INSTYTUT BADAŃ SYSTEMOWYCH
POLSKIEJ AKADEMII NAUK

# TECHNIKI INFORMACYJNE
# TEORIA I ZASTOSOWANIA

Wybrane problemy
Tom 5 (17)

*poprzednio*

ANALIZA SYSTEMOWA W FINANSACH
I ZARZĄDZANIU

Pod redakcją
Andrzeja MYŚLIŃSKIEGO

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# EUROPEAN OPTION PRICING BY NON-LINEAR SCHRÖDINGER EQUATION

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**Abstract.** Heisenberg uncertainty principle is defined for the economic world. We want to find out why there are difficulties in forecasting stock and option prices at the basis of historical data. We will consider stochastic noise that is determined by human based psychological inclinations. As a improvement of our last, linear based Schrödinger model, we would like to propose new model based on nonlinear Schrödinger equation. Model was calibrated using market data. Calibration was performed using Levenberg-Marquardt algorithm.

**Keywords:** European option pricing, Black-Scholes equation, linear and non-linear Schrödinger equation, Heisenberg's uncertainty principle, econophysics.

## 1 INTRODUCTION

Options are usually priced using Black-Scholes equation. This non-linear parabolic equation is based on geometric Brownian motion model of the stock price stochastic process. Similar processes appear also among quantum particles and are described by time-dependent Schrödinger equation. In this paper, we would like to consider non-linear Schrödinger equation in terms of option pricing.

### 1.1 Option pricing using linear Schrödinger equation

In our previous papers [27] we made assumption that option price is given by equation below:

$$\psi(y(t),t) = \exp(-kt - \frac{2\sqrt{(}r/2-k)}{\sigma}y(t) + g), \tag{1}$$

with y(t) defined as:

$$y(t) = \ln(S(t)) - (r - \frac{\sigma^2}{2})t, \tag{2}$$

where $S(t)$ is an onset price, $k$ stands for particle expected energy and $\sigma$ is the asset price (stock) volatility [5]. Constant $r$ is the rate of return [22] and parameter $g$ is a constant (obtained by data calibration). Our previous calculations showed that Black-Scholes [2] equation for economy is the same as Schrödinger [26] equation for free particle that interacts with constant potential [15]. From physical point of view, this gives interesting interpretation on the basis of quantum physics.

The assumption that free particle (for example electron in hydrogen atom) reacts with external environment by a constant potential is a rough approximation. We know that constant potential, is reserved for interacting free particle with barrier potential. Thus we think that Schrödinger equation that stands for the option pricing, needs to have more complicated form, especially the potential energy that describes relation between particle and the entourage. Nevertheless, if described Schrödinger equation varies from real form, it gives additional information about market forecasting. Especially about trader leverage for the market. We think that one of the reasons that we cannot predict stock or option price at the basis of historical data, is the influence of measuring impact (trader or market participant) on measured object (stock, option). In physics this is well known as Uncertainty principle [1].

### 1.2 Econphysics uncertainty principle

The uncertainty principle appeared in a paper by Werner Heisenberg [16], a German physicist who was working at Neils's institute in Copenhagen. He wrote a paper "On the Perceptual Content of Quantum Theoretical Kinematics and Mechanics". The more familiar form of the equation came a few years later when he had further refined his thoughts in subsequent lectures and papers. The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of the uncertainties of these two measurements. There is likewise a minimum for the product of the uncertainties of the energy and time. It can be described as follows:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{3}$$

or:

$$\Delta E \Delta t \geq \frac{\hbar}{2}, \tag{4}$$

where $\Delta x$ is measurement uncertainty for position, $\Delta p$ is measurement uncertainty for momentum, $\Delta E$ is measurement uncertainty for energy and $\Delta t$ is measurement uncertainty for time.

The uncertainty principle says that we cannot measure the position $x$ and the momentum $p$ of a particle with absolute precision. The more accurately we know one of these values, the less accurately we know the other. Multiplying together the errors in the measurements of these values, has to give a number greater than or equal to half of $\hbar$ [11] reduced Planck constant. This is not a statement about the inaccuracy of measurement instruments nor a reflection on the quality of experimental methods; it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things. In physics the Heisenberg uncertainty principle asserts a limit in our ability to simultaneously know certain facts, such as the position and speed of a particle. The theory that captures this idea is a probabilistic theory. In quantum mechanics only the probabilities of outcomes can be known in advance. However, from economy point of view, much is made of the inability to forecast the current economy crisis. On the contrary, it is a fundamental principle that there can be no reliable way of predicting a crisis. The analogy with physics is instructive.

