POLSKA AKADEMIA NAUK
Instytut Badań Systemowych

BADANIA OPERACYJNE I SYSTEMOWE: ŚRODOWISKO NATURALNE, PRZESTRZEŃ, OPTYMALIZACJA

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Jan Studziński
red.

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ON MIP FORMULATION OF THE ADJUSTMENT PROBLEM FOR THE MST AND MHP PROBLEMS

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We consider a pair of optimization problems for a given weighted undirected graph: the minimum spanning tree (MST) problem and its restriction, the minimum Hamiltonian path (MHP) problem. For this pair of problems the adjustment problem consists in finding such minimum norm perturbations of weights of edges, which guarantee that the set of optimal solutions of the MST problem in the perturbed graph contains a Hamiltonian path. In this paper we consider a mixed integer programming (MIP) formulation of the adjustment problem and describe computational results obtained for randomly generated graphs.

1. Introduction

The adjustment problem described in Libura (2007) is formulated for a pair of optimization problems: an initial optimization problem with linear objective function

$$v(c, X) = \max\{c^T x : x \in X\}, \tag{1}$$

where $c \in \mathbb{R}^n$ and $X \subseteq \mathbb{R}^n$, and its restriction for a given subset $F \subseteq X$ of feasible solutions:

$$\max\{c^T x : x \in F\}. \tag{2}$$

Given such a pair of problems one wants to adjust the objective function coefficients in problem (1), such that an optimal solution of the perturbed problem is also a feasible solution for the restriction (2). The adjustment problem seeks among admissible perturbations, one that is least costly according to some given norm.

Let $\Delta \subseteq \mathbb{R}^n$ be the set of all admissible perturbations of the original vector of weights c, and let $\|\delta\|$ denotes a norm of $\delta \in \mathbb{R}^n$. The adjustment problem related to F and Δ is formally stated as follows:

$$a(F, \Delta) = \min\{\|\delta\| : v(c + \delta, X) = v(c + \delta, F), \delta \in \Delta\}. \tag{3}$$

Given sets $\Delta \subseteq \mathbb{R}^n$ and $F \subseteq X$, the optimal value $a(F, \Delta)$ of the problem (3) is called the adjustment cost with respect to F and Δ. In this paper we will consider only l_1 norm, i.e., for $\delta = \delta^+ - \delta^-$, $\delta^+, \delta^- \in \mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x \geq 0\}$, $\|\delta\| = \mathbf{1}^T\delta^+ - \mathbf{1}^T\delta^-$, where $\mathbf{1} \in \mathbb{R}^n$ is a vector of ones. Moreover, we will take $\Delta = \mathbb{R}^n$.

In the following we assume that the initial optimization problem (1) is a linear programming problem:

$$\max c^T x \tag{4a}$$

$$A'x \leq b' \tag{4b}$$

$$A''x = b'' \tag{4c}$$

$$x \geq 0, \tag{4d}$$

where $A' \in \mathbb{R}^{m' \times n}$, $A'' \in \mathbb{R}^{m'' \times n}$, $b' \in \mathbb{R}^{m'}$ and $b'' \in \mathbb{R}^{m''}$.

A restriction of this problem will be defined by adding the new linear constraints

$$Cx \leq d, \tag{5}$$

where $C \in \mathbb{R}^{p \times n}$, $d \in \mathbb{R}^p$, and requiring that the feasible solutions belong to the set $\mathbb{B}^n = \{0, 1\}^n$. Thus, the restricted problem (2) considered in the paper has the form:

$$\begin{aligned} &\max c^T x \\ &A'x \leq b' \\ &A''x = b'' \\ &Cx \leq d \\ &x \in \mathbb{B}^n. \end{aligned} \tag{6}$$

