Developments in Fuzzy Sets,
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Volume I: Foundations

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Dedicated to Professor Beloslav Riečan on his 75th anniversary

A method of construction of intuitionistic fuzzy tolerances based on a similarity measure between intuitionistic fuzzy sets

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Abstract

This paper presents an approach to constructing an intuitionistic fuzzy tolerance matrix. Basic concepts of Atanassov's intuitionistic fuzzy set theory and some distances between intuitionistic fuzzy sets are considered. A method to calculate the intuitionistic fuzzy tolerance degrees between intuitionistic fuzzy sets on the basis of the Hausdorff distance is developed. Finally an illustrative example used to compare the proposed similarity measure with an existing method.

Keywords: intuitionistic fuzzy sets, intuitionistic fuzzy tolerance, Hausdorff distance, similarity measure.

1 Introduction

The intuitionistic fuzzy set theory, originated by Atanassov [1] has been used in a wide range of applications, such as decision making, logic programming, medical diagnosis, image processing, and pattern recognition. Applications of the intuitionistic fuzzy set theory to pattern recognition problems are outlined by Vlachos and Sergiadis in [7]. Moreover, several intuitionistic fuzzy clustering methods were proposed by different researchers.

Firstly, fuzzy clustering method based on intuitionistic fuzzy tolerance relations was proposed by Hung, Lee and Fuh in [2]. An intuitionistic fuzzy similarity relation matrix is obtained by beginning with an intuitionistic fuzzy tolerance relation matrix using the extended n-step procedure by using the composition of intuitionistic fuzzy relations. A hard partition for corresponding thresholds values α and β is the result of classification.

Secondly, concepts of the association matrix and the equivalent association matrix were defined by Xu, Chen and Wu [10]. Thus, methods for calculating the association coefficients of intuitionistic fuzzy sets were introduced in [10]. The proposed in [10] clustering algorithm uses the association coefficients of intuitionistic fuzzy sets to construct an association matrix, and utilizes a procedure to transform it into an equivalent association matrix. The α-cutting matrix of the equivalent association matrix is used to clustering the given intuitionistic fuzzy sets. So, a hard partition for some value of α is the result of classification. That is why the proposed in [10] clustering method is similar to the clustering technique which was proposed by Hung, Lee and Fuh in [2].

Thirdly, a method to constructing an intuitionistic fuzzy tolerance matrix from a set of intuitionistic fuzzy sets and a netting method to clustering of intuitionistic fuzzy sets via the corresponding intuitionistic fuzzy tolerance matrix are developed by Wang, Xu, Liu and Tang in [9]. A hard partition is the result of classification and the clustering result depends on the chosen value of a confidence level $\alpha \in [0,1]$.

So, these relational algorithms cannot provide the information about membership degrees and non-membership degrees of the objects to each cluster. From other hand, the intuitionistic generalization of a heuristic method of possibilistic clustering was introduced in [8], where the D-PAIFC-algorithm was also described. The unique principal allotment among the unknown least number of fully separate intuitionistic fuzzy clusters and corresponding values of the tolerance threshold α and the difference threshold β, $0 \le \alpha + \beta \le 1$ are results obtained from the D-PAIFC-algorithm. It should be noted, that membership degrees and non-membership degrees of intuitionistic fuzzy clusters obtained from the D-PAIFC-algorithm correspond to each object.

A matrix of an intuitionistic fuzzy tolerance relation is the matrix of initial data for all relational intuitionistic fuzzy clustering methods. Hovewer, the data can be presented as a family of intuitionistic fuzzy sets, where each object is represented by some intuitionistic fuzzy set which is defined on the set of attributes. So, an intuitionistic fuzzy tolerance matrix should be constructed from the family of intuitionistic fuzzy sets.

The main goal of the present paper is a proposition of the new method of constructing an intuitionistic fuzzy tolerance relation from intuitionistic fuzzy sets. The method can be useful for the data preprocessing for relational algorithms of intuitionistic fuzzy clustering. The Hausdorff distance is the basis of the method. For this purpose, a short consideration of basic concepts of Atanassov's intuitionistic fuzzy set theory and distances between intuitionistic fuzzy sets is given, a similarity measure which was proposed by Wang, Xu, Liu and Tang in [9] is considered and a new similarity measure based on the Hausdorff distance is proposed. The new similarity measure is illustrated by a short example [5]. Some properties of the new similarity measure are investigated. An illustrative example used to compare the proposed similarity measure with the similarity measure which was proposed by Wang, Xu, Liu and Tang [9].