The Heisenberg uncertainty principle arises because the observer interferes with the system. This is more pronounced in economics: an analyst whose forecasts are believed in will have an impact on the behaviour of the people he is analysing. This can be called uncertainty principle in economics. There exists a maligned theory of rational expectations. It is the tool that economists use to account for the relationship between analyst and analyzed. A reliable method of predicting a crisis must require that anyone (or at least anyone with the requisite technical expertise) can apply and reach the same correct conclusion using the same method. The uncertainty principle in economics arises from the fact that we are all actors in the economy and the models we use determine our behavior. If a model is discovered to be correct, then we will change our behaviour to reflect our new understanding and when enough number of us does so, the original model stops being correct. We assume that human behaviour bears the markings of uncertainty. Forecast made by analyst will have an impact on the decisions made by the other traders. We want to determine which factors taken from the market can show us how to measure that impact. Then we want to apply it in our theoretical model to predict asset price and related option price.

Some major fluctuations related to asset price can be determined by natural hazards like volcano eruption, floods, tornadoes. But in that case humans don't have any control on that. Other example that has huge impact on market are political events like the Ukrainian crisis.

In this paper we want to concentrate on the impacts that are rather analyst related than related to politics or natural hazards.

Analysts can give recommendation, suggestion that can be used to make decision if trader should keep, sell or buy selected trade. We can say that their decisions has huge impact on trader's decisions and on the whole market. Analyst can give few recommendations. Typical recommendations are: positive, negative and neutral. We have provided similar to above principle which occurs in Econophysics. We performed our calculations using atomic units (so where $h$ constant is equal to 1). As we said before, Black-Scholes transformation to Schrödinger equation gave additional interpretation, that $k$ can be treated as system energy. We can give Econophysic's uncertainty principle definition:

$$\Delta k \Delta t \geq \frac{1}{4\pi} \tag{5}$$

Equation (5) explains why it is so difficult to forecast option or stock price. We understand that market participants have impact on the prices that are forecasted. This happens because market participants are communicating (so sending and receiving information) with other market participants. Participants try to predict the stock or option price. We also know that stock prices are described by stochastic processes, we would like to know if this stands behind Econophysic's uncertainty principle.

### 1.3 Stochastic processes

In probability theory, a stochastic process (random process) [18], is the counterpart to a deterministic [6] process. Instead of obtaining with only one possible reality of how the process might evolve under time, in a stochastic or random process there is some indeterminacy in its future evolution described by probability distributions. This means that even if the initial condition (or starting point) is known, there are many possibilities the process might go to, but some paths may be more probable and others are less.

In the simplest possible case, a stochastic process amounts to a sequence of random variables known as a time series. One approach to stochastic processes treats them as functions of one or several deterministic arguments. Those values are random variables: non-deterministic quantities which have certain probability distributions. Random variables corresponding to various times may be completely different. The main requirement is that these different random quantities all have the same type. Random values of a stochastic process at different times may be independent random variables, in most commonly considered situations they exhibit complicated statistical correlations.

Familiar examples of processes modeled as stochastic time series include stock market and exchange rate fluctuations. Wiener process is defined as continuous-time stochastic process. It is observed that stock price follows Wiener process with drift. It can be visualized as a particle suspended in water which is being bombarded by water molecules. The temperature of the water will influence the force of the bombardment, and thus we need a parameter $\sigma$ to characterize this. Moreover, there is a water current which drives the particle in a certain direction, and we will assume a parameter $\mu$ to characterize the drift [17]. To describe the displacements of the particle, the Wiener process can be generalized to the process:

$$dS_t = \mu dt + \sigma dW_t, \tag{6}$$

with solution given by:

$$S_t = S_0 + \mu t + \sigma W_t, \tag{7}$$

where: $W_t$ is Wiener process [7], and $S_t = S(t)$ is trade price observed in the market. In next part of this article, we would like to approximate the realization of $S_t$ function using deterministic function. As it is stated in (7), $S_t$ consists from deterministic part:

$$S_0 + \mu t, \tag{8}$$

and non-deterministic (stochastic) part:

$$\sigma W_t. \tag{9}$$

That is why $S_t$ function can be written in additive form:

$$S(t) = S_d(t) + S_s(t). \tag{10}$$

We define $S_d(t)$ as function that is corresponding to drift so deterministic process, and $S_s(t)$ as part related to stochastic process. We assume that $S(t)$ is a product of $S_d(t)$ (signal) and $S_s(t)$ (noise). Before we go through our model, we would like to understand what process is behind that noise.