In Libura (2007) it is shown that the adjustment problem for a pair of problems (4) and (6) and $\|\delta\| = l_1$, $\Delta = \mathbb{R}^n$, can be stated as follows:

$$\min(\mathbf{1}^T \delta^+ - \mathbf{1}^T \delta^-) \tag{7a}$$

$$\begin{cases} (A')^T y' - \delta^+ + \delta^- \geq c \\ (A'')^T y'' - \delta^+ + \delta^- \geq c \\ y' \geq 0, y'' \in \mathbb{R}^{m''} \end{cases} \tag{7b}$$

$$(b')^T y' + (b'')^T y'' - c^T x - (\delta^+)^T x + (\delta^-)^T x = 0 \tag{7c}$$

$$\begin{cases} A'x \leq b' \\ A''x = b'' \\ Cx \leq d \\ x \in \mathbb{B}^n \end{cases} \tag{7d}$$

$$\delta^+, \delta^- \geq 0. \tag{7e}$$

In the above problem the constraints (7b) and the variables y', y'', correspond to the dual problem for the linear program (4), whereas constraints (7d) and variables x correspond to the restricted problem (6).

The nonlinear constraint (7c) states that the value of the objective function in the perturbed initial linear program (4) is equal to the value of the objective function in its dual. To solve problem (7) with standard optimization packages for MIP it is necessary to linearize the nonlinear terms $(\delta^+)^T x$ and $(\delta^-)^T x$. In the next section we describe this step and give all details for MIP formulation of the adjustment problem for a pair of problems: the minimum spanning tree (MST) problem, regarded as the initial optimization problem (4), and the minimum Hamiltonian path problem – as its restriction (6).

2. MST, MHP and the adjustment problems

Let $G = (V, E, c)$ be a connected weighted graph, where $V = \{1, \ldots, n\}$, $E \subseteq \{\{i, j\} : i, j \in V\}$, $c \in \mathbb{R}^{|E|}$. Our goal will be to find such perturbations $\delta \in \mathbb{R}^{|E|}$ of the weight vector c, that some minimum spanning tree in the perturbed graph is a Hamiltonian path in this graph. First, we will formulate the minimum spanning tree problem in G as the initial linear program (4). Then, we describe additional constraints to get the minimum Hamiltonian path problem as the restriction (6). Finally, we give the adjustment problem (7) for this pair of optimization problems.

We use the following notation:

$$\begin{aligned} K &= V \setminus \{1\}, \\ D &= \{(i, j) \in V \times V : \{i, j\} \in E\}. \end{aligned} \tag{8}$$

Define for $(i, j) \in D$ nonnegative variables $x_{ij} \in \mathbb{R}$ and introduce for $k \in K$ and $(i, j) \in D$ auxiliary nonnegative variables $f_{ij}^k \in \mathbb{R}$. It is known (see e.g. Magnanti and Wolsey (1995)), that the following continous linear programming problem (9) may be used to calculate the minimum weight spanning tree T^* in graph $G = (V, E, c)$. Namely, given an optimal solution x^*, f^* of problem (9), the optimal tree T^* is composed of the edges $\{i, j\} \in E$ for which $x_{ij}^* + x_{ji}^* > 0$.

$$\max \sum_{\{i,j\} \in E} -c_{ij} \cdot (x_{ij} + x_{ji}) \tag{9a}$$

$$x_{ij} - f_{ij}^k \geq 0, \ k \in K, (i, j) \in D \tag{9b}$$

$$\begin{cases} \sum_{(i,j) \in D} x_{ij} = n - 1 \\ \sum_{(j,1) \in D} f_{j1}^k - \sum_{(1,j) \in D} f_{1j}^k = -1, \ k \in K \\ \sum_{(j,k) \in D} f_{jk}^k - \sum_{(k,j) \in D} f_{kj}^k = 1, \ k \in K \\ \sum_{(j,i) \in D} f_{ji}^k - \sum_{(i,j) \in D} f_{ij}^k = 0, \ i, k \in K, i \neq k \end{cases} \tag{9c}$$

$$\begin{cases} x_{ij} \geq 0, \ (i, j) \in D \\ f_{ij}^k \geq 0, \ k \in K, (i, j) \in D. \end{cases} \tag{9d}$$

Parts (9a), (9b), (9c) in the above formulation of the MST problem correspond, respectively, to (4a), (4b), (4c) in the initial linear program (4).

