Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Dedicated to Professor Beloslav Riečan on his 75th anniversary

Transformations of intuitionistic fuzzy relations

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Abstract

This article is devoted to examination on transformations of intuitionistic fuzzy relations in the context of preservation of the given properties of these relations in the process of transformation. The considered transformations are lattice operations and some other operations defined by Atanassov and in addition, the complement, the converse and composition of relations are taken into account. Among others, semi-properties of intuitionistic fuzzy relations, namely semi-reflexivity, semi-irreflexivity, semi-symmetry, semi-connectedness, semi-asymmetry, semi-transitivity are examined.

Keywords: intuitionistic fuzzy relations, composition, dual composition, relation classification, basic properties, semi-properties, preservation of properties, invariant transformations.

1 Introduction

Atanassov intuitionistic fuzzy sets and relations (originally called intuitionistic fuzzy sets and relations, cf. [1], [2]) are applied for example in group decision making, optimization problems, graph theory and neural networks (cf. [6]) so it is worth dealing with such concepts both for theoretical and practical reasons. The concept of an Atanassov intuitionistic fuzzy set and an Atanassov intuitionistic fuzzy relation (intuitionistic fuzzy relation for short) generalize the concept of a fuzzy set and a fuzzy relation introduced by Zadeh (cf. [19], [20]). Namely, not

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only the degree of membership to a given set (relation) is considered but also the degree of non-membership to this set (relation) is taken into account in such way that the sum of both values is less than or equal to one. Therefore, a pair $\rho = (R, R^d)$ is called an intuitionistic fuzzy relation, where $R,\ R^d : X \times X \to [0,1]$ are fuzzy relations fulfilling the condition $R(x,y) + R^d(x,y) \leq 1,\ \ x,y \in X$, $X \neq \emptyset$. Intuitionistic fuzzy relations may have diverse types of properties (cf. [5], [7], [13]) and there are many interesting problems to deal with in this area (see for example [8]). We consider the problem of preservation of intuitionistic fuzzy relation properties by transformations, especially we take into account the complement, the converse and composition of a relation and we also examine lattice operations. Generally, the regarded transformations are of the type $\mathfrak{F} : AIFR(X)^n \to AIFR(X)$, $n \in \mathbb{N}$, where AIFR(X) stands for the family of all intuitionistic fuzzy relations described in a given set X. Among others, semi-properties of intuitionistic fuzzy relations are examined. These properties are important because of its possible applications for preference procedure which is of great interest nowadays (see e.g. [10], [14], [15], [18]). In Section 2, some necessary concepts and properties are recalled. In Section 3, preservation of the following properties is examined: reflexivity, irreflexivity, connectedness, asymmetry, transitivity, dual transitivity. In Section 4, preservation of semi-reflexivity, semi-irreflexivity, semi-symmetry, semi-connectedness, semi-asymmetry, semi-transitivity is considered.

2 Basic definitions

Let us first recall, useful in our further considerations, the definition of the composition and the dual composition of fuzzy relations considered in the family $FR(X) = \{R | R : X \times X \to [0,1]\}$, $X \neq \emptyset$ of all fuzzy relations (cf. [20]) in a given set $X \neq \emptyset$. If $card\, X = n$, $n \in \mathbb{N}$, then a fuzzy relation $R : X \times X \to [0,1]$ may be represented by a matrix belonging to $[0,1]^{n \times n}$, where $R = [r_{ij}]$ and $r_{ij} = R(x_i, x_j)$.

Definition 1 (cf. [20])**.** Let $R, S \in FR(X)$. The composition of fuzzy relations R and S is the fuzzy relation $(R \circ S) \in FR(X)$ such that

$$(R \circ S)(x,z) = \sup_{y \in X} \min(R(x,y), S(y,z)), \quad (x,z) \in X \times X. \tag{1}$$

If $card\, X = n$, $R = [r_{ij}]$, $S = [s_{jk}]$, then

$$R \circ S = [t_{ik}],\ \text{where } t_{ik} = \max_{1 \leqslant j \leqslant n} \min(r_{ij}, s_{jk}), \quad i,k = 1,...,n,\ n \in \mathbb{N}. \tag{2}$$

The dual composition of fuzzy relations R and S is the fuzzy relation $(R \circ' S) \in FR(X)$ such that

$$(R \circ' S)(x,z) = \inf_{y \in X} \max(R(x,y), S(y,z)), \quad (x,z) \in X \times X. \tag{3}$$

If $card\, X = n$, $R = [r_{ij}]$, $S = [s_{jk}]$, then

$$R \circ' S = [t_{ik}], \text{ where } t_{ik} = \min_{1 \leqslant j \leqslant n} \max(r_{ij}, s_{jk}), \quad i,k = 1,...,n, \ n \in \mathbb{N}. \tag{4}$$

Now we put the definition of an intuitionistic fuzzy relation.