### 1.4 Noise and signal separation

We have estimated stock price taken from Polish market by analysing bankier.pl website. This website contains daily trading for stocks and options. We used n-polynomial approximation to estimate stock price:

$$S(t) \sim W(t) \tag{11}$$

where $W(t)$ is $n$-degree polynomial that is used to fit market data, n is a parameter that is often between 3 and 5, as it gives satisfied approximation. Mentioned approach is not to approximate stochastic process, but to approximate the realization of S(t). Approximation level is measured at the basis of Pearson correlation coefficient [14] and also at the basis of residual sum (rest) of squares factor. The parameter $n$ is acceptable if correlation tends to 1 and residual sum of squares [9] tends to 0. Our mathematical model is modified to the following form:

$$S(t) = W(t). \tag{12}$$

Approximating realization of S(t) with deterministic function doesn't give accurate results, however we can compare approximated results with market data. At the basis of that comparison, we can say, that better view of the $S(t)$ function can be given by equation bellow:

$$S(t) = W(t) + n(t), \tag{13}$$

where $n(t)$ is time dependent noise function and is representing stochastic part of (10) equation. $W(t)$ is n-degree polynomial that was calculated in previous iteration. We assume that $n(t)$ function corresponds to analysts recommendations and to the uncertainty principle. We want to designate this function for selected trade and check what is its nature. To figure out that, we have analyzed over 20 trades. We chosen those trades that are stable, and haven't reached any huge fluctuations. On Fig 1, we showed noise and signal separation for KGHM trade. Additionally, we have created table that contains comparison for $S(t)$, $W(t)$ and $n(t)$. Now we want to show how $S_s(t)$ was changing in time. It is presented in the table below. As it was showed in Table 1, difference between measured data, and approximated data are equal to (max) $\pm 10\%$ with $n_{average} = 0.99$. We think that disorders made by $n(t)$ function came from investor's psychological background. In case of a stock price we know that it is determined by decisions made by market members and it stands behind the Econophysic's uncertainty principle. Those decisions are dependent from market analysis using technical and fundamental analysis but also are based on emotions, psychological state and psychological inclinations.

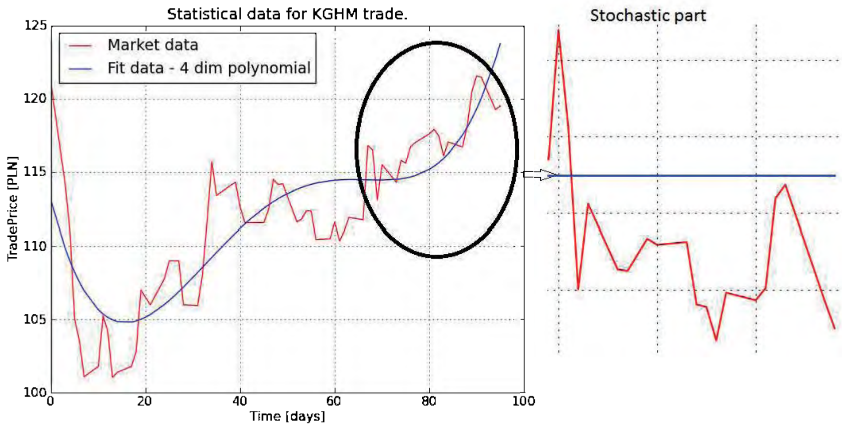


**Fig. 1.** KGHM stock price. Noise and signal separation.

**Table 1.** Comparison for stock price - $S(t)$, approximated price - $W(t)$ and noise function $n(t)$.

| Date | $S(t)$ | $W(t)$ | $n(t)$ | $n_{average}$ |
|---|---|---|---|---|
| 2014-03-24 | 101.80 | 93.339 | 1.091 | |
| 2014-03-25 | 102.80 | 98.035 | 1.049 | |
| 2014-06-09 | 119.25 | 124.605 | 0.957 | |
| 2014-06-10 | 119.50 | 125.801 | 0.950 | |
| | | | | 0.990 |