2 Preliminary remarks

The first subsection of the section provides a consideration of some basic concepts of the Atanassov's intuitionistic fuzzy set theory. In the second subsection, a brief review of distances between intuitionistic fuzzy sets is given.

2.1 Basic concepts of the intuitionistic fuzzy set theory

Let us remind some basic definitions of the Atanassov's intuitionistic fuzzy set theory [1]. All concepts will be considered for a finite universe $X = \{x_1, \ldots, x_n\}$.

Definition 1. *An intuitionistic fuzzy set* IA *in* X *is given by ordered triple* $IA = \left\{ \langle x_i, \mu_{IA}(x_i), \nu_{IA}(x_i) \rangle \mid x_i \in X \right\}$, *where* $\mu_{IA}, \nu_{IA} : X \to [0,1]$ *should satisfy a condition*

$$0 \le \mu_{IA}(x_i) + \nu_{IA}(x_i) \le 1, \tag{1}$$

for all $x_i \in X$. *The values* $\mu_{IA}(x_i)$ *and* $\nu_{IA}(x_i)$ *denote the degree of membership and the degree of non-membership of element* $x_i \in X$ *to* IA, *respectively.*

For each intuitionistic fuzzy set IA in X an intuitionistic fuzzy index [1] of an element $x_i \in X$ in IA can be defined as follows

$$\rho_{IA}(x_i) = 1 - \left(\mu_{IA}(x_i) + \nu_{IA}(x_i) \right). \tag{2}$$

The intuitionistic fuzzy index $\rho_{IA}(x_i)$ can be considered as a hesitancy degree of x_i to IA. It is seen that $0 \le \rho_{IA}(x_i) \le 1$ for all $x_i \in X$.

Obviously, when $\nu_{IA}(x_i) = 1 - \mu_{IA}(x_i)$ for every $x_i \in X$, the intuitionistic fuzzy set IA is an ordinary fuzzy set A in X. For each fuzzy set A in X, we have $\rho_A(x_i) = 0$, for all $x_i \in X$.

Definition 2. *Let* $X = \{x_1, \ldots, x_n\}$ *be an ordinary non-empty set. The binary intuitionistic fuzzy relation* IR *on* X *is an intuitionistic fuzzy subset* IR *of* $X \times X$, *which is given by the expression*

$$IR = \left\{ \langle (x_i, x_j), \mu_{IR}(x_i, x_j), \nu_{IR}(x_i, x_j) \rangle \mid x_i, x_j \in X \right\}, \tag{3}$$

where $\mu_{IR} : X \times X \to [0,1]$ *and* $\nu_{IR} : X \times X \to [0,1]$ *satisfy the condition* $0 \le \mu_{IR}(x_i, x_j) + \nu_{IR}(x_i, x_j) \le 1$ *for every* $(x_i, x_j) \in X \times X$.

Let $\mathrm{IFR}(X)$ denote the set of all intuitionistic fuzzy relations on some universe X. An intuitionistic fuzzy relation $IR \in \mathrm{IFR}(X)$ is reflexive if for every $x_i \in X$, $\mu_{IR}(x_i, x_i) = 1$ and $\nu_{IR}(x_i, x_i) = 0$. An intuitionistic fuzzy relation $IR \in \mathrm{IFR}(X)$ is called symmetric if for all $(x_i, x_j) \in X \times X$, $\mu_{IR}(x_i, x_j) = \mu_{IR}(x_j, x_i)$ and $\nu_{IR}(x_i, x_j) = \nu_{IR}(x_j, x_i)$. An intuitionistic fuzzy relation IT in X is called an intuitionistic fuzzy tolerance if it is reflexive and symmetric. So, any intuitionistic fuzzy tolerance can be presented by a matrix $r_{n\times n} = [\mu_{IT}(x_i, x_j), \nu_{IT}(x_i, x_j)]$, $i, j = 1, \ldots, n$, where a tolerance coefficient $r(x_i, x_j) = (\mu_{IT}(x_i, x_j), \nu_{IT}(x_i, x_j))$, $i, j \in \{1, \ldots, n\}$ is called a closeness degree of x_i and x_j [9].

