True \end{array}$$

where

$W_{P_{TD2}, S_{FB}}$  = “There is a token in place  $P_{TD2}$ ”.

The  $\alpha$ -tokens obtain characteristics according to (2).

$$\begin{aligned}
 Z_1 &= \langle \{S_{p^1}, S_{w^{11}}, S_{w^{13}}, S_{b^1}, S_{w_{p^1}}\}, \{S_{n^1}, S_{w_{p^1}}\}, R_1, \\
 &\wedge(\vee(S_{p^1}, S_{w^{11}}, S_{w^{13}}), \vee(S_{b^1}, S_{w_{p^1}})) \rangle,
 \end{aligned}$$

where

$$R_1 = \begin{array}{c|cc} & S_{n^1} & S_{w_{p^1}} \\ S_{p^1} & False & True \\ S_{w^{11}} & False & True, \\ S_{w^{13}} & False & True \\ S_{b^1} & True & False \\ S_{w_{p^1}} & True & False \end{array}$$

Tokens  $\alpha^1, \beta^1, \beta^3$  and  $\gamma^1$  merge into the  $\chi^1$ -token in place  $S_{w_{p^1}}$ .

$$Z_2 = \langle \{S_{n^1}, S_{F^1}\}, \{S_{a^1}\}, R_2, \wedge(S_{n^1}, S_{F^1}) \rangle,$$

where

$$R_2 = \begin{array}{c|c} & S_{a^1} \\ S_{n^1} & True, \\ S_{F^1} & True \end{array}$$

Tokens  $\delta^1$  and  $\chi^1$  merge into the  $\sigma^1$ -token according to [5].

$$Z_3 = \langle \{S_{a^1}, S_{W^2}, S_{b^2}, S_{Wp^2}\}, \{S_{n^2}, S_{Wp^2}\}, R_3, \wedge(\vee(S_{a^1}, S_{W^2}), \vee(S_{b^2}, S_{Wp^2})) \rangle,$$

where

$$R_3 = \begin{array}{c|cc} & S_{n^2} & S_{Wp^2} \\ \hline S_{a^1} & False & True \\ S_{W^2} & False & True \\ S_{b^2} & True & False \\ S_{Wp^2} & True & False \end{array},$$

Tokens  $\sigma^l$ ,  $\beta^2$  and  $\gamma^2$  merge into the  $\chi^2$ -token according to [5].

$$Z_4 = \langle \{S_{n^2}, S_{F^2}, S_{41}\}, \{S_{a^1}, S_{a^{21}}, S_{41}\}, R_4, \wedge(S_{n^2}, S_{F^2}, S_{41}) \rangle,$$

where

$$R_4 = \begin{array}{c|ccc} & S_{a^2} & S_{a^{21}} & S_{41} \\ \hline S_{n^2} & False & False & True \\ S_{F^2} & False & False & True \\ S_{41} & W_{S_{41}, a^2} & W_{S_{41}, a^{21}} & False \end{array},$$

where

$$W_{S_{41}, a^2} = W_{S_{41}, a^{21}} = \text{“The value is calculated “}.$$

Tokens  $\delta^2$  and  $\chi^2$  union into the  $\sigma^2$ -token, using [5].

### 3 Conclusion

The dynamic neural networks have different structures and properties. The proposed GN model present work of the Nonlinear Autoregressive Network with Exogenous Inputs (NARX). This is the one of the recurrent dynamic neural networks with feedback connection with several layers. Instead of other types neural network now we are describing the process during the time.

This paper is one of the series of papers that describe neural networks in terms of generalized nets.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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