To get the MHP problem, being a restriction of (9), it is necessary to add the following constraints:

$$\sum_{j \in V} (x_{ij} + x_{ji}) \leq 2, \ i \in V, \tag{10}$$

$$x_{ij} \in \mathbb{B}, \ (i, j) \in D. \tag{11}$$

Observe that constraints (10) correspond to linear constraints (5). The obtained optimization problem corresponds to (6).

Let $w_{ij}^k \geq 0$ for $k \in K$, $(i, j) \in D$, be dual variables for inequalities (9b), and $l \in \mathbb{R}$, $v_1^k \in \mathbb{R}$ for $k \in K$, $v_k^k \in \mathbb{R}$ for $k \in K$, $v_i^k \in \mathbb{R}$ for $i, k \in K$, $i \neq k$ denote dual variables for consecutive groups of constraints in problem (9c). The dual problem of linear program (9) can be stated as follows:

$$\min \sum_{k\in K} -(v_k^k - v_1^k) - (n-1)\cdot l \tag{12a}$$

$$\begin{cases} v_j^k - v_i^k - w_{ij}^k \le 0,\ k\in K, (i,j)\in D \\ \sum_{k\in K} w_{ij}^k + l \le c_{ij},\ \{i,j\}\in E \\ \sum_{k\in K} w_{ji}^k + l \le c_{ij},\ \{i,j\}\in E \\ w_{ij}^k \ge 0,\ k\in K, (i,j)\in D \\ v_i^k \in \mathbb{R},\ i\in V, k\in K \\ l\in\mathbb{R}. \end{cases} \tag{12b}$$

We may now formulate the adjustment problem (7) for the considered pair of problems. It is enough to use constraints (12b) as constraints (7b) and constraints (9b), (9c), (9d), (10), (11) as constraints (7d). The constraint (7c) corresponds now to the objective functions (9a), (12a), and has the following form:

$$\sum_{k\in K}(v_k^k - v_1^k) + (n-1)\cdot l - \sum_{\{i,j\}\in E} c_{ij}(x_{ij}+x_{ji}) - \sum_{(i,j)\in D}(\delta_{ij}^+ - \delta_{ij}^-)x_{ij} = 0, \tag{13}$$

where $\delta_{ij}^+, \delta_{ij}^- \ge 0$ for $(i,j)\in D$.

Finally, the adjustment problem for a pair: the MST and the MHP problems is stated as follows:

$$\min \sum_{(i,j)\in D}(\delta_{ij}^+ - \delta_{ij}^-) \tag{14a}$$

$$\begin{cases} v_j^k - v_i^k - w_{ij}^k \le 0,\ k\in K, (i,j)\in D \\ \sum_{k\in K} w_{ij}^k + l - \delta_{ij}^+ + \delta_{ij}^- \le c_{ij},\ \{i,j\}\in E \\ \sum_{k\in K} w_{ji}^k + l - \delta_{ji}^+ + \delta_{ji}^- \le c_{ij},\ \{i,j\}\in E \\ w_{ij}^k \ge 0,\ k\in K, (i,j)\in D \\ v_i^k \in \mathbb{R},\ i\in V, k\in K \\ l\in\mathbb{R} \end{cases} \tag{14b}$$

$$\sum_{k\in K}(v_k^k - v_1^k) + (n-1)\cdot l - \sum_{\{i,j\}\in E} c_{ij}(x_{ij}+x_{ji}) - \sum_{(i,j)\in D}(\delta_{ij}^+ - \delta_{ij}^-)x_{ij} = 0 \tag{14c}$$