### 1.5 Psychological background

Most of the investors are capable to predict future stock price using technical [12] and fundamental analysis [20]. Technical analysis is used to forecast the changes of prices through the study of past market data, especially stock price. Investors are using different factors to obtain information if current stock price tends to increase or to decline. One of the example is overbought/oversold [8] factor which is also called stochastic price oscillator. Fundamental analysis is the examination of the underlying forces that affect the well being of the economy, industry groups, and companies. The investor wants to consider the overall growth rate, market size, and analyze it's importance to the economic system. Both methods are crucial in stock pricing, but are impaired by other psychological influences. We can say that financial market is efficient if changes in prices reflect the incoming information. We can distinguish three types of market's efficiency. Week efficiency means that information taken from the past have no effect on stock prices, and stock price meets the random walk criteria. Average efficiency means that all public informations are reflected in current stock prices. Strong efficiency means that all public and confidential informations are reflected in current stock prices. If decisions are made on the basis of technical and fundamental analysis then market is at least average efficient, but if investors are making decisions only at the basis of their emotions and psychological inclinations, then we can observe that prices are capable with random walk process. There are few exceptions [28] that occur on markets:

- On Mondays there is lower rate of return than in other days.
- Similar situation is in January - rate of return is often lower than in other months.
- Rate of returns depends on financial size of the company (higher rate for smaller companies).
- Information drift rates - rates are moving in the same direction in which they were changed.

There is also multiple heuristics that are causing to lower market efficiency:

- Overconfidence heuristics that is described as illusion of control and excessive optimism.
- Availability heuristics - assigning a higher probability for the better-known items.
- Anchoring heuristic - recognition of insignificant values as those who are important.
- Representativeness heuristics - inference on the basis of too few trials.
- Conservatism - underestimation of the importance of the new information due to strong attachment to the opinions.
- Positive effect of freshness - forecasting the continuation of the observed trend.
- Negative effect of freshness - forecasting the reversal of the observed trend.

The above mentioned rules affect the decisions made by the investor. Investor rationality thus depends on the normative layer (often described by technical and fundamental analysis) and also depends on descriptive layer described by behavioural processes. In the light of psychological inclinations it is difficult to consider that the investor behaves rationally, but on the other hand it seems that discovered anomalies have fragile nature and it is quite impossible to achieve over average rate of return. That is why we should interpret $n(t)$ as function that corresponds with behavioural processes and should be treated as noise that is disturbing signal.

Econophysic's uncertainty principle explains why it is so difficult to predict stock/ option price. We also can say that stochastic noise is behind of that. We assume that this noise is determined by trader's psychological inclinations. We know that it will never be possible to predict stock/option price with high accuracy, however we can always try to find better results by using more complicated potential energy that is described in Schrödinger equation. We want to find Schrödinger equation that better reflects option price than it was given in our previous calculations. Our phenomenological assumption is that non-linear Schrödinger equation can extend linear model. In particular, non-linear model can be simplified into linear form. That is why, we want to apply non-linear Schrödinger equation into option pricing. We want to check if it can describe option price more precisely than linear model.

## 2 CREATING THE NEW MODEL

### 2.1 Option pricing using non-linear Schrödinger equation

We want to present another option pricing model based on Schrödinger equation. As a result of our previous digressions we want to propose, more difficult model based on non-linear Schrödinger equation. Non-linear Schrödinger equation [4] describes two identical particles having the same state. We can say that microword contains two types of particles: fermions, and bosons. Fermions [25] particles follow the Pauli exclusion principle. Fermions include all quarks and leptons, as well as any composite particle made of an odd number of these, such as all baryons and many atoms and nuclei. A fermion can be an elementary particle, such as the electron, or it can be a composite particle, such as the proton. only one fermion can occupy a particular quantum state at any given time. If multiple fermions have the same spatial probability distribution, then at least one property of each fermion, such as its spin, must be different. It means that we cannot have two fermions in the same state. Bosons [21] are different from fermions, because they can occupy the same state. An important characteristic of bosons is that their statistics do not restrict the number of them that occupy the same quantum state. Since bosons with the same energy can occupy the same place in space, bosons are often force carrier particles. Thus fermions are sometimes said to be the constituents of matter, while bosons are said to be the particles that transmit interactions (force carriers), or the constituents of radiation. The properties of lasers and masers [24], superfluid helium-4 [23] and Bose-Einstein condensates are all consequences of statistics of bosons. We would like to use Schrödinger equation for boson gas to forecast option price. Bose-Einstein condensate is described with following non-linear Schrödinger equation:

$$i\frac{\partial\psi(x,t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) + \beta|\psi(x,t)|^2\psi(x,t), \quad (14)$$

with initial condition : $\psi(x, t = 0) = \psi_0(x)$,

where, $x$ denotes position, $t$ denotes time, $V(x) + \beta|\psi(x,t)|^2$ stands for potential energy for condensate. Coupling constant $\beta$ is proportional to the scattering length of two interacting bosons. In this form this equation is also known as Gross-Pitaevskii [10] equation. We assume that $V(x) = 0$. After substituting $V(x) = 0$ into equation (14) we get:

$$i\frac{\partial\psi(x,t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + \beta|\psi(x,t)|^2\psi(x,t). \quad (15)$$

