Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

SRI PAS IBS PAN

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Systems Research Institute
Polish Academy of Sciences

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 83-894-7540-5

Dedicated to Professor Beloslav Riečan on his 75th anniversary

A note on bijective correspondence between intuitionistic fuzzy sets and intuitionistic fuzzy sets of p-th type

Parvathi Rangasamy[1],
Peter Vassilev[2] **and Krassimir Atanassov**[3]

[1] Department of Mathematics, Vellalar College for Women,
Erode - 638 012, Tamilnadu, India
paarvathis@rediffmail.com

[2] Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Block 105, 1113 Sofia, Bulgaria
peter.vassilev@gmail.com

[3] Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Block 105, 1113 Sofia, Bulgaria
krat@bas.bg

Abstract

In this paper, we provide mappings that map an IFS of p-th type into IFS and vice versa. Some examples are given to illustrate the transformation. Finally, such mapping is shown between two arbitrary IFS sets of p-th and q-th type (where $p \in [1, +\infty)$, $q \in [1, +\infty)$ are real numbers,p and q are distinct).

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy sets of p-th type, bijection.

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2012.*

1 Introduction

The intuitionistic fuzzy sets (IFSs) were introduced as a generalization and extension to the concept of Fuzzy sets by K. Atanassov (see [1]). Here, we briefly recollect some of the basic definitions and notions. Let E be a universe set (i.e. the set of all the *relevant* elements that will be considered). Let $A \subset E$. An intuitionistic fuzzy set is a set

$$A^* \stackrel{\text{def}}{=} \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

where the functions

$$\mu_A : E \to [0, 1]$$

and

$$\nu_A : E \to [0, 1]$$

reflect the degrees of membership (belongingness) and non-membership (non-belongingness) of the element x from E to the set A, respectively, and every x satisfies that:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

The function π_A which is given by

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x),$$

defines the degree of uncertainty of the belongingness of the element x to the set A. In [2] a new type of IFS, namely IFS of second type(IFSST), was proposed for the first time. There (1) was substituted with

$$0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1 \tag{2}$$

It is worth noting that IFS of second type allows the membership and non-membership functions to take values for which $\mu + \nu > 1$ but (2) holds. Subsequent investigation lead to exploration of IFS of p-th type (IFSpT) (where $p \in [1, +\infty)$ is a real number). For such sets (1) is substituted by the following equation (see e.g. [3])

$$0 \leq \mu_A^p(x) + \nu_A^p(x) \leq 1 \tag{3}$$

One may immediately see that for $p = 1$ we have IFS and for $p = 2$ we have IFSST. The question of how to transform one type into another arises naturally (of course, to do so they must be defined over the same universe). An idea which is proposed in [4] provides the basis for this transformation.

2 Bijective correspondence between IFS and IFS of second type

Here, the mapping which transforms IFS to IFSST is described, and then, the inverse mapping from IFSST to IFS is shown. It is obvious that the proposed mapping is a bijection.

Theorem 1 *Let $A \in IFS$ and*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}.$$

Then the set

$$A' = \{\langle x, \mu'_A(x), \nu'_A(x)\rangle | x \in E\},$$

where

$$\mu'_A(x) = \sqrt{\mu_A(x)};\ \nu'_A(x) = \sqrt{\nu_A(x)} \tag{4}$$

is an IFS of the second type.

Indeed, from (1) for A and (4), it follows that (2) for A' is true for all $x \in E$. Thus, (4) defines a transformation from the class of all IFSs to the class of all IFSsST.

Theorem 2 *Let $A \in$ IFSST and*

$$A^* = \{\langle x, \mu_{A^*}(x), \nu_{A^*}(x)\rangle | x \in E\}.$$

Then the set

$$A'' = \{\langle x, \mu''_A(x), \nu''_A(x)\rangle | x \in E\},$$

where

$$\mu''_A(x) = \mu^2_{A^*}(x);\ \nu''_A(x) = \nu^2_{A^*}(x) \tag{5}$$

is an IFS.

Indeed, from (2) for A^* and (5), it follows that (1) for A'' is true for all $x \in E$. Thus, (5) defines a transformation from the class of all IFSsST to the class of all IFSs. It is clear that (4) is inverse mapping to (5) and that with the two transformations, we have established a bijective correspondence between the two classes.

For example, let us consider the IFS:

$$\{\langle x_1, 0.36, 0.49\rangle, \langle x_2, 0.16, 0.53\rangle \langle x_3, 0.3, 0.96\rangle\}$$

then using (4) we obtain that the corresponding IFSST is:

$$\{\langle x_1, 0.6, 0.7\rangle, \langle x_2, 0.4, \sqrt{0.53}\rangle \langle x_3, \sqrt{0.3}, \sqrt{0.96}\rangle\}$$

3 Bijection between IFSpT and IFSqT

In this Section, we provide a generalization of Theorem 1 and Theorem 2 from the previous section with the assumption that p and q are distinct.

Theorem 3 *Let $p \in [1, +\infty)$ and let $A \in$ IFSpT and*

$$A^{p^*} = \{\langle x, \mu_{A^{p^*}}(x), \nu_{A^{p^*}}(x)\rangle | x \in E\}.$$

Then the set

$$A''' = \{\langle x, \mu'''_A(x), \nu'''_A(x)\rangle | x \in E\},$$

where

$$\mu'''_A(x) = \mu_{A^*}^{\frac{p}{q}}(x);\ \nu'''_A(x) = \nu_{A^*}^{\frac{p}{q}}(x); \tag{6}$$

is an IFSqT (where $q \in [1, +\infty)$).

Obviously, we obtain Theorem 1 for $p = 1$ and $q = 2$ and we obtain Theorem 2 for $p = 2$ and $q = 1$. The Theorem follows from (6) and (3) applied to A^{p^*} and A'''.

4 Conclusion

In the present paper, we have shown a bijective correspondence between all classes of IFSpT. These transformations may be used, for example, in cases when an operator may not be applied directly on a certain IFSpT but can be applied on IFSqT. We have thus shown that between any two classes of IFSpT (for two distinct values of $p \in [1, +\infty)$) there is one-to-one correspondence. In particular, there is a bijective correspondence between IFS and IFSpT.

Acknowledgement

The authors would like to thank the Department of Science and Technology, NewDelhi, India and Ministry of Education and Science, Sofia, Bulgaria, for their financial support to the Bilateral Scientific Cooperation Research Programme INT/Bulgaria/B-2/08 and BIn - 02 - 09.

References

[1] Atanassov, K. Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg, 1999.

[2] Atanassov, K. A Second Type of Intuitionistic Fuzzy Sets, BUSEFAL, Vol. 56, 1993, 66-70.

[3] Vassilev, P.,R. Parvathi, K. Atanassov, Note on Intuitionistic Fuzzy Sets of p-th Type. Issues in Intuitionistic Fuzzy Sets and Generalized Nets, Vol. 6, 2008, 43-50.

[4] Vassilev, P. On reassessment of expert evaluations in the case of intuitionistic fuzzines. Advanced Studies in Contemporary Mathematics, Vol. 20, No. 4, 2010, 569-574.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2011

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475404
ISBN 838947540-5
9 788389 475404