Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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ISBN 83-894-7540-5

Dedicated to Professor Beloslav Riečan on his 75th anniversary

Conditional probability on the family of IF-events with product

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Abstract

The paper contains a new proof of existence of the conditional probability on intuitionistic fuzzy sets.

Keywords: MV-algebra, IF-events, product, conditional probability.

1 Introduction

General idea of this work is to transfer theorems from MV-algebras to IF-events. In this work we construct the conditional probability. Although the main result is known (see [4], [6], [7]) we present a new proof.

2 Conditional probability on the family of IF-events with product

MV-algebras

By the Mundici theorem ([2]) MV-algebra can be characterized by the help of l-groups.

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2012.*

Definition 1 *An l-group is and algebraic system*

$$(G, +, \leq)$$

such that

$(G, +)$ *is and Abelian group*
$(G, \leq)$ *is a partially ordered set being a lattice*
$a \leq b \Longrightarrow a + c \leq b + c$ *for any* a, b, c *in* G.

Definition 2 *An MV-algebra is an algebraic system*

$$(M, \oplus, \odot, \leq, 0, u)$$

where

$M = [0, u]$ *is an interval in an l-group* $G = (G, +, \leq)$
0 *is the neutral element of G (i.e.* $a + 0 = a$ *for any* $a \in G$*)*
u *is the strong unit of G (i.e. to any* $a \in G$ *there exists* $n \in N$
such that $a \leq u + u + ... + u(n$*-times))*
$a \oplus b = (a + b) \wedge 1,$
$a \odot b = (a + b - 1) \vee 0.$

Definition 3 *A state on an MV-algebra M is a mapping* $m : \mathcal{M} \to \langle 0, 1\rangle$ *satisfying the following conditions:*

(i) $m(u) = 1$, $m(0) = 0$;

(ii) $a \odot b = 0 \Rightarrow m((a \oplus b)) = m(a) + m(b)$;

(iii) $a_n \nearrow a \Longrightarrow m(a_n) \nearrow m(a)$.

Definition 4 *An MV-algebra with product is a pair$(\mathcal{M}, .)$, where $\mathcal{M}$ is an MV-algebra and . is a commutative and associative binary operation on $\mathcal{M}$ satisfying the following conditions:*

(i) $1 \cdot a = a$ *for any* $a \in M$

(ii) $a \cdot (b \odot \neg c) = (a \cdot b) \odot \neg (a \cdot c)$ *for any* $a, b, c \in \mathcal{M}$

IF-events

Let (Ω, S) be a measurable space. By the Atanassov theory ([1]) IF-set is a pair $A = (\mu_A, \nu_A)$ of functions $\mu_A, \nu_A : \Omega \to \langle 0, 1\rangle$ such that $\mu_A + \nu_A \leq 1$. If μ_A, ν_A are S-measurable functions, than A is called IF- event. $\mathcal{F}$ denotes the family of all IF-events.

We shall use the Lukasiewicz connectives ([5]):
$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B - 1) \vee 0)$
$A \odot B = ((\mu_A + \mu_B - 1) \vee 0, (\nu_A + \nu_B) \wedge 1)$
$A \leq B \Leftrightarrow \mu_A \leq \mu_B, \nu_A \geq \nu_B$

Definition 5 *A mapping $m : \mathcal{F} \to \langle 0, 1\rangle$ is called a state if the following properties are satisfied:*

(i) $m\,(1, 0) = 1$*,* $m\,(0, 1) = 0$*;*

(ii) $A \odot B = 0 \Rightarrow m\,((A \oplus B)) = m\,(A) + m\,(B)\,;$

(iii) $A_n \nearrow A \Longrightarrow m(A_n) \nearrow m(A).$

Definition 6 *Let $\mathcal{J} = \{(-\infty, t); t \in R\}$. An observable on $\mathcal{F}$ is any mapping $x : \mathcal{J} \to \mathcal{F}$ satisfying the conditions:*

(i) $t_n \nearrow \infty \Longrightarrow x((-\infty, t_n)) \nearrow (1, 0)\,;$

(ii) $t_n \searrow -\infty \Longrightarrow x((-\infty, t_n)) \searrow (0, 1)\,;$

(iii) $t_n \nearrow t \Longrightarrow x((-\infty, t_n)) \nearrow x((-\infty, t_n)) \nearrow x((-\infty, t)).$

Definition 7 *An observable $x : \mathcal{J} \to \mathcal{F}$ be called to be integrable if there exists*

$$E(x) = \int_R t dF(t),$$

where $F : R \to \langle 0, 1\rangle$ is distribution function of the observable x, $F(t) = m(x((-\infty, t))), t \in R$.

We define the product on $\mathcal{F}$:

Definition 8 *If $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$ are events then*
$(\mu_A, \nu_A) \cdot (\mu_B, \nu_B) = (\mu_A \mu_B, \nu_A + \nu_B - \nu_A \nu_B).$

Proposition 1 *Let* $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in \mathcal{F}$ *and* $\boldsymbol{1} = (1, 0)$. *Then*

(i) $\boldsymbol{1} \cdot A = A$ *for any* $A \in \mathcal{F}$

(ii) $A \cdot (B \odot \neg C) = (A \cdot B) \odot \neg (A \cdot C)$ *for any* $A, B, C \in \mathcal{F}$

Proof.

