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Random yield limit of stochastically non-homogeneous elements in tension

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THE CORRECTED strength of a rod in tension R_d' is defined as the quantile (1.4) of the random yield limit of the rod, the parameters of probability distributions $\bar{R}'$, v_R' differing from the parameters $\bar{R}$, v_R of probability distributions of the yield limit measured in standard specimens owing to the stochastic non-homogeneity of the material. The size factor m_s, i.e., the ratio of R_d' to the conventional value of R_d is formulated (2.6) by means of a modified Weibull theory. An improved formula for the size factor is then derived (3.9) taking into account the autocorrelation of the local yield limit (3.1) and assuming it to be a stationary log-normal stochastic function.

Skorygowaną wytrzymałość obliczeniową pręta rozciąganego R_d' określa się jako kwantyl (1.4) losowej granicy plastyczności pręta, przy czym parametry rozkładu prawdopodobieństw $\bar{R}'$, v_R' różnią się od parametrów $\bar{R}$, v_R, rozkładu prawodpodobieństw granicy plastyczności, mierzonej na znormalizowanych próbkach, na skutek stochastycznej niejednorodności materiału. Współczynnik skali m_s czyli stosunek R_d' do konwencjonalnej wartości R_d dla pręta o długości L i przekroju A sformułowano (2.6) na podstawie zmodyfikowanej teorii Weibulla. Następnie wyprowadzono dokładniejszy wzór na współczynnik skali (3.9), uwzględniając autokorelację lokalnej granicy plastyczności (3.1) i zakładając, że jest ona stacjonarną logarytmo-normalną funkcją stochastyczną.

Исправленная расчетная прочность растягиваемого стержня $\bar{R}_d'$ определяется как квантил (1.4) случайного предела пластичности стержня, причем параметры распределения вероятностей $\bar{R}'$, v_R' отличаются от параметров $\bar{R}$, v_R распределения вероятностей предела пластичности измеренного на стандартных образцах, вследствие стохастической неоднородности материала. Масштабный фактор m_s, т. е. отношение R_d' к обыкновенному значению R_d для стержня длины L и сечения A сформулирован (2.6) на основе модифицированной теории Вейбулла. Затем выведена более точная формула для масштабного фактора (3.9) учитывая автокорреляцию локального предела пластичности (3.1) и предполагая, что является он стационарной логарифмо-нормальной стохастической функцией.

1. Deterministic and stochastic non-homogeneity

A REALISTIC estimation of elastic-plastic deformation or the load carrying capacity of structures often requires that the material non-homogeneity be taken into account in static calculations. W. OLSZAK with his co-workers [10] indicated and classified the most important sources of non-homogeneity of structural materials and solved numerous boundary value problems of mechanics of non-homogeneous bodies which had been earlier limited to perfectly homogeneous media. In the new solutions, it was usually assumed that the elastic and plastic moduli were completely defined functions of position within the body, continuous or stepwise, whose parameters could be specified on the basis of experimental investigations of the structural material. The main trend of development of non-homogeneous media mechanics on the basis of deterministic formalism was accompanied by a series of papers dealing with the statistical or microscopic non-homogeneity [5], where the mechanical properties of materials were characterized by random — i.e., stochastic functions of position in the body. Parameters of the stochastic functions may be specified on the basis of statistical testing of specimens made of the material to be used in the structure.

Both the deterministic and statistical approaches to the problems of material non-homogeneity extend the class of solutions which may be used in practical design of structures. They also make possible to formulate a number of questions of theoretical interest. For instance, in solving the problems of functional nonhomogeneity of elastic-plastic plates in bending [8], the problem arises as to under what conditions the stresses remain continuous at the interface between the elastic and plastic regions, and how the problem is influenced by compressibility, incompressibility or plastic orthotropy of the material.