$$\begin{cases} \sum_{(i,j)\in D} x_{ij} = n-1 \\ \sum_{(j,1)\in D} f_{j1}^k - \sum_{(1,j)\in D} f_{1j}^k = -1,\ k\in K \\ \sum_{(j,k)\in D} f_{jk}^k - \sum_{(k,j)\in D} f_{kj}^k = 1,\ k\in K \\ \sum_{(j,i)\in D} f_{ji}^k - \sum_{(i,j)\in D} f_{ij}^k = 0,\ i,k\in K, i\ne k \\ x_{ij} - f_{ij}^k \ge 0,\ k\in K, (i,j)\in D \\ x_{ij}\ge 0,\ (i,j)\in D \\ f_{ij}^k \ge 0,\ k\in K, (i,j)\in D \end{cases} \tag{14d}$$

$$\begin{cases} \sum_{j\in V}(x_{ij}+x_{ji})\le 2,\ i\in V\\ x_{ij}\in\{0,1\},\ (i,j)\in D\\ \delta^+_{ij}=\delta^+_{ji}\ge 0,\ (i,j)\in D\\ \delta^-_{ij}=\delta^-_{ji}\ge 0,\ (i,j)\in D. \end{cases} \tag{14e}$$

To linearize terms $\delta^+_{ij}x_{ij}$, $\delta^-_{ij}x_{ij}$, $(i,j)\in D$, we use a standard technique (see e.g. Williams (1993)). Namely, we introduce new nonnegative variables z^+_{ij}, z^-_{ij}, $(i,j)\in D$ satisfying the following conditions:

$$z^+_{ij}=\delta^+_{ij}x_{ij},\ (i,j)\in D, \tag{15a}$$

$$z^-_{ij}=\delta^-_{ij}x_{ij},\ (i,j)\in D. \tag{15b}$$

Now we can replace the constraint (14c) in the problem (14), with the following linear constraint:

$$\sum_{k\in K}(v^k_k-v^k_1)+(n-1)\cdot l-\sum_{\{i,j\}\in E}c_{ij}(x_{ij}+x_{ji})-\sum_{(i,j)\in D}z^+_{ij}+\sum_{(i,j)\in D}z^-_{ij}=0. \tag{16}$$

For any new variable z^+_{ij}, z^-_{ij}, $(i,j)\in D$, we have to add also constraints which would guarantee that equations (15a) and (15b) hold. Let us take for example the equation $z^+_{ij}=\delta^+_{ij}x_{ij}$ for some ordered pair $(i,j)\in D$. This equation is equivalent to two implications:

$$\begin{aligned} x_{ij}=0 &\Rightarrow z^+_{ij}=0,\\ x_{ij}=1 &\Rightarrow z^+_{ij}=\delta^+_{ij}, \end{aligned} \tag{17}$$

which can be modeled by adding the following new constraints:

$$\begin{aligned} &z^+_{ij}-Mx_{ij}\le 0,\\ &-\delta^+_{ij}+z^+_{ij}\le 0,\\ &\delta^+_{ij}-z^+_{ij}+Mx_{ij}\le M, \end{aligned} \tag{18}$$

where M is a sufficiently large constant satisfying the inequality $\delta^+_{ij}\le M$ for any $(i,j)\in D$. It can be shown (see Libura (2007)) that for $\Delta=\mathbb{R}^n$ we can take $M=\sum_{\{i,j\}\in E}|c_{ij}|$. So, to linearize all nonlinear terms in (14c) it is necessary to add for any $(i,j)\in D$ the following inequalities:

$$\begin{aligned} &z^+_{ij}-Mx_{ij}\le 0\\ &-\delta^+_{ij}+z^+_{ij}\le 0\\ &\delta^+_{ij}-z^+_{ij}+Mx_{ij}\le M\\ &z^-_{ij}-Mx_{ij}\le 0\\ &-\delta^-_{ij}+z^-_{ij}\le 0\\ &-\delta^-_{ij}-z^-_{ij}+Mx_{ij}\le M\\ &z^+_{ij},z^-_{ij}\ge 0. \end{aligned} \tag{19}$$

3. Computational results

In this section we describe computational experiment provided to test our MIP approach for the adjustment problem.