(i) $\mathbf{1} \cdot A = (1, 0) \cdot (\mu_A, \nu_A) = (1 \cdot \mu_A, 0 + \nu_A - 0 \cdot \nu_A) = (\mu_A, \nu_A) = A$

(ii)

L: $$\begin{aligned} A \cdot (B \odot \neg C) &= (\mu_A, \nu_A) \cdot ((\mu_B, \nu_B) \odot \neg (\mu_C, \nu_C)) = \\ &= (\mu_A, \nu_A) \cdot ((\mu_B, \nu_B) \odot (1 - \mu_C, 1 - \nu_C)) = \\ &= (\mu_A, \nu_A) \cdot ((\mu_B + 1 - \mu_C - 1) \vee 0, (\nu_B + 1 - \nu_C)) = \\ &= (\mu_A, \nu_A) \cdot (\mu_B - \mu_C, \nu_B + 1 - \nu_C) = \\ &= (\mu_A\mu_B - \mu_A\mu_C, \nu_A + \nu_B + 1 - \nu_C - \nu_A\nu_B - \nu_A + \nu_A\nu_C) = \\ &= (\mu_A\mu_B - \mu_A\mu_C, \nu_B - \nu_C - \nu_A\nu_B + \nu_A\nu_C + 1) \end{aligned}$$

R: $$\begin{aligned} (A \cdot B) \odot \neg (A \cdot C) &= ((\mu_A, \nu_A) \cdot (\mu_B, \nu_B)) \odot \neg ((\mu_A, \nu_A) \cdot (\mu_C, \nu_C)) = \\ &= (\mu_A\mu_B, \nu_A + \nu_B - \nu_A\nu_B) \odot (1 - \mu_A\mu_C, 1 - \nu_A - \nu_C + \nu_A\nu_B) = \\ &= ((\mu_A\mu_B + 1 - \mu_A\mu_C - 1) \vee 0, \\ &\quad (\nu_A + \nu_B - \nu_A\nu_B + 1 - \nu_A - \nu_C + \nu_A\nu_C) \wedge 1) = \\ &= (\mu_A\mu_B - \mu_A\mu_C, \nu_B - \nu_C - \nu_A\nu_B + \nu_A\nu_C + 1) \end{aligned}$$

L=R □

MV-algebras with product and IF-events with product

Theorem 1 *Let* $(\mathcal{F}, \cdot)$ *be a IF-events with product. Let map* $m : \mathcal{F} \to \langle 0, 1 \rangle$ *be a state on* $\mathcal{F}$. *Then there exist map* $\bar{m} : \mathcal{M} \to \langle 0, 1 \rangle$, *where* $\mathcal{M}$ *is MV-algebra with product* $(\mathcal{M}, \cdot)$ *and* $\mathcal{F} \subset \mathcal{M}$ *such that* $m = \bar{m}|\mathcal{F}$.

Proof.

Let $(\mu_A, \nu_A) \in \mathcal{M}$. Then

$$(\mu_A, \nu_A) \oplus (0, 1 - \nu_A) = (\mu_A, 0)\,.$$

Define

$$\bar{m}\,(\mu_A, \nu_A) = m\,(\mu_A, 0) - m\,(0, 1 - \nu_A)\,.$$

Evidently $\bar{m}$ is a state on $\mathcal{M}$ and $\bar{m}(\mu_A, \nu_A) = m(\mu_A, \nu_A)$ if $A \in \mathcal{F}$.
If $(\mu_A, \nu_A) \in \mathcal{F}$ then

$$m(\mu_A, \nu_A) + m(0, 1 - \nu_A) = m(\mu_A, 0) \qquad \square$$

We shall use Theorem 1 in application for conditional probability.

Conditional probability

If m is a state on $\mathcal{F}$ and x is an observable on $\mathcal{F}$, then the mapping $m_x = m \circ x : \mathcal{B}(\mathbf{R}) \to \mathcal{F}$ is a probability measure.

In [6] the conditional probability is defined on MV-algebras.

Theorem 2 *Let $a \in \mathcal{M}$, $y : \mathcal{B}(R) \to \mathcal{M}$ be an observable and $\bar{m} : \mathcal{M} \to \langle 0, 1\rangle$ be a state on $\mathcal{M}$. Then there exist the conditional probability $\bar{p}(a/y) : R \to R$ such that*

$$\int_B \bar{p}(a/y) d\bar{m}_y = \bar{m}(a \cdot y(B)),$$

for any $B \in \mathcal{B}(R)$.

Proof. See [6]. $\square$

Theorem 3 *Let $A = (\mu_A, \nu_A) \in \mathcal{M}$. Let $y : \mathcal{B}(R) \to \mathcal{F}$ be an observable and $m : \mathcal{F} \to \langle 0, 1\rangle$ be a state on $\mathcal{F}$. Then there exist the conditional probability $p(A/y) : R \to R$ such that*

$$\int_B p(A/y) dm_y = m(A \cdot y(B)),$$

for any $B \in \mathcal{B}(R)$.

Proof. Let $A = (\mu_A, \nu_A) \in \mathcal{M}$.
Exist the conditional probability $\bar{p}(a/y) : R \to R$ and state $\bar{m} : \mathcal{M} \to \langle 0, 1\rangle$, such that $\forall B \in \mathcal{B}(R)$

$$\int_B \bar{p}(a/y) d\bar{m}_y = \bar{m}(a \cdot y(B)).$$

Observable y is a mapping from $\mathcal{B}(R)$ to $\mathcal{F}$ and $\mathcal{F} \subset \mathcal{M}$. We use to Theorem 1.
Then $m = \bar{m}$ and $p(A/y) = \bar{p}(A/y)$ and

$$\int_B p(A/y) dm_y = m(A \cdot y(B)). \qquad \square$$

3 Conclusions

In the paper we presented a method for obtaining some result for probability on IF-events by some results holding for probability on MV-algebras.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475404
ISBN 838947540-5
9 788389 475404