Another question consists in determining the types of non-homogeneity (continuous, layered, axi-symmetric etc.) of an elastic-plastic wedge loaded by a concentrated force [9] for which the stress distribution is qualitatively the same as in an elastic homogeneous medium (i.e. it remains radial). A very important and interesting problem of the theory of stochastically non-homogeneous media is the problem of what is called size effect, which may be explained shortly as the problem of dependence of unit strength upon the volume of the body. More generally, the problem consists in finding the deviations of the mean values of deformation or load carrying capacity from the values following from the well-grounded laws of model similarity. The present paper offers a certain theoretical contribution to the problem of size effect. Our considerations are confined, however, to the very simple case of a prismatic element acted upon by an axial force of extension.

In this manner, we shall concentrate our attention on the most important problems; moreover, such a method proves to be suitable from the point of view of practical calculations. The current stage of discussion concerning the necessary modifications of standard methods of dimensioning the building structures is connected with the introduction of simplified, semiprobabilistic methods, in particular the limit state method [3]. According to this method, separate, partial safety factors are introduced, such as the non-homogeneity factor k which enables determination of the design strength R_d in terms of the characteristic strength R_k,

$$R_d = kR_k. \tag{1.1}$$

For structural steels, the characteristic strength R_k equals what is called "minimum yield limit" which is determined for any kind of steel by the corresponding metallurgical standards.

The factor k is given by the formula:

$$k = \frac{\overline{R}}{R_k}(1-3v_R), \tag{1.2}$$

where $\overline{R}$ is the mean value of the yield limit and v_R — the coefficient of variability of the yield limit. Parameters $\overline{R}$, v_R are determined in standard laboratory tests.

In the case of a normal — that is, Gaussian yield limit of the material — definition (1.2) allows for interpretation of the design strength as a distribution quantile on the probability level

$$\omega = 1.35‰.$$

The value of the design strength can be established more precisely by introducing the size factor m_s:

$$R'_d = m_s R_d. \tag{1.3}$$

According to this suggestion, the size factor would introduce certain corrections to the design strength of rods subject to tension and having dimensions different from those which served to measure the yield limit of the material. The corrected design strength is also defined as a quantile, and hence in the case of normal probability distribution and $\omega = 1.35‰$.

$$R'_d = \overline{R'}(1-3v'_R), \tag{1.4}$$

though here $\overline{R'}$, v'_R are distribution parameters of the random carrying capacity of the entire structural element.

The problem consists in deriving the formulae for calculating the parameters $\overline{R'}$, v'_R, and consequently the size factor m_s, on the basis of known values of the strength distribution parameters of the specimens and of the dimensions of structural elements. Such formulae are obtained by means of the analysis of random non-homogeneities of the material. The well-known WEIBULL and other formulae concern the problem of strength of brittle materials [2]; they express the design strength in terms of the volume of the structural element. The size effect of ductile materials such as structural steels requires a separate theory. Paper [7] modifies the Weibull theory by taking into account the fact that in the critical, weakest cross-section of the rod in tension made of ductile, elastic-plastic material, a full redistribution of stresses (to the limit random level) precedes the plastic flow. These assumptions imply that the expected strength of a steel rod decreases with its length and the strength variance is smaller for larger cross-sectional areas. In that paper, the formulae were derived for the scale factor of alloy treated steels, as also its values were established for the L-beams most frequently applied in support structures of electric transmission lines. The corresponding study was prepared for the "Energoprojekt" design office as a part of a larger project concerning the probabilistic justification of safety factors; it requires further continuation, since the size factor calculated from the modified Weibull hypothesis for ductile materials follow from a very simplified theoretical model and do not take into account the yield limit autocorrelation in the neighbouring points of the medium. Thus it is a singular case of discrete non-homogeneity in which the yield limit assumes stepwise, stochastically independent values at separate points of the medium. Attempts were made to introduce a more general model by assuming the yield limit to be stationary stochastic function with a non-degenerated autocorrelation function. The material non-homogeneity was assumed to be influenced by (in addition to random factors) systematic factors following from different plastic hardening occurring in rolled profiles of different dimensions.