We used randomly generated graphs with the number of vertices $|V| = 11, \ldots, 20$. The following families of benchmarks were considered:

1. *complete1* – complete graphs with the number of vertices $|V| = 11, \ldots, 20$, in which the weight of each edge is an integer random variable uniformly distributed in the interval $[0, |V|^2]$. For each number of vertices 10 instances were generated.

2. *complete2* – complete graphs with the number of vertices $|V| = 11, \ldots, 20$, in which the weight of each edge is taken with the same probability from the set $\{0, 1\}$. For each number of vertices 10 instances were generated.

3. *density* – graphs with 20 vertices in which each edge appears with the probability $d = 0.2, 0.3, \ldots, 1.0$. The weight of an edge is an integer random variable uniformly distributed in the interval $[0, |V|^2]$. For each density considered 10 instances were generated.

4. *data* – a family of three groups of benchmarks (*integer*, *boolean*, *real*), constructed to investigate the influence of data (discrete, continous). In the *integer* and *real* group the weight of each edge is an integer or a continous random variable uniformly distributed in the interval $[0, |V|^2]$, respectively. In the *boolean* group the weight of each edge is taken with the same probability from the set $\{0, 1\}$. For each group considered 10 instances of graphs with $|V| = 20$ vertices were generated.

A computer program in Java was used to generate all benchmarks and related MIP problems. To solve these MIP problems we use in our program ILOG CPLEX 10 solver libraries. For all instances optimal solutions for the adjustment problem were obtained. Resulting MIP problems for $|V| = 20$ had roughly 17000 variables (including 380 binaries), 17500 constraints, and 66500 nonzeros.

Computational times (in seconds) for AMD Opteron Processor 248 with 4 GB RAM for families *complete1*, *complete2*, *density* and *data* are presented in the following tables and figures. For any 10 instances from these families, we computed the following statistic parameters:

1. *min* – minimum computational time,

2. *median* – median of computational times,

3. *avg* – average of computational times,

4. *stddev* – standard deviation of computational times,

5. *max* – maximum computational time.

In Table 1 and Figure 1, Table 2 and Figure 2, Table 3 and Figure 3 computational results for families *complete1*, *complete2*, *density* are summarized, respectively (for sake of space in the tables the results for the smallest instances $|V| = 11, 12$ and for densities

complete1 benchmarks	Number of vertices							
	13	14	15	16	17	18	19	20
min	0.27	0.42	1.1	1.59	2.09	2.99	7.29	10.37
median	2.47	1.62	3.92	2.63	31.23	9.49	13.8	27.2
avg	3.78	6.17	10.78	25.53	87.64	42.77	101.41	252.49
stddev	3.6	7.69	17.66	35.44	148.78	54.35	198.91	516.4
max	9.95	22.39	58.52	103.08	492.37	158.64	644.21	1552.4

Tablica 1: Computational times (in seconds) for the *complete1* family.

complete2 benchmarks	Number of vertices							
	13	14	15	16	17	18	19	20
min	0.56	0.8	1.82	2.69	5.96	9.08	11.89	16.94
median	1.55	2.02	3.45	14.83	33.74	83.57	115.31	300.77
avg	1.57	2.61	6.56	23.73	37.27	88.67	192.18	332.26
stddev	0.86	2.16	5.68	28.43	19.01	54.84	265.18	309.52
max	2.92	6.77	18.15	95.04	75.12	205.92	794.86	1042.16