2.2 Distances between intuitionistic fuzzy sets

Distances between fuzzy sets or similarity measures are used for constructing the matrix of fuzzy tolerance in the case of ordinary fuzzy sets. Different distances between intuitionistic fuzzy sets and similarity measure were also proposed by different researchers. A review of distances between intuitionistic fuzzy sets is given, for example, in Todorova and Vassilev in [6].

Two ways of measuring distances between intuitionistic fuzzy sets exist. Some researchers use the memberships and non-memberships only in the formulae whereas the others researchers use all three parameters, such as degree of membership, degree of non-membership and intuitionistic fuzzy index, which are characterizing any intuitionistic fuzzy set. Some negative effects of using two parameters are shown by Szmidt and Kacprzyk in [3], [4] and [5]. Let us consider some well-known examples of distances between intuitionistic fuzzy sets.

In the first place, a direct generalization of distances between ordinary fuzzy sets for intuitionistic fuzzy sets was made by Atanassov in [1]. In particular, the generalization of the normalized Hamming distance between two intuitionistic fuzzy sets IA and IB on $X = \{x_1, \ldots, x_n\}$ was formulated as follows:

$$l_I(IA, IB) = \frac{1}{2n}\sum_{i=1}^{n}\left[\left|\mu_{IA}(x_i) - \mu_{IB}(x_i)\right| + \left|\nu_{IA}(x_i) - \nu_{IB}(x_i)\right|\right], \; i = 1, \ldots, n. \qquad (4)$$

From other hand, Szmidt and Kacprzyk proposed to take into account the three parameter characterization of intuitionistic fuzzy sets. Here is the definition of the generalization of the normalized Hamming distance between two intuitionistic fuzzy sets IA and IB on $X = \{x_1, \ldots, x_n\}$ given by Szmidt and Kacprzyk in [3]:

$$l_I^{'}(IA,IB)=\frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{l}|\mu_{IA}(x_i)-\mu_{IB}(x_i)|+|\nu_{IA}(x_i)-\nu_{IB}(x_i)|+\\+|\rho_{IA}(x_i)-\rho_{IB}(x_i)|\end{array}\right],\ i=1,\ldots,n. \quad (5)$$

The normalized Hamming distance between intuitionistic fuzzy sets IA and IB on $X=\{x_1,\ldots,x_n\}$ based on the Hausdorff distance was defined by Szmidt and Kacprzyk in [4] as follows:

$$h_I^{'}(IA,IB)=\frac{1}{2n}\sum_{i=1}^{n}\max\left\{\begin{array}{l}|\mu_{IA}(x_i)-\mu_{IB}(x_i)|,|\nu_{IA}(x_i)-\nu_{IB}(x_i)|,\\|\rho_{IA}(x_i)-\rho_{IB}(x_i)|\end{array}\right\},\ i=1,\ldots,n. \quad (6)$$

It should be noted, that for separate elements the Hausdorff distances reduce just to the ordinary Hamming distance.

3 Outline of the approach

The present section describes an approach to constructing an intuitionistic fuzzy tolerance relation based on measurement of similarities between intuitionistic fuzzy sets. In the first subsection a similarity measure which was proposed by Wang, Xu, Liu and Tang [9] is considered. The second subsection of the section includes the consideration of a new similarity measure based on the Hausdorff distance.

3.1 A way to measure the intuitionistic fuzzy similarity degrees between intuitionistic fuzzy sets

The method for constructing the intuitionistic fuzzy tolerance relation was proposed by Wang, Xu, Liu and Tang in [9]. The corresponding similarity measure is based on the normalized Hamming distance and the similarity measure can be expressed by a formula

$$r(IA,IB)=\begin{cases}(1,0), & IA=IB\\ \left(\begin{array}{l}1-\frac{1}{n}\sum_{i=1}^{n}|\nu_{IA}(x_i)-\nu_{IB}(x_i)|-\frac{1}{n}\sum_{i=1}^{n}|\rho_{IA}(x_i)-\rho_{IB}(x_i)|,\\ \frac{1}{n}\sum_{i=1}^{n}|\nu_{IA}(x_i)-\nu_{IB}(x_i)|\end{array}\right), & IA\neq IB\end{cases}, \quad (7)$$

for all $i,j=1,\ldots,n$. That is why the closeness degree $r(IA,IB)=(\mu_{IT}(IA,IB),\nu_{IT}(IA,IB))$ of intuitionistic fuzzy sets IA and IB can be constructed according to the formula (7). Obviously, if all the differences of values of the non-membership degree and the differences of values of the intuitionistic fuzzy index of two objects IA and IB with respect to attributes x_i, $i = 1, \ldots, n$ get smaller, then the two objects are more similar to each other.