2. Size factor according to the modified Weibull hypothesis

The Weibull theory is now considered as a classical theory presented in numerous handbooks [1, 2, 6] and is concerned with the size effect in problems of strength of brittle materials, in which "the weakest link effect" may be justified — i.e., the hypothesis according to which the strength of the weakest point determines the strength of the entire body.

Modification of the theory for rods made of ductile materials [7] leads to the conclusion that the scale factor is not a function of volume V of the body, like in Weibull's original theory, but depends on two variables: cross-sectional area A and length L of the rod.

Without going into detailed derivations let us quote the final formula for the distribution function of strength R of a steel rod. This is a Weibull type distribution — i.e., extremum distribution

$$(2.1)\qquad \begin{aligned} F(R) &= 0 \quad \text{for} \quad R < 0,\\ F(R) &= 1-\exp\left(-\sqrt[u]{R/\check{R}}\right) \quad \text{for} \quad R \geqslant 0, \end{aligned}$$

where the distribution parameters are: u — Weibull variability index, $\check{R}$ — the characteristic minimum of strength.

The parameters are expressed in terms of the normal variability coefficient v and the expected strength of the rod $\bar{R}$ in the following manner:

$$\frac{\Gamma(1+2u)}{\Gamma(1+u)} = 1+v, \text{ or approximately } u \approx 0.8v,$$

$$\check{R}\Gamma(1+u) = \bar{R}, \quad \text{or approximately } \check{R} \approx \bar{R}(1-0.5u).$$

$\Gamma(x)$ is the Euler gamma function tabulated in [12]. The approximate formulae can be applied for small values of $v < 15\%$ which is true e.g., in steel.

Parameters v, R depend on the rod dimensions A, L, as follows from the Weibull theory [7], in the following manner:

$$(2.3)\qquad v = v_0\sqrt{A_0/A}, \quad \bar{R} = \bar{R}_0(L_0/L)^u,$$

where A_0 and L_0 are the dimensions of a hypothetical, elementary grain of the material, and $\bar{R}_0$ — the mean yield limit of the grains.

The elementary grains in Weibull's theory do not coincide with the grains of the metallographic steel structure and should be understood as regions whose mechanical properties are stochastically independent of the properties of other regions. The values of L_0, A_0, $\bar{R}_0$, v_0 is determined indirectly on the basis of two series of tension tests performed on steel specimens of dimensions A_1, L_1 and A_2, L_2, respectively. After estimating the mean values $\bar{R}_1$, $\bar{R}_2$ and the Weibull variability indices u_1, u_2 in these tests, parameters L_0, $\bar{R}_0$ are found from the formulae:

$$(2.4)\qquad L_0 = L_1\sqrt[u_1-u_2]{(\bar{R}_1/\bar{R}_2)(L_1/L_2)^{u_2}}, \quad \bar{R}_0 = \bar{R}_1(L_1/L_0)^{u_1},$$

Parameters A_0, v_0 do not have to be determined separately; it suffices to evaluate the expression:

$$(2.5)\qquad v_0\sqrt{A_0} = v_1\sqrt{A_1} \quad \text{or} \quad v_0\sqrt{A_0} = v_2\sqrt{A_2}.$$

The equality $v_1\sqrt{A_1} = v_2\sqrt{A_2}$ = const serves as a control test of the theory. Obviously, small discrepancies may occur following from the fact that v_1, v_2 are estimated on the basis of a limited number of statistical tests; then $v_0\sqrt{A_0}$ should be averaged.

The form of Eq. (2.4) is simpler than that given in [7], the formulae being equivalent and obtained by algebraic transformations. The formulae will be used to specify the Weibull distribution parameters, starting from two tension tests series performed on steel 18G2A specimens described in [4].

