

# EFFECTS OF NONLINEARITIES IN INERTER ON THE PERFORMANCE OF TUNED MASS DAMPER

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## 1. Introduction

Tuned mass dampers (TMD) are widely used for damping of unwanted oscillations of mechanical and structural systems. In this paper we investigate the effects of adding an inerter to the TMD taking into account the influence of inerter nonlinearities of different types. Inerter has been introduced in early 2000s by Smith [1] and it is a two terminal element which has the property that the force generated at its ends is proportional to the relative acceleration of its terminals.

## 2. Model of the system

Model consists of the base oscillator that can move in vertical direction and the TMD connected to its top (see Fig. 1). The base oscillator has the mass  $M$  and is connected with the support via the spring of stiffness  $K$  and a viscous damper described by the damping coefficient  $C$ . It is excited by a harmonic force of amplitude  $F$  and frequency  $\omega$ . The TMD is used to mitigate the vibrations of the base structure. The TMD has the mass  $m$  and is connected with the main body via four links: spring of stiffness  $k$ , viscous damper with damping coefficient  $c$ , element that corresponds to dry friction described by parameter  $d_f$  and the last one is the inerter. We use the model of inerter with play that is described by four parameters: inertance  $I$ , stiffness  $k_i$ , viscous damping coefficient  $c_i$  and dimension of the backlash gap  $\varepsilon$ .

The motion of the system is described by four generalized coordinates two of which are used to describe the dynamics of inerter with play. The vertical displacement of the base oscillator is given by coordinate  $x$ . To describe the position of the TMD we use coordinate  $y$ . The model of play uses coordinates  $r$  and  $u$ , where  $r$  describes the distance between the two nodes of inerter while  $u$  defines the actual gap in the system.

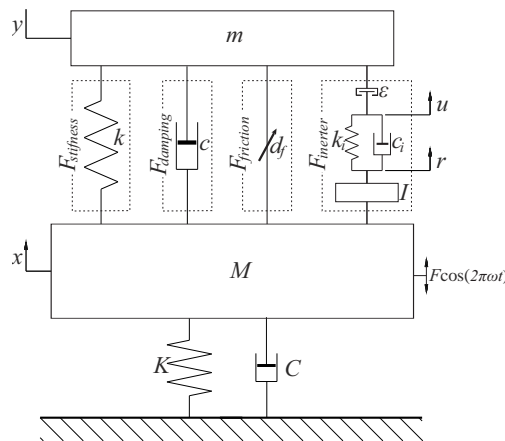


Figure 1: The model of the considered system and notation of system’s parameters.

Using Lagrange equations of the second type we obtain the system’s equations of motion:

$$(1) \quad M\ddot{x} + Kx + C\dot{x} + F_{stiffness} + F_{damping} + F_{friction} + F_{inerter} = F \cos(\omega t),$$

$$(2) \quad m\ddot{y} - F_{stiffness} - F_{damping} - F_{friction} - F_{inerter} = 0,$$

### 3. Full model of TMD with inerter

We compare the effects introduced by each element and investigate how they changes in the presence of the others. Results are presented in Figure 2 are calculated for  $I_{ratio} = 0.1$ ). In Figure 2 (a) we show 5 FRC curves calculated for the following conditions: (1)  $c = 0 [\frac{Ns}{m}]$ ,  $d_f = 0 [N]$ ,  $\varepsilon = 0 [m]$ ,  $k_i \rightarrow \infty [\frac{N}{m}]$ ,  $c_i = 0 [\frac{Ns}{m}]$  - (black line marked as "1"). (2)  $c = 0 [\frac{Ns}{m}]$ ,  $d_f = 0 [N]$ ,  $\varepsilon = 0.0001 [m]$ ,  $k_i = 2 \cdot 10^7 [\frac{N}{m}]$ ,  $c_i = 0.01 [\frac{Ns}{m}]$  - (yellow line marked as "2" (overlapped by the black line)). (3)  $c = 10 [\frac{Ns}{m}]$ ,  $d_f = 0 [N]$ ,  $\varepsilon = 0 [m]$ ,  $k_i \rightarrow \infty [\frac{N}{m}]$ ,  $c_i = 0 [\frac{Ns}{m}]$  - (blue line marked as "3"). (4)  $c = 0 [\frac{Ns}{m}]$ ,  $d_f = 10 [N]$ ,  $\varepsilon = 0 [m]$ ,  $k_i \rightarrow \infty [\frac{N}{m}]$ ,  $c_i = 0 [\frac{Ns}{m}]$  - (red line marked as 4) (5)  $c = 10 [\frac{Ns}{m}]$ ,  $d_f = 10 [N]$ ,  $\varepsilon = 0.0001 [m]$ ,  $k_i = 2 \cdot 10^7 [\frac{N}{m}]$ ,  $c_i = 0.01 [\frac{Ns}{m}]$  (green line marked as 5).