The corresponding intuitionistic fuzzy relation possesses the symmetry property and the reflexivity property. Moreover, the condition $0 \le \mu_{IT}(IA, IB) + \nu_{IT}(IA, IB) \le 1$ is met for any intuitionistic fuzzy sets IA and IB. These facts were proved in [9].

3.2 A new similarity measure between intuitionistic fuzzy sets based on the Hausdorff metric

Let us consider the definition of the Hausdorff distance, which was given by Szmidt and Kacprzyk in [5].

Definition 3. *Let* $A = \{a_1, \ldots, a_p\}$ *and* $B = \{b_1, \ldots, b_q\}$ *be two finite set. The Hausdorff distance* $H(A, B)$ *is defined as:*

$$H(A, B) = \max\big(h(A, B), h(B, A)\big), \tag{8}$$

where

$$h(A, B) = \max_{a \in A} \min_{b \in B} d(a, b), \tag{9}$$

and following conditions are met:

– a *and* b *are elements of sets* A *and* B *respectively,*

– $d(a, b)$ *is any metric between these elements,*

– *the two distances* $h(A, B)$ *and* $h(B, A)$ *(10) are called directed Hausdorff distances.*

It should be noted, that the directed distances $h(A, B)$ and $h(B, A)$ are not symmetric. So, following the formula (7), the Hausdorff-distance-based similarity measure for intuitionistic fuzzy sets can be defined as follows.

Definition 4. *Let* IA *and* IB *be two intuitionistic fuzzy sets on* $X = \{x_1, \ldots, x_n\}$ *and* IT *be a binary intuitionistic fuzzy relation on* X. *The closeness degree* $r(IA, IB) = \big(\mu_{IT}(IA, IB), \nu_{IT}(IA, IB)\big)$ *of intuitionistic fuzzy sets* IA *and* IB *can be constructed according to a formula*

$$r^{'}(IA, IB) = \begin{pmatrix} 1 - \frac{1}{n}\sum_{i=1}^{n} \max\big\{|\nu_{IA}(x_i) - \nu_{IB}(x_i)|, |\rho_{IA}(x_i) - \rho_{IB}(x_i)|\big\}, \\ \frac{1}{n}\sum_{i=1}^{n} \max|\nu_{IA}(x_i) - \nu_{IB}(x_i)| \end{pmatrix}, \tag{10}$$

for all $i, j = 1, \ldots, n$.

The proposed similarity measure should be explained by an illustrative example which was given by Szmidt and Kacprzyk. The data originally to appear in [5]. Let us consider the following one-element intuitionistic fuzzy sets: $IA, IB, ID, IG, IE \in X = \{x\}$

$$IA = \{\langle x,1,0\rangle\},\ IB = \{\langle x,0,1\rangle\}, ID = \{\langle x,0,0\rangle\},\ IG = \left\{\left\langle x,\frac{1}{2},\frac{1}{2}\right\rangle\right\},\ IE = \left\{\left\langle x,\frac{1}{4},\frac{1}{4}\right\rangle\right\}.$$

The results obtained from (10) are:

$r^{'}(IA,IB) = \left(1-\max\{|0-1|,|0-0|\},\max\{|0-1|\}\right) = (0,1)$,

$r^{'}(IA,ID) = \left(1-\max\{|0-0|,|0-1|\},\max\{|0-0|\}\right) = (0,0)$,

$r^{'}(IB,ID) = \left(1-\max\{|1-0|,|0-1|\},\max\{|1-0|\}\right) = (0,1)$,

$r^{'}(IA,IG) = \left(1-\max\{|0-1/2|,|0-0|\},\max\{|0-1/2|\}\right) = (1/2,1/2)$,

$r^{'}(IA,IE) = \left(1-\max\{|0-1/4|,|0-1/2|\},\max\{|0-1/4|\}\right) = (1/4,3/4)$,

$r^{'}(IB,IG) = \left(1-\max\{|1-1/2|,|0-0|\},\max\{|1-1/2|\}\right) = (1/2,1/2)$,

$r^{'}(IB,IE) = \left(1-\max\{|1-1/4|,|0-1/2|\},\max\{|1-1/4|\}\right) = (1/4,3/4)$,

$r^{'}(ID,IG) = \left(1-\max\{|0-1/2|,|1-0|\},\max\{|0-1/2|\}\right) = (0,1/2)$,

$r^{'}(ID,IE) = \left(1-\max\{|0-1/4|,|1-1/2|\},\max\{|0-1/4|\}\right) = (1/4,3/4)$,

$r^{'}(IG,IE) = \left(1-\max\{|1/2-1/4|,|0-1/2|\},\max\{|1/2-1/4|\}\right) = (1/2,1/4)$.