From our previous calculations, we know that in our linear model, particle mas $m$ is $\sim \frac{1}{\sigma}$. Also we remember that position $x$ can be interpreted as stock price $S$. When $\beta$ tends to 0, non-linear Schrödinger equation turns into linear. Our new option pricing model is based on the following:

$$i\frac{\partial\psi(S,t)}{\partial t} = -\frac{1}{2}\sigma\frac{\partial^2\psi(S,t)}{\partial S^2} + \beta|\psi(S,t)|^2\psi(S,t), \quad (16)$$

where: $S$ is the asset price, $t$ is time, $\sigma$ is the asset price volatility and $\beta$ is the parameter that denotes market impact on option. We will find solutions for equation (16) by using following function:

$$\psi(S,t) = \phi(\xi)\exp i(kS - \omega t), \quad (17)$$

where $\phi(\xi)$ is unknown function and $\xi = S - \sigma kt$. Function $\exp i(kS - \omega t)$ is solution of linear Schrödinger equation for free particle. After substituting (17) into (16) we obtain following equations:

$$i\frac{\partial\psi(S,t)}{\partial t} = e^{i(kS-t\omega)}\left(\omega\phi(\xi) - ik\sigma\frac{\partial\phi(\xi)}{\partial\xi}\right), \quad (18)$$

$$-\frac{1}{2}\sigma\frac{\partial^2\psi(S,t)}{\partial S^2} = \frac{1}{2}e^{i(kS-t\omega)}\sigma\left(k^2\phi(\xi) - 2ik\frac{\partial\phi(\xi)}{\partial\xi} - \frac{\partial^2\phi(\xi)}{\partial\xi^2}\right), \quad (19)$$

$$\beta|\psi(S,t)|^2\psi(S,t) = e^{i(kS-t\omega)-2Im(kS-t\omega)}\beta|\phi(\xi)|^2\phi(\xi), \quad (20)$$

where $Im$ denotes imaginary part. We assume that: $k, S, \omega, t$ are real:

$$Im(kS - t\omega) = 0. \tag{21}$$

For sake of simplicity, we also assume that:

$$|\phi(\xi)|^2\phi(\xi) = \phi(\xi)^3. \tag{22}$$

Substituting (21) and (22) into (20), we get:

$$\beta|\psi(S,t)|^2\psi(S,t) = e^{i(kS-t\omega)}\beta\phi(\xi)^3. \tag{23}$$

After inserting (18), (19) and (23) into equation (16), we get final equation to solve:

$$\frac{\partial^2\phi(\xi)}{\partial\xi^2} + (\omega - \frac{1}{2}\sigma k^2)\phi(\xi) - \beta(\xi)^3 = 0. \tag{24}$$

We expect to find solutions in the following form:

$$\phi(\xi) = a_0 + a_1 sn(\xi), \tag{25}$$

where $a_0$ and $a_1$ are constants, and $sn(\xi)$ is defined as Jacobi elliptic [3] function:

$$sn(\xi) = sn(\xi, m), \tag{26}$$

where $m \in (0, 1)$ is a real parameter. Jacobi elliptic functions properties are given below:

$$sn(\xi, 0) = sin(\xi), \tag{27}$$

$$sn(\xi, 1) = tanh(\xi), \tag{28}$$

$$cn(\xi, 0) = cos(\xi), \tag{29}$$

$$dn(\xi, 0) = 1, \tag{30}$$

$$\frac{d}{d\xi}(sn(\xi)) = cn(\xi)dn(\xi), \tag{31}$$

$$\frac{d}{d\xi}(cn(\xi)) = -sn(\xi)dn(\xi). \tag{32}$$

First and second order derivatives for (25) equation are below:

$$\frac{\partial\phi(\xi)}{\partial\xi} = a_1 cn(\xi)dn(\xi), \tag{33}$$

$$\frac{\partial^2\phi(\xi)}{\partial\xi^2} = -a1\left(sn(\xi)[1 - m^2sn^2(\xi)]\right) + m^2sn(\xi)[1 - sn^2(\xi)]. \tag{34}$$

Few graphical examples of Jacobi elliptic functions (for different parameters $\xi$ and $m$) are given below:

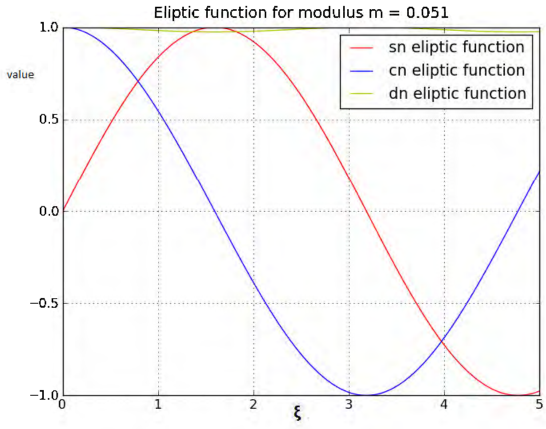


**Fig. 2.** Elliptic functions: $sn(\xi, m), cn(\xi, m), dn(\xi, m)$, for different parameters $m$ and $\xi$.

After substituting (25) into (24), we get:

$$\phi(\xi) = \pm m\sqrt{(\frac{\sigma}{\beta})}sn(\xi), \quad m \in (0,1), \tag{35}$$

and

$$\phi(\xi) = \pm\sqrt{(\frac{\sigma}{\beta})}tanh(\xi), \quad m = 1. \tag{36}$$

Solution (35) is an general solution, and solution (36) is given for $m = 1$. Substituting (35) and (36) into (17), we get the full analytical solution for equation (16):

$$\psi(S,t) = \pm m\sqrt{(\frac{\sigma}{\beta})}sn(s-\sigma kt)\exp i(kS-\frac{1}{2}\sigma(1+m^2+k^2)), \quad m \in (0,1), \tag{37}$$

and

$$\psi^d(S,t) = \pm\sqrt{(\frac{\sigma}{\beta})}tanh(s-\sigma kt)\exp i(kS-\frac{1}{2}\sigma(2+k^2)), \quad m = 1. \tag{38}$$

Where $\psi^d(S,t)$ denotes dark-soliton solution. Equation (37) is a general solution for equation (16), however its special form presented in (38) is called dark soliton. We expect that $\psi(S,t)$ function to be real. This function presented in equation (37) contains also imaginary part. Keeping in mind that any complex function can be split into real and imaginary part:

$$\exp ix = \cos x + i \sin x. \tag{39}$$

Considering that $\beta < 0$, we see that real part of equation (37) is given by:

$$\psi_r(S,t) = \pm m\sqrt{(\frac{\sigma}{\beta})}sn(s-\sigma kt)\cos(kS - \frac{1}{2}\sigma(1+m^2+k^2)), m \in (0,1) \tag{40}$$

and $\psi_r^d(S,t)$ denotes the real part of equation (38):

$$\psi_r^d(S,t) = \pm\sqrt{(\frac{\sigma}{\beta})}tanh(s-\sigma kt)\cos(kS - \frac{1}{2}\sigma(2+k^2)), m = 1. \tag{41}$$

Equations (40) and (41) will be calibrated with market data. We will check correlation between the model that uses abowe equations and compare it to the market data.

### 2.2 Numerical computations

In this chapter we want to fit equation (40) with market data. We have chosen OW20F3280 option that is based on WIG20 asset (stock). Calibration was performed using Levenberg-Marquardt [13] algorithm that was implemented using within Python application. It was included in $scipy.optimize$ package. The Levenberg-Marquardt algorithm is an iterative technique that locates the minimum of a function that is expressed as the sum of squares of non-linear functions. It has become a standard technique for non-linear least-squares problems and can be thought of as a combination of steepest descent and the Gauss-Newton [19] method. Pearson correlation coefficient was calculated using $scipy.stats$ package. Charts were performed using $matplotlib$ library. Few examples of usage are below:

```
def func_s(t, p1, p2, p3):
  return p1 + p2*t+ p3*t*t
popt, pcov = curve_fit(func_s,tdata,sdata,p0=(1.0,0.8,0.2))
```

where $func\_s(t, p1, p2, p3)$ is one variable function ($t$) with three constants ($p1, p2, p3$).
Function $curve\_fit(func_s, tdata, sdata, p0 = (1.0, 0.8, 0.2))$ fits $sdata$ (values for function) and $tdata$ (values for variable $t$) using $func\_s$ function. The initial values for parameters $p1, p2, p3$ are defined in $p0$ parameter.