Fivefold flat specimens (according to Polish Standards PN-62/H-04310):

$$A_1 = 20\times 30 = 600\,\text{mm}^2, \qquad L_1 = 140\,\text{mm},$$
$$A_2 = 27{,}5\times 30 = 825\,\text{mm}^2, \qquad L_2 = 160\,\text{mm}.$$

Results (empirical mean, standard deviation, normal variability coefficient):

$$\overline{R}_1 = 38{,}31\,\text{kG/mm}^2, \quad s_1 = 3{,}72\,\text{kG/mm}^2, \quad v_1 = 0{,}0710,$$
$$\overline{R}_2 = 38{,}87\,\text{kG/mm}^2, \quad s_2 = 2{,}49\,\text{kG/mm}^2, \quad v_2 = 0{,}0641.$$

Checking the control condition $v_1\sqrt{A_1} = v_2\sqrt{A_2}$:

$$\left.\begin{aligned} 0.0710\sqrt{600} &= 1.74,\\ 0.0641\sqrt{825} &= 1.84,\end{aligned}\right. \qquad \text{mean value } 1.79\,\text{mm}.$$

Averaged normal variability coefficients

$$v_1 \approx 0.0710\cdot\frac{1{,}79}{1{,}74} = 0{,}0731,$$

$$v_2 \approx 0{,}0641\cdot\frac{1{,}79}{1{,}84} = 0.0623$$

and Weibull's variability coefficients

$$u_1 \approx 0.8\times 0{,}0731 = 0.0585,$$
$$u_2 \approx 0{,}8\times 0{,}0623 = 0.0498.$$

Parameters of "elementary grains"

$$L_0 = 140\left[\frac{38.31}{38.87}\left(\frac{140}{160}\right)^{0.0498}\right]^{\frac{1}{0.0598-0.0498}} = 140[(0.984(0.876)^{0.0498}]^{115} =$$
$$= 140\times 0{,}984^{115}\times 0.876^{5.73} = \mathbf{10.2\ mm.}$$
$$\overline{R}_0 = 38.31\left(\frac{140}{10.2}\right)^{0.0585} = 38.31\times 13.7^{0.0585} = \mathbf{44.6\ kG/mm^2.}$$

This example enables us to construct the approximate Table of scale factors for the alloy treated steel 18G2A. Unfortunately, the corresponding experimental data concerning low carbon structural steels St0S, St3S are not yet available.

In the Table are given some values of the scale factor m_s for L-beams most frequently applied in support structures of electric transmission lines, of various lengths L, and for the quantile value $\omega = 1.35\times 10^{-3}$, which correspond to the strength calculated according to the limit state method.

The size factor is formulated in the following manner:

$$m_s = \frac{\bar{R}_0}{R_d}\left(\frac{L_0}{L}\right)^{u'} \frac{\left(\ln\frac{1}{1-\omega}\right)^{u'}}{\Gamma(1+u')} \approx \frac{\bar{R}_0}{R_d}\left(\frac{L_0}{L}\right)^{u'} \frac{\omega^{u'}}{1-0.5u'}, \tag{2.6}$$

where

$$u' \approx 0.8v' = 0.8\sqrt{v^2+v_A^2} = 0.8\sqrt{\frac{v_0^2 A_0}{A}+v_A^2}$$

is the global Weibull variability index of the random yield limit and random cross-sectional area (varying due to the dimension tolerance).

$R_d = \breve{R}_1 \exp(-t_\omega \nu_1') \approx \bar{R}_1(1-t_\omega v_1')$ is the conventional design strength determined for log-normal distributions of the yield limit for the steel specimens with parameters $LN(\breve{R}_1, \nu_1)$, with the random deviations of the cross-sectional area of the rod $\nu_1' = \sqrt{\nu_1^2+\nu_A^2}$, and for the safety level $1-\omega$; here $(1/2)+(1/2)\mathrm{erf}(t_\omega/2) = 1-\omega$, $\mathrm{erf}\,x$ denotes the error function, t_ω is the normal tolerance coefficient.