It should be noted, that values of non-memberships $\nu_{IT}(IA,IB)$ are not equal to dissimilarity values $h_I^{'}(IA,IB)$ for all elements obtained from the distance (6).

Let us consider some basic properties of the proposed similarity measure (10) and the corresponding binary intuitionistic fuzzy relation IT. In the first place, we need to check whether $0 \le \mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) \le 1$ holds or not.

Lemma 1. *Let IA and IB be two intuitionistic fuzzy sets on $X = \{x_1,\ldots,x_n\}$ and IT be a binary intuitionistic fuzzy relation on X. The condition $0 \le \mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) \le 1$ is met for the closeness degree of intuitionistic fuzzy sets IA and IB which is constructed according to the formula (10)*

Proof. Since

$$\mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) = 1 - \frac{1}{n}\sum_{i=1}^{n}\max\{|\nu_{IA}(x_i)-\nu_{IB}(x_i)|, |\rho_{IA}(x_i)-\rho_{IB}(x_i)|\} \ge$$

$$\ge 1 - \frac{1}{n}\sum_{i=1}^{n}\max\{|\nu_{IA}(x_i)-\nu_{IB}(x_i)|\} - \frac{1}{n}\sum_{i=1}^{n}\max\{|\rho_{IA}(x_i)-\rho_{IB}(x_i)|\} +$$

$$+\frac{1}{n}\sum_{i=1}^{n}\max\{|\nu_{IA}(x_i)-\nu_{IB}(x_i)|\} = 1 - \frac{1}{n}\sum_{i=1}^{n}\max\{|\rho_{IA}(x_i)-\rho_{IB}(x_i)|\} \le 1$$

we have $0 \le \mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) \le 1$ with $\mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) = 1$ if and only if $\rho_{IA}(x_i) = \rho_{IB}(x_i)$ for all $x_i \in X$, and $\mu_{IT}(IA,IB) + \nu_{IT}(IA,IB) = 0$ if and only if either $\rho_{IA}(x_i) = 1$ and $\rho_{IA}(x_i) = 0$ for all $x_i \in X$, or $\rho_{IA}(x_i) = 0$ and $\rho_{IA}(x_i) = 1$ for all $x_i \in X$. So, the lemma is proved. □

Lemma 2. *The binary intuitionistic fuzzy relation* IT *on* X *which is constructed according to the formula (10) is the reflexive intuitionistic fuzzy relation on* X.

Proof. Let $IA = \{\langle x_i, \mu_{IA}(x_i), \nu_{IA}(x_i)\rangle \mid x_i \in X\}$ be an intuitionistic fuzzy set on some universe $X = \{x_1, \ldots, x_n\}$. Let us consider the closeness degree $r'(IA,IA) = (\mu_{IT}(IA,IA), \nu_{IT}(IA,IA))$ which is constructed according to the formula (11). By the formula (2) we have $\rho_{IA}(x_i) = 1 - \mu_{IA}(x_i) - \nu_{IA}(x_i)$, $\mu_{IA}(x_i) = 1 - \nu_{IA}(x_i) - \rho_{IA}(x_i)$, $\nu_{IA}(x_i) = 1 - \mu_{IA}(x_i) - \rho_{IA}(x_i)$. So, we obtain

$$\mu_{IT}(IA,IA) = 1 - \frac{1}{n}\sum_{i=1}^{n} \max\left\{ \begin{array}{l} |1 - \mu_{IA}(x_i) - \rho_{IA}(x_i) - 1 + \mu_{IA}(x_i) + \rho_{IA}(x_i)|, \\ |1 - \mu_{IA}(x_i) - \nu_{IA}(x_i) - 1 + \mu_{IA}(x_i) + \nu_{IA}(x_i)| \end{array} \right\} =$$

$= 1 - 0 = 1.$

The value of the membership degree $\mu_{IT}(IA,IA)$ obtained from the formula (10) is equal 1 for any intuitionistic fuzzy set IA, $\mu_{IT}(IA,IA) = 1$. The condition $\nu_{IT}(IA,IA) = 0$ can be shown in similar manner.