Correlation is calculated at the basis of estimated function and market data comparison:

```
corr = pearsonr(sdata,func_s(tdata,p1,p2,p3))[0]
```

### 2.3 Calibrating the model - stock price (WIG20)

We have created python script to estimate 3-order polynomial parameters for approximating WIG20 trade using Levenberg-Marquardt algorithm. Our calculation are presented on Figure 3:

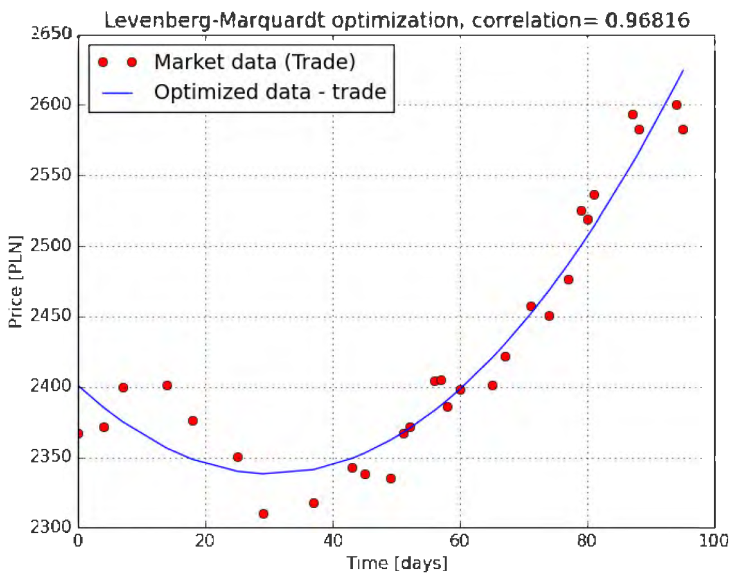


**Fig. 3.** WIG20 stock approximation using 3-order polynomial.

As we see on Fig. 3, we have achieved correlation coefficient equal to $0.97$ when did fit market data (WIG20 stock) using 3-order polynomial. Now we will perform similar probes with fitting dark soliton (41). Then we will perform similar calibration but for the general solution given by (40) equation. We will perform computations for different $m$ values.

### 2.4 Calibrating the model - option price (OW20F3280)

We have used Levenberg-Marquardt algorithm to calibrate equation (40), however we have simplified (40) and (41) equations to the following form:

$$\psi_r(S,t) = mzsn(s-\sigma kt)\cos(kS(t)-\frac{1}{2}\sigma(1+m^2+k^2)), m \in (0,1) \quad (42)$$

where,

$$z = \pm\sqrt{(\frac{\sigma}{\beta})}. \quad (43)$$

and $S(t)$ is stock price (WIG20), approximated using 3-order polynomial. The same simplicity is proposed for (41) equation:

$$\psi_r^d(S,t) = ztanh(s-\sigma kt)\cos(kS-\frac{1}{2}\sigma(2+k^2)). \quad (44)$$

Calibrating (44) equation with market data, gives results presented on figure below. We see that calibrating dark soliton solution (44) with the mar-

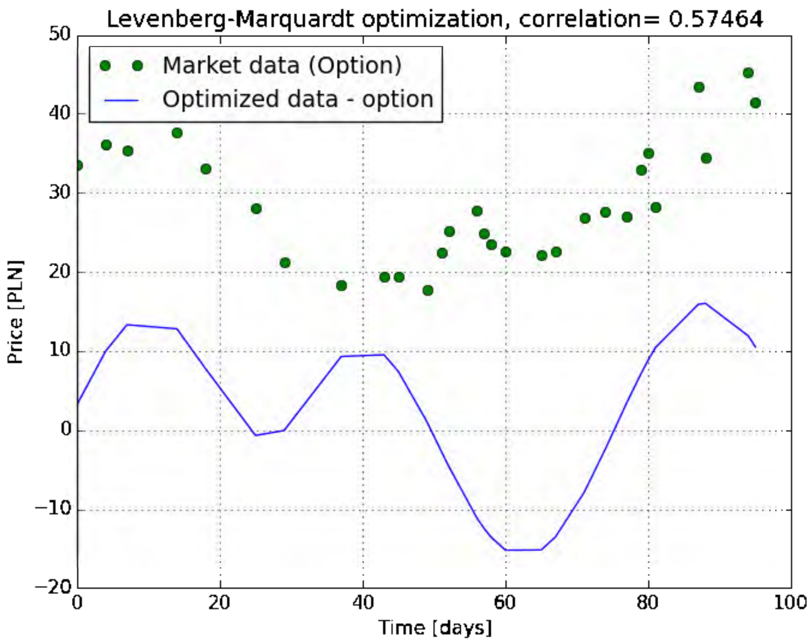


**Fig. 4.** Calibrating dark soliton equation (44) with OW20F3280 option market data.

ket data, gives correlation coefficient equal to 0.57. Now, we will check if calibration of (42) equation can give different results. We have created a set of values (sequence from 0.01 to 0.99 with 0.01 step) for parameter $m$, and for each value, we have calculated parameters: $z$, $\sigma$ and $k$. Stock price (WIG20) $S(t)$ was approximated using 3-order polynomial. The result of calibration are presented on the figure below: On the Fig. 5 we have