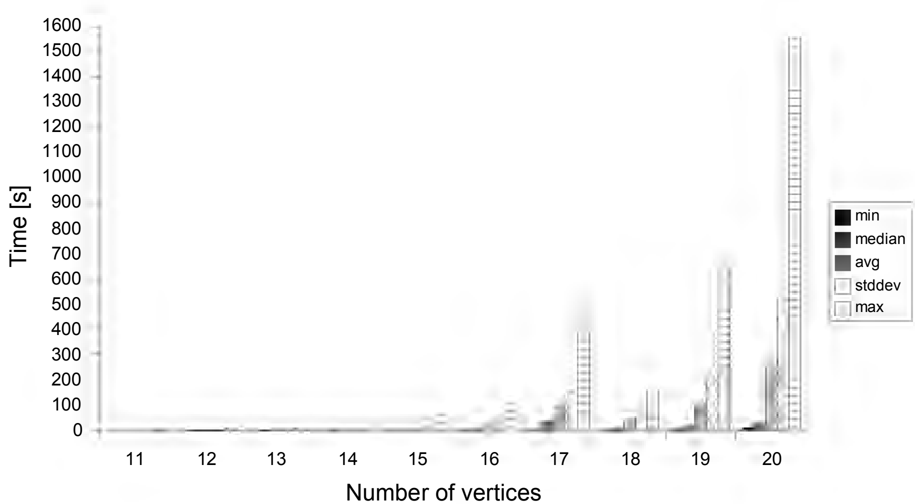
Tablica 2: Computational times (in seconds) for the *complete2* family.

density benchmarks	Graphs density						
	0.4	0.5	0.6	0.7	0.8	0.9	1.0
min	0.91	1.5	1.96	2.94	3.63	13.68	10.37
median	13.58	15.34	106.31	26.65	82.61	327.24	27.2
avg	29.06	38.03	164.58	46.43	215.98	683.23	252.49
stddev	37.33	47.62	173.37	54.04	275.54	734.15	516.4
max	98.11	139.64	493.52	182.67	830.16	1627.78	1552.35

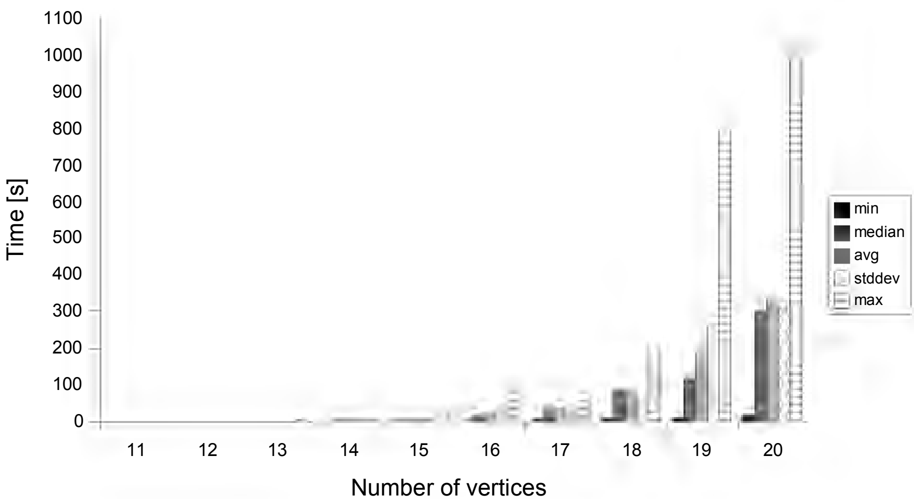
Tablica 3: Computational times (in seconds) for the *density* family.

data benchmarks	Data type		
	boolean	*integer*	*real*
min	16.94	10.37	9.19
median	300.77	27.2	124.14
avg	332.26	252.49	534.27
stddev	309.52	516.4	764.78
max	1042.16	1552.35	1980.44