Equation (2.6) is derived from the assumption that the effective strength R_d' corrected by the Weibull theory is the quantile of the minimum strength distribution of the rod cross-sections, at a fixed length of the rod L and random cross-sections A,

$$F(R_d') = \omega, \quad R_d' = m_s R_d. \tag{2.7}$$

Table 1. Size factors according to the modified Weibull hypothesis for single angle beams made of 18G2A steel; quantile level = 0.00135

Profile dimensions (mm) and area (cm²) / Length L (cm)	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000
L 35×35×4 2.67	0.519	0.509	0.500	0.490	0.484	0.480	0.475	0.471	0.466	0.462	0.458	0.455	0.453	0.451	0.449	0.447	0.454
L 40×40×4 3.08	0.552	0.541	0.531	0.524	0.518	0.516	0.512	0.506	0.501	0.498	0.495	0.493	0.490	0.486	0.485	0.481	0.480
L 45×45×4 3.48	0.611	0.600	0.590	0.585	0.581	0.573	0.569	0.563	0.560	0.559	0.553	0.551	0.549	0.545	0.543	0.540	0.540
L 50×50×4 3.89	0.602	0.593	0.581	0.576	0.570	0.567	0.560	0.558	0.552	0.550	0.548	0.543	0.541	0.539	0.537	0.535	0.531
L 45×45×5 4.29	0.631	0.623	0.616	0.609	0.601	0.599	0.592	0.591	0.584	0.581	0.578	0.575	0.572	0.570	0.568	0.565	0.563
L 60×60×6 6.91	0.744	0.731	0.725	0.719	0.711	0.709	0.701	0.699	0.696	0.692	0.690	0.684	0.682	0.681	0.680	0.677	0.673
L 75×75×5 7.38	0.749	0.740	0.730	0.724	0.720	0.712	0.710	0.705	0.702	0.700	0.697	0.693	0.686	0.682	0.680	0.678	0.678
L 100×100×10 19.2	0.916	0.908	0.898	0.891	0.889	0.884	0.880	0.875	0.871	0.869	0.865	0.863	0.861	0.858	0.856	0.855	0.854
L 120×120×10 23.3	0.965	0.955	0.950	0.944	0.939	0.935	0.931	0.927	0.925	0.920	0.918	0.915	0.914	0.911	0.910	0.907	0.905

It should be mentioned that the simple method of taking into account the random deviations of the cross-sectional area is accurate provided:

(a) the random cross-section A is stochastically independent of the yield limit R of the cross-section;

(b) the least distance between the cross-sections whose areas are stochastically independent equals the size L_0 of the elementary grain determined from the analysis of the variability of random yield limits;

(c) the variability indices v (normal) and ν (log-normal) are small; then $v \approx \nu$.

3. Size factor according to the hypothesis of log-normal stochastic functions

The applications of stationary stochastic functions to the problems of dynamic loads are well-known; more precisely, the applications concern the calculation of the probability of exceeding a definite level by the random loading process [11]. The solution will now be transferred to the calculation of exceeding a definite stress level by the stochastic function of yield limits; the deterministic argument is not the time (thus we are not faced with a stationary random process) but the coordinates of a material point of the medium whose strength is being calculated; we are dealing with a homogeneous random field.

Let us introduce the following assumptions specified for the case of calculation of strength of thin-walled steel elements subject to tensile forces.

(a) The single phase (i.e., at one point of the medium) probability distribution of yield limits R is log-normal with parameters $LN(\breve{R}_g, \nu)$. This assumption is equivalent to the assumption that the logarithms of yield limits have a Gaussian distribution with parameters $N(\overline{\ln R} = \ln \breve{R}_g, v)$, $\breve{R}_g$ denoting the median of the local yield limit.

(b) The autocorrelation function of the yield limit is isotropic in the middle planes of walls of the steel element — i.e., it depends on the distance y of points in the plane (measured after developing the walls). The hypothetical form of the correlation function is assumed in the form

$$K(y) = \nu_g^2 \exp\left(-\frac{2y^2}{\pi^2 \delta_g^2}\right), \tag{3.1}$$

where δ_g is the so-called effective period of return of the expected value given by the formula

$$\delta_g = -\pi \left[K(y) \bigg/ \frac{d^2K(y)}{dy^2}\right]_{y\to 0}, \tag{3.2}$$

which is easily found to coincide with Eq. (3.1).