So, the condition $r'(IA,IA) = (\mu_{IT}(IA,IA), \nu_{IT}(IA,IA)) = (1,0)$ is met for any intuitionistic fuzzy set IA on the universe X. That is why conditions $\mu_{IR}(x_i,x_i) = 1$ and $\nu_{IR}(x_i,x_i) = 0$ are met for every $x_i \in X$. The lemma is proved. □

Lemma 3. *The binary intuitionistic fuzzy relation* IT *on* X *which is constructed according to the formula (10) is the symmetric intuitionistic fuzzy relation on* X.

Proof. Let us consider the closeness degree which is constructed according to the formula (10). Since the condition $|\nu_{IA}(x_i) - \nu_{IB}(x_i)| = |\nu_{IB}(x_i) - \nu_{IA}(x_i)|$ and the condition $|\rho_{IA}(x_i) - \rho_{IB}(x_i)| = |\rho_{IA}(x_i) - \rho_{IB}(x_i)|$ are met, then $r'(IA,IB) = r'(IB,IA)$. So, the lemma is proved. □

The consequence of these lemmas is the proposition that the intuitionistic fuzzy relation IT constructed according to the formula (10) is the intuitionistic fuzzy tolerance relation.

4 An illustrative example

The introduced similarity measure should be explained. For the purpose, let us consider an example which was described by Wang, Xu, Liu, and Tang in [9]. Five different cars x_i, $i=1,\ldots,5$ must be classified into several kinds. Each car has six evaluation attributes which represent the oil consumption, coefficient of friction, price, comfortable degree, design and safety coefficient evaluated for five cars. Denote oil consumption by x^1, coefficient of friction by x^2, price by x^3, comfortable degree by x^4, design by x^5 and safety coefficient by x^6. The characteristics information of the cars is presented in Table 1.

Table 1: The initial data set

Objects	Attributes					
	x^1	x^2	x^3	x^4	x^5	x^6
x_1	(0.3, 0.5)	(0.6, 0.1)	(0.4, 0.3)	(0.8, 0.1)	(0.1, 0.6)	(0.5, 0.4)
x_2	(0.6, 0.3)	(0.5, 0.2)	(0.6, 0.1)	(0.7, 0.1)	(0.3, 0.6)	(0.4, 0.3)
x_3	(0.4, 0.4)	(0.8, 0.1)	(0.5, 0.1)	(0.6, 0.2)	(0.4, 0.5)	(0.3, 0.2)
x_4	(0.2, 0.4)	(0.4, 0.1)	(0.9, 0.0)	(0.8, 0.1)	(0.2, 0.5)	(0.7, 0.1)
x_5	(0.5, 0.2)	(0.3, 0.6)	(0.6, 0.3)	(0.7, 0.1)	(0.6, 0.2)	(0.5, 0.3)

In fact, the matrix of attributes is presented by Table 1. So, each car can be considered as an intuitionistic fuzzy set x_i, $i=1,\ldots,5$, and $\mu_{x_i}(x^t)\in[0,1]$, $i=1,\ldots,5$, $t=1,\ldots,6$ are their membership degrees and $\nu_{x_i}(x^t)$, $i=1,\ldots,5$, $t=1,\ldots,6$ are their non-membership degrees. In other words, each intuitionistic fuzzy set x_i, $i=1,\ldots,5$ is defined on the universe of attributes $\{x^t \mid t=1,\ldots,6\}$. That is why the membership degree $\mu_{x_i}(x^t)$ can be interpreted as the degree of expressiveness of some attribute x^t, $t\in\{1,\ldots,6\}$ for the object x_i, $i\in\{1,\ldots,5\}$ and the non-membership degree $\nu_{x_i}(x^t)$ can be considered as the degree of non-expressiveness of the attribute.