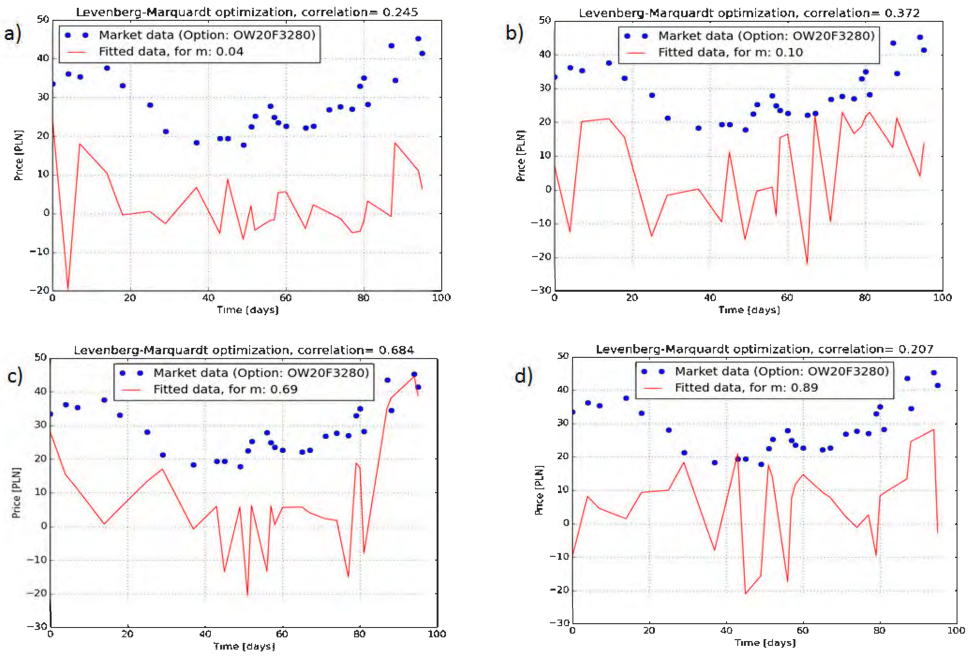


**Fig. 5.** Calibrating equation (42) with OW20F3280 option market data, for different parameter $m$.

presented computations for four cases. We see that highest correlation was achieved for point $c$, where parameter $m$ was equal to $0.69$. For that case, correlation is equal to $0.68$ and is higher than it was for dark soliton solution.

## 3 CONCLUDING REMARKS

We have defined Econophysic's uncertainty principle, and identified psychological processes that determine its nature. To understand it on the ground of social behaviour, we made research related to psychological inclinations and also to stochastic processes. We have impression that it is not possible to predict option price at the basis of historical data, however it is always possible to invent mathematical model to find better option price prediction. That is why we have proposed new model, based on non-linear Schrödinger equation. We have tested its general and special solution, and performed computations to test correlation between the model, and the market data. We also think that due to lack of market data (we gathered stock/option data for the period of three months), we could not perform reliable computations. In the future, we would like to compare our results with full numerical computations for non-linear Schrödinger model.

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## WYCENA OPCJI EUROPEJSKICH Z WYKORZYSTANIEM RÓWNANIA SCHRÖDINGERA

**Streszczenie.** Zasada nieoznaczoności Heisenberg'a została rozszerzona na zagadnienia świata ekonomii. Stochastyczny szum, spowodowany psychologicznymi reakcjami inwestorów powoduje, że prognozowanie trendów ekonomicznych na podstawie danych historycznych obarczone jest dużym błędem. W pracy zaproponowano nowy, ulepszony model wyceny opcji, oparty na nieliniowym równaniu Schrödingera i uwzględniający różnego typu perturbacje. Model został skalibrowanu przy użyciu rzeczywistych danych giełdowych. Do kalibracji danych użyto algorytmu Levenberg'a - Marquardt'a.

**Słowa kluczowe:** wycena opcji europejskich, równanie Blacka-Scholesa, liniowe i nieliniowe równanie Schrdingera, zasada nieoznaczoności Heisenberga, ekonofizyka

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