DETERMINATION OF PROPERTIES OF A TUNED MASS ABSORBERS USING A BASIN STABILITY METHOD

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1. Introduction

In mechanical and structural systems the knowledge of all possible solutions is crucial for safety and reliability. Due to nonlinearity, for the same set of parameters more then one stable solution may exist [?, ?]. This phenomenon is called multistability. As an example, we point out the classical tuned mass absorber [?]. This device is well known and widely used to absorb energy and mitigate unwanted vibrations. However, the best damping ability is achieved in the neighbourhood of the multistability zone [?, ?]. Among all coexisting solutions only one mitigates oscillations effectively. Practically, in nonlinear dynamical systems with more then one degree of freedom it is impossible to find all existing solutions without huge effort and using classical methods of analytical and numerical investigation. That is why we use here a new method basing on the idea of basin stability [?].

2. Model of systems and resuluts

The example is a system with a Duffing oscillator and a tuned mass absorber presented in Figure **??**. The main body consists of mass M fixed to the ground with nonlinear spring (hardening characteristic $k_1 + k_2y^2$) and a viscous damper (damping coefficient c_1). The main mass is forced externally by a harmonic excitation with amplitude F and frequency ω. The absorber is modelled as a mathematical pendulum with length l and mass m. A small viscous damping is present in the pivot of the pendulum.

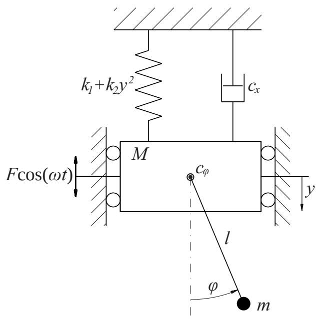

Figure 1: The model of the first considered system.

The dimensionless equations are as follows:

$$\ddot{x} - ab\ddot{\gamma}\sin\gamma - ab\dot{\gamma}^2\cos\gamma + x + \alpha x^3 + d_1\dot{x} = f\cos\mu\tau, \tag{1}$$
$$\ddot{\gamma} - \tfrac{1}{b}\ddot{x}\sin\gamma + \sin\gamma + d_2\dot{\gamma} = 0,$$

where μ is the frequency of the external forcing and we consider it as controlling parameter. The dimensionless parameters have the following values: $f = 0.5$, $a = 0.091$, $b = 3.33$, $\alpha = 0.031$, $d_1 = 0.132$ and $d_2 = 0.02$.

We focus on three solutions. First is $2:1$ periodic oscilations, second is solution where pendulum is in hanging down postion (HDP) and third is $1:1$ rotations (for details see [?, ?]). In Figure **??** we show the probability of reaching the three aforementioned solutions obtained using basin stability method. The initial conditions are random numbers drawn from the following ranges: $x_0 \in [-2,\ 2]$, $\dot{x}_0 \in [-2,\ 2]$, $\gamma_0 \in [-\pi,\ \pi]$ and $\dot{\gamma}_0 \in [-2.0,\ 2.0]$. The frequency of excitation is within a range $\mu \in [0,\ 3.0]$. We take 15 equally spaced subsets of μ and in each subset we calculate the probability of reaching a given solution. For each subset we calculate 1000 trials each time drawing initial conditions of the system and a value of μ from the appropriate range. Then we plot the dot in the middle of the subset which indicate the probability of reaching a given solution in each considered range. Lines that connect the dots are shown just to emphasize the tendency.

In Figure **??** we mark the probability of reaching the $2 : 1$ resonance using blue dots. As we expected, for $\mu < 1.4$ and $\mu > 2.2$ the solution does not exist for details see [**?**]). In the range $\mu \in [1.4,\ 2.2]$ the maximum value of probability $p(2 : 1) = 0.971$ is reached in the subset $\mu \in [1.8,\ 2.0]$ and outside that range the probability decreases. A similar analysis is performed for HDP. The values of probability is indicated by the red dots. As one can see for $\mu < 0.8$, $\mu \in [1.2,\ 1.4]$ and $\mu \in [2.6,\ 2.8]$, the HDP is the only existing solution. The rapid decrease close to $\mu \approx 1.0$ indicates the $1 : 1$ resonance and the presence of other coexisting solutions in this range (see [**?**]). In the range $\mu \in [1.2,\ 1.4]$ the probability $p(\mathrm{HDP}) = 1.0$ which corresponds to a border between solutions born from $1 : 1$ and $2 : 1$ resonance. Hence, up to $\mu = 2.0$ the probability of the HDP solution is a mirror refection of $p(2 : 1)$. Finally, for $\mu > 2.0$ the third considered solution comes in and we start to observe an increase of probability of the rotating solution $S(\mu,\ \mathrm{HDP})$ as shown in Figure **??**(b). However, the chance of reaching the rotating solution remains small and never exceeds $p(1 : 1) = 8 \times 10^{-2}$.

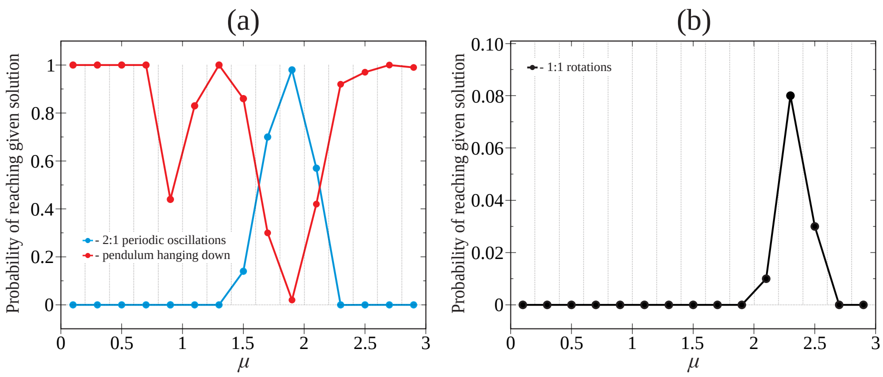

Figure 2: Probability of reaching given solutions in system giben by Eq. (1).

3. Conclusions

The presented method let us detect solutions in nonlinear dynamical systems. It is robust and can be used not only for mechanical and structural systems but also for any system given by differential equations where the knowledge about existing solutions is crucial.

Acknowledgement

This work is funded by the National Science Center Poland based on the decision number 2015/17/B/ST8/03325.

References

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