Thus, if $X=\{x_1,\ldots,x_n\}$ is the set of objects which are defined on the universe of attributes $\{x^t \mid t=1,\ldots,m\}$ then the formula (7) can be rewritten as follows:

$$r(x_i, x_j) = \begin{cases} (1, 0), & x_i = x_j \\ \begin{pmatrix} 1 - \frac{1}{m}\sum_{t=1}^{m} \left| \nu_{x_i}(x^t) - \nu_{x_j}(x^t) \right| - \frac{1}{m}\sum_{t=1}^{m} \left| \rho_{x_i}(x^t) - \rho_{x_j}(x^t) \right|, \\ \frac{1}{m}\sum_{t=1}^{m} \left| \nu_{x_i}(x^t) - \nu_{x_j}(x^t) \right| \end{pmatrix}, & x_i \neq x_j \end{cases}, \quad (11)$$

for all $i, j = 1, \ldots, n$. So, the formula (10) can be rewritten as follows:

$$r'(x_i, x_j) = \begin{pmatrix} 1 - \frac{1}{m}\sum_{t=1}^{m} \max\left\{ \left| \nu_{x_i}(x^t) - \nu_{x_j}(x^t) \right|, \left| \rho_{x_i}(x^t) - \rho_{x_j}(x^t) \right| \right\}, \\ \frac{1}{m}\sum_{t=1}^{m} \max \left| \nu_{x_i}(x^t) - \nu_{x_j}(x^t) \right| \end{pmatrix}. \quad (12)$$

for all $i, j = 1, \ldots, n$.

By applying the formula (11) to the data set, the matrix of intuitionistic fuzzy tolerance relation was obtained [9]. The matrix is presented in Table 2.

Table 2: An intuitionistic fuzzy tolerance

IT	x_1	x_2	x_3	x_4	x_5
x_1	(1.00, 0.00)				
x_2	(0.80, 0.10)	(1.00, 0.00)			
x_3	(0.72, 0.12)	(0.82, 0.08)	(1.00, 0.00)		
x_4	(0.75, 0.13)	(0.72, 0.10)	(0.70, 0.05)	(1.00, 0.00)	
x_5	(0.65, 0.22)	(0.68, 0.18)	(0.63, 0.23)	(0.63, 0.25)	(1.00, 0.00)

Let us consider an application the formula (12) to the intuitionistic data matrix of Table 1. For example, by applying the formula (12) to intuitionistic fuzzy sets x_3 and x_4, we calculate

$$\mu_{IT}(x_3, x_4) = 1 - \frac{1}{6}\sum_{t=1}^{6} \max\left\{ \left| \nu_{x_3}(x^t) - \nu_{x_4}(x^t) \right|, \left| \rho_{x_3}(x^t) - \rho_{x_4}(x^t) \right| \right\} =$$

$$= 1 - \frac{1}{6}\begin{pmatrix} \max\{0.0, 0.2\} + \max\{0.0, 0.4\} + \max\{0.1, 0.3\} + \max\{0.1, 0.2\} + \\ + \max\{0.0, 0.2\} + \max\{0.1, 0.5\} \end{pmatrix} =$$

$$= 1 - \frac{1}{6}(0.2 + 0.4 + 0.3 + 0.2 + 0.2 + 0.5) = 1 - 0.3 = 0.7;$$

$$\nu_{IT}(x_3,x_4)=\frac{1}{6}\sum_{t=1}^{6}\max\left|\nu_{x_3}(x^t)-\nu_{x_4}(x^t)\right|=$$

$$=\frac{1}{6}\left(\max\{0.0\}+\max\{0.0\}+\max\{0.1\}+\max\{0.1\}+\max\{0.0\}+\max\{0.1\}\right)=$$

$$=\frac{0.3}{6}=0.05.$$

Other values of the membership degree $\mu_{IT}(x_i,x_j)$ and values of of the non-membership degree $\nu_{IT}(x_i,x_j)$ can be calculated for all $i,j=1,\ldots,6$ in a similar way. So, applying the formula (12) we obtain the intuitionistic fuzzy tolerance relation which is equal to the intuitionistic fuzzy tolerance relation of Table 2. As we can see from the example, the proposed Hausdorff-metric-based similarity measure and the similarity measure based on the normalized Hamming distance give fully consistent results.

5 Concluding remarks

A new method for the similarity measurement between intuitionistic fuzzy sets is presented in the paper. The method is based on the Hausdorff distance. This distance was used to generate a new similarity measure to calculate the degree of similarity and degree of dissimilarity between intuitionistic fuzzy sets. Some properties of the proposed similarity measure are considered.

In the paper we have considered the similarity measure between intuitionistic fuzzy sets on the finite universe only which are usually used in different applications. The consideration of the proposed similarity measure in the case of the infinite universe of discourse is the perspective of future investigations.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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