Comparing the FRCs from Figure 2 (a) we see that the effects caused by viscous damping and dry friction are qualitatively the same while the play itself does not cause macroscopic changes to the shape of FRC. In literature authors often use simple model of TMD with viscous damper because it is simple and mathematically convenient. Our results prove that such model can also well simulate the behaviour of TMDs with inerters but only when viscous damping coefficient and dry friction parameters are relatively small (see previous sections). Practically this means that simplified model is sufficient to model the device without additional damper (TMD which consists only of mass connected with the main body via spring and inerter.

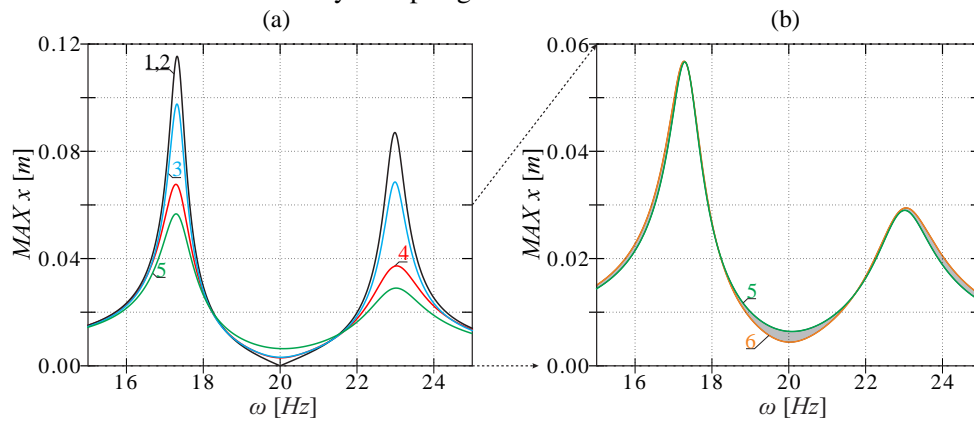


Figure 2: Comparison between the effects caused by four investigated elements. In subplots (a) we present FRC calculated for models with each investigated factor and the full model that contains all of them. In subplots (b) we magnify and compare the results obtained for the full model (green curves) and the simplified model (orange curves).

### 4. Conclusions

We analyzed the full model of TMD that contains all four factors. For the investigation we pick parameter values that corresponds to TMD with typical inerter. So we assume relatively small energy dissipation via viscous damping and dry friction, and reference play gap. After comparing the results we can say that the effects introduced by viscous damping and dry friction are qualitatively comparable while play has not macroscopic influence on the system's dynamics. For details see full paper [2]

### Acknowledgements

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### References

- [1] M.C. Smith. Synthesis of mechanical networks, the inerter. *Automatic Control, IEEE Transactions on*, 47(10), 1648–1662 (2002).
- [2] P. Brzeski, P. Perlikowski. Effects of play and inerter nonlinearities on the performance of tuned mass damper. *Nonlinear Dynamics* 88 (2), 1027-1041 (2017).