Making no assumptions concerning the autocorrelation in the third dimension (across the wall thickness), we presume that the statistical investigation will proceed separately for each thickness g, and the parameters R_g, ν_g, δ_g will be determined according to the thickness. It is known that the size effect for steel sheets of various thickness is strongly influenced by technological factors (different rates of cooling after rolling, influence of plastic deformation etc.) and may be estimated empirically;

(c) The mechanism of fracture of the rod has a plastic flow character — i.e., all points of the cross-section must reach the yield limit, but yielding of a single cross-section is equivalent to fracture of the entire rod. We confine ourselves to plane cross-sections perpendicular to the axis of the rod.

The expected value of the yield limit logarithm in a cross-section with global length of walls $2a$ equals the expected value of the logarithm of the local yield limit, $\overline{\ln R} = \ln \breve{R}_g$, and the variance of the yield limit in the cross-section is calculated from the formula [11],

$$(3.3)\qquad \nu^2 = \frac{1}{a}\int_0^{2a}\left(1-\frac{y}{2a}\right)K(y)\,dy = \frac{\nu_g}{a}\int_0^{2a}\left(1-\frac{y}{2a}\right)\exp\left(-\frac{2y^2}{\pi^2\delta_g^2}\right)dy$$

$$= \nu_g^2\left[\left(\frac{\pi}{2}\right)^{3/2}\frac{\delta_g}{a}\,\mathrm{erf}\frac{2\sqrt{2a}}{\pi\delta_g} + \frac{\pi^2\delta_g^2}{8a^2}\left(1-\exp\left(-\frac{8a^2}{\pi^2\delta_g^2}\right)\right)\right],$$

whence (Fig. 1) follows the formula:

$$(3.4)\qquad \nu \approx \nu_g\sqrt{1{,}98\frac{\delta_g}{a} + 1{,}24\left(\frac{\delta_g}{a}\right)^2} \approx 1{,}4\nu_g\sqrt{\frac{\delta_g}{a}}$$

for $\delta_g/a \ll 1$.

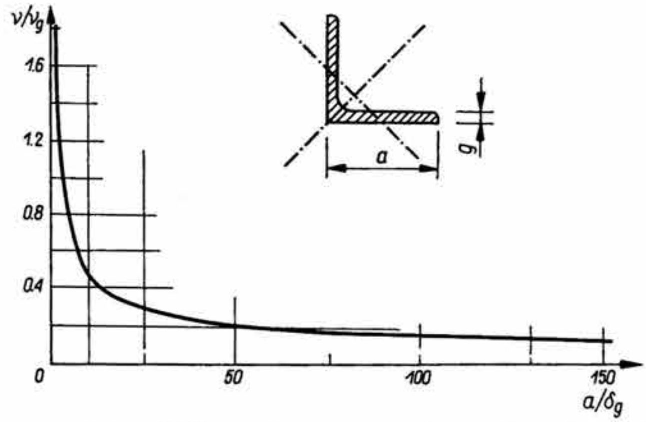

FIG. 1. Dependence of the variability coefficient ν on the width a of legs of an steel angle beam.