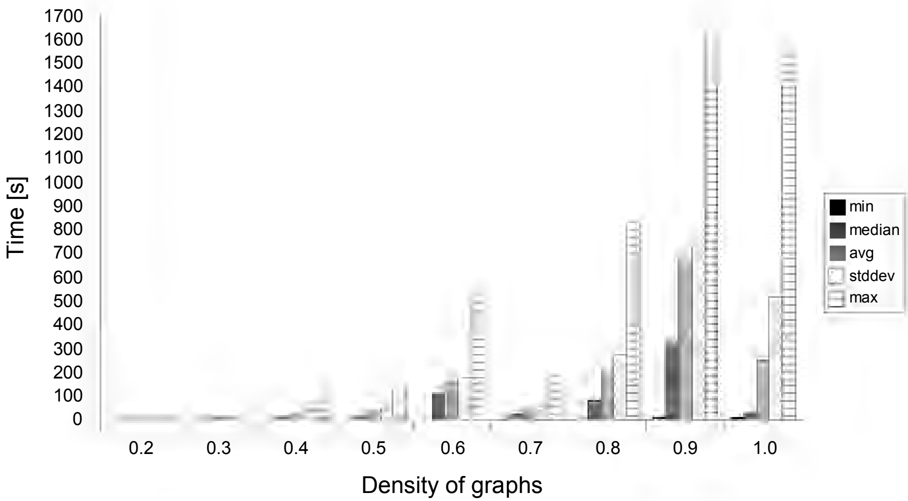
Tablica 4: Computational times (in seconds) for the *data* family.



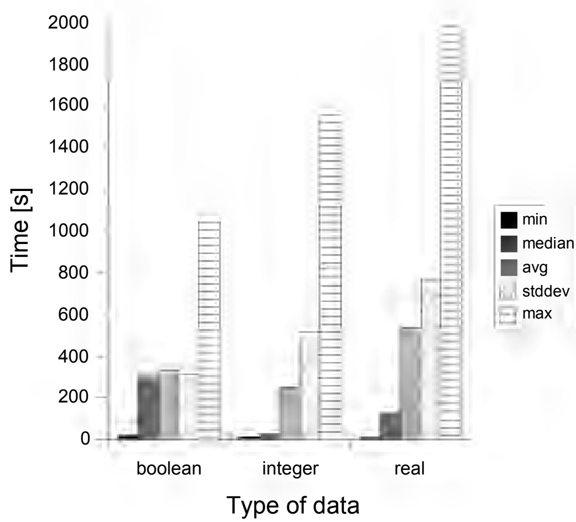
Rysunek 1: Computational times (in seconds) for the *complete1* family.



Rysunek 2: Computational times (in seconds) for the *complete2* family.



Rysunek 3: Computational times (in seconds) for the *density* family.



Rysunek 4: Computational times (in seconds) for the *data* family.

0.2 and 0.3 are omitted). One can observe very fast grow of the computational time as a function of the number of vertices (*complete1* and *complete2* families) and the density (*density* family) of graphs. Only for benchmarks with 18 vertices for the *complete1* family, 17 vertices for the *complete2* family, 0.7 and 1.0 densities for the *density* family, a decrease of the computational time appeared; probably it is related to changes in ILOG CPLEX optimization strategies, but this problem requires further investigations.

In Table 4 and Figure 4 computational results for *data* family of three groups of benchmarks (*integer*, *boolean*, *real*) are summarized. The relations of the computational times for these groups are rather complicated. The *max* and *stddev* values are the smallest for problems from *boolean* group, and the largest for *real* group, but the *median* value appeared the smallest for *integer* group, and the largest for *boolean* group.

Our computational results were rather limited, for example only standard ILOG CPLEX optimization strategies were used, but all the results indicate that the MIP formulation of the adjustment problem for the considered pair of problems (MST and MHP) leads to difficult optimization problems and is limited to graphs with small number of vertices.

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BADANIA OPERACYJNE I SYSTEMOWE: ŚRODOWISKO NATURALNE, PRZESTRZEŃ, OPTYMALIZACJA

Książka składa się z artykułów przedstawiających wyniki prac z dziedziny badań operacyjnych i systemowych, poświęconych środowisku naturalnemu i zarządzaniu nim, zwłaszcza w zakresie ochrony atmosfery, globalnego ocieplenia i walki z nim, jakości i zaopatrzenia w wodę. Tematyka ta jest rozszerzona o aspekty przestrzenne, regionalne i samorządowe, a także planowanie i funkcjonowanie infrastruktury. Tom zamykają prace metodyczne, dostarczające technik, będących podstawą prezentowanych zastosowań.

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