The risk of exceeding an arbitrarily settled value R' by a usually higher stationary log-normal stochastic function of the yield limit $R(x)$ is, according to the Rice formula [11]

$$(3.5)\qquad r = \frac{1}{\delta_g}\exp\left[-\frac{(\ln\breve{R}_g - \ln R')^2}{2\nu^2}\right] = \frac{1}{\delta_g}\exp\left[-\frac{\ln^2(\breve{R}_g/R')}{2\nu^2}\right],$$

and the probability of not exceeding R' on the length L, according to the "reliability at a constant risk" formula [6]

$$(3.6)\qquad F(R) = P[\ln R(x) < \ln R' | 0 \leqslant x \leqslant L] = 1-\exp\left\{-\frac{L}{\delta_g}\exp\left[-\frac{\ln^2(\breve{R}_g/R')}{2\nu_g^2}\right]\right\}$$

$$= 1-\exp\left\{-\exp\left[\ln\frac{L}{\delta_g} - \frac{\ln^2(\breve{R}_g/R')}{2\nu_g^2}\right]\right\}.$$

This is the formula for the yield limit distribution function R of a rod of length L. It contains three parameters $\breve{R}_g$, ν_g, δ_g for each thickness g of the L-profile. The parameters may be determined from two series of tests performed on specimens having different cross-sectional areas and lengths.

The calculation strength containing the scale coefficient follows from equating the distribution function to the probability assumed:

$$F(R_d') = \omega.$$

Solution of this equation for R_d' (Fig. 2) yields

$$R_d' = \breve{R}_g e^{-\lambda\nu}, \tag{3.7}$$

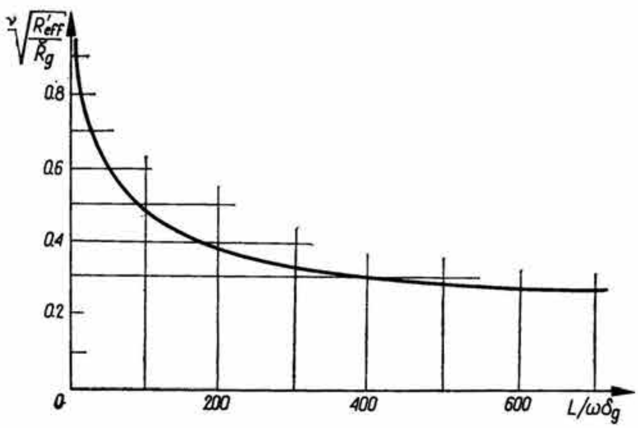

FIG. 2. Dependence of the design strength R_d' on the length L of a steel rod in tension.

where

$$\lambda = \sqrt{2}\sqrt{\ln\frac{L}{\delta_g} - \ln\ln\frac{1}{\lambda-\omega}} \approx \sqrt{2\ln\frac{L}{\delta_g\omega}} = 2.14\sqrt{\lg(L/\delta_g\omega)}.$$

The size factor, defined as the ratio of the corrected and conventional design strengths

$$m_s = R_d'/R_d \tag{3.8}$$

for log-normal distributions of yield limits R_1 measured on standard specimens, is expressed by

$$m_s = \frac{\breve{R}_g}{R_1} e^{-\lambda\nu + t_\omega \nu_1}. \tag{3.9}$$

t_ω denoting the normal tolerance factor.

Taking into account the fact that the cross-sectional area $A(x)$ is a stationary stochastic function of x, independent of the yield limit $R(x)$, the log-normal variability indices are corrected

$$\nu' = \sqrt{\nu^2 + \nu_A^2}, \quad \nu' = \sqrt{\nu_1^2 + \nu_A^2}, \tag{3.10}$$

Here ν_A is the log-normal variability index of the cross-sectional area.

The size effect analysis presented here is based on more general assumptions than in the Weibull theory, and hence the results should enable a better fitting of the theoretical formulae to experimental data.

Some examples of the scale factors calculated from the modified Weibull hypothesis (Table 1), as also the parameters derived from the theory of stationary stochastic functions (Figs. 1, 2), indicate that the scale effect is rather considerable and should not be disregarded in practical calculations: it might be responsible for decreasing the safety factor below the conventionally predicted values, as was actually observed in full-scale tests performed on certain steel support structures.

Analysis of stochastic nonhomogeneities of materials is closely connected with the probabilistic theory of safety and methods of structural design which take into account differentiated safety requirements, depending on the destination of the particular structure, it may be used for converting the effective strength to other safety classes.

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