Definition 2 (cf. [2]). Let $X \neq \emptyset$, $R,\ R^d : X \times X \to [0,1]$ be fuzzy relations fulfilling the condition

$$R(x,y) + R^d(x,y) \leqslant 1, \ \ x,y \in X.$$

A pair $\rho = (R, R^d)$ is called an intuitionistic fuzzy relation. The family of all intuitionistic fuzzy relations described in a given set X is denoted by $AIFR(X)$.

Example 1. The identity relation $\iota \in AIFR(X)$ has the following form $\iota = (I, I^d)$:

$$I = \begin{cases} 1 & \text{for } x = y \\ 0 & \text{for } x \neq y \end{cases}, \quad I^d = \begin{cases} 0 & \text{for } x = y \\ 1 & \text{for } x \neq y \end{cases}, \quad x,y \in X. \tag{5}$$

With each intuitionistic fuzzy relation $\rho \in AIFR(X)$ the fuzzy relation $\pi_\rho : X \times X \to [0,1]$ is associated (cf. [16]), where

$$\pi_\rho(x,y) = 1 - R(x,y) - R^d(x,y), \quad x,y \in X. \tag{6}$$

The number $\pi_\rho(x,y)$ is called an index of an element (x,y) in the intuitionistic fuzzy relation ρ. Each fuzzy relation R can be expressed in the language of intuitionistic fuzzy relations. In this case we put $R^d(x,y) = 1 - R(x,y)$ for all $x,y \in X$. As a result $\pi_\rho(x,y) = 0$ for all $x,y \in X$.

We use the notation $\rho(x,y) = (R(x,y), R^d(x,y))$, where $\rho \in AIFR(X)$. Basic transformations of the type $\mathfrak{F} : AIFR(X)^n \to AIFR(X)$, $n \in \mathbb{N}$, and relations for intuitionistic fuzzy relations are defined in the following way

Definition 3 (cf. [2], [7]). For arbitrary $\rho = (R, R^d),\ \sigma = (S, S^d) \in AIFR(X)$ we define:

- the sum: $\rho \vee \sigma = (R \vee S, R^d \wedge S^d)$,
- the intersection: $\rho \wedge \sigma = (R \wedge S, R^d \vee S^d)$, where

$$(R \vee S)(x,y) = \max(R(x,y), S(x,y)), \quad x,y \in X,$$

$$(R \wedge S)(x,y) = \min(R(x,y), S(x,y)), \quad x,y \in X,$$

• the converse relation: $\rho^{-1} = (R^{-1}, (R^d)^{-1})$, where $R^{-1}(x,y) = R(y,x)$, for each $R \in FR(X)$ and $x, y \in X$,
• the complement: $\rho' = (R^d, R)$.
And for a set $P \subset X \times X$:

$$\bigvee_{(x,y)\in P} \rho(x,y) = (\bigvee_{(x,y)\in P} R(x,y), \bigwedge_{(x,y)\in P} R^d(x,y)), \tag{7}$$

$$\bigwedge_{(x,y)\in P} \rho(x,y) = (\bigwedge_{(x,y)\in P} R(x,y), \bigvee_{(x,y)\in P} R^d(x,y)), \tag{8}$$

where for each $R \in FR(X)$

$$\bigvee_{(x,y)\in P} R(x,y) = \sup_{(x,y)\in P} R(x,y) \quad \text{and} \quad \bigwedge_{(x,y)\in P} R(x,y) = \inf_{(x,y)\in P} R(x,y).$$

We consider the composition and the dual composition of elements of the *AIFR*(X) (cf. [7]). However, we only concentrate on the basic form of this composition, where operations maximum and minimum are involved.
Let $\rho, \sigma \in AIFR(X)$. Thus:
• the composition of relations ρ, σ is the relation

$$\rho \circ \sigma = (R \circ S, R^d \circ' S^d) \in AIFR(X),$$

• the dual composition of relations ρ, σ is the relation

$$\rho \circ' \sigma = (R \circ' S, R^d \circ S^d) \in AIFR(X),$$

where operations $\circ$ and $\circ'$ are described by the formulas (1) and (3). We define also the following relations between elements of the family $AIFR(X)$:

$$\rho \leqslant \sigma \Leftrightarrow (R \leqslant S,\ S^d \leqslant R^d),$$

$$\rho = \sigma \Leftrightarrow (R = S,\ R^d = S^d).$$

The pair $(AIFR(X), \leqslant)$ is a partially ordered set. Operations $\vee, \wedge$ are the supremum and the infimum in $AIFR(X)$, respectively. As a result the family $(AIFR(X), \vee, \wedge)$ is a lattice (for a definition of a lattice and other related concepts see [4]) which is a consequence of the fact that $([0,1], \max, \min)$ is a lattice. The lattice $AIFR(X)$ is complete. There exist the bottom and the top elements in $AIFR(X)$. We will denote these elements by **0**, **1**, respectively, where

$\mathbf{0} = (0, 1)$, $\mathbf{1} = (1, 0)$ and $0, 1 \in FR(X)$ are the constant fuzzy relations. For each subset P of $X \times X$ there exist the values (7) and (8). This fact follows from the definition of supremum $\bigvee$ and infimum $\bigwedge$ and from the fact that the values of fuzzy relations are from the interval $[0, 1]$ which with the operations maximum and minimum forms a complete lattice. Operation $'$ is de Morgan complement in this lattice because $(\rho')' = \rho$ and $\rho \leqslant \sigma \Rightarrow \sigma' \leqslant \rho'$. We consider only the originally proposed operations on relations in the family $AIFR(X)$ [3]. There are also generalizations of these operations with the use of triangular norms and conorms, see [9]. Some other operations in the intuitionistic environment were also introduced by Atanassov ([3], p. 9).

3 Preservation of the basic properties

There are many particular properties of intuitionistic fuzzy relations. We apply the following ones which are modifications of the ones applied in [7] and [13]. In the sequel we will see that such way of defining these properties guarantee the analogy between results on fuzzy relations in the family $FR(X)$ and the results on intuitionistic fuzzy relations in the family $AIFR(X)$.

Definition 4. Relation $\rho \in AIFR(X)$ is:
• reflexive, if
$$\underset{x \in X}{\forall} \rho(x, x) = \mathbf{1}, \tag{9}$$
• irreflexive, if
$$\underset{x \in X}{\forall} \rho(x, x) = \mathbf{0}, \tag{10}$$
• symmetric, if
$$\underset{x,y \in X}{\forall} \rho(x, y) = \rho(y, x), \tag{11}$$
• asymmetric, if
$$\underset{x,y \in X}{\forall} \rho(x, y) \wedge \rho(y, x) = \mathbf{0}, \tag{12}$$
• antisymmetric, if
$$\underset{x,y \in X, x \neq y}{\forall} \rho(x, y) \wedge \rho(y, x) = \mathbf{0}, \tag{13}$$
• totally connected, if
$$\underset{x,y \in X}{\forall} \rho(x, y) \vee \rho(y, x) = \mathbf{1}, \tag{14}$$
• connected, if
$$\underset{x,y \in X, x \neq y}{\forall} \rho(x, y) \vee \rho(y, x) = \mathbf{1}, \tag{15}$$

• transitive, if

$$\underset{x,y,z\in X}{\forall}\ \rho(x,y)\wedge\rho(y,z)\leqslant\rho(x,z), \tag{16}$$

• dually transitive, if

$$\underset{x,y,z\in X}{\forall}\ \rho(x,y)\vee\rho(y,z)\geqslant\rho(x,z). \tag{17}$$

In this paper only the classical transitivity property is considered. In general, instead of operations $\vee$ and $\wedge$, some other operations may be applied (see [7]).

As a consequence of the previous definition one has (cf. [5])

Theorem 1. *Let $\rho=(R,R^d)\in AIFR(X)$. Relation ρ is:*
• *reflexive if and only if $\iota\leqslant\rho$ ([5]),*
• *irreflexive if and only if $\rho\wedge\iota=\mathbf{0}$,*
• *symmetric if and only if $\rho=\rho^{-1}$,*
• *asymmetric if and only if $\rho\wedge\rho^{-1}=\mathbf{0}$,*
• *antisymmetric if and only if $\rho\wedge\rho^{-1}\leqslant\iota$,*
• *totally connected if and only if $\rho\vee\rho^{-1}=\mathbf{1}$,*
• *connected if and only if $\rho\vee\rho^{-1}\vee\iota=\mathbf{1}$,*
• *transitive if and only if $\rho\circ\rho\leqslant\rho$ (cf. [7]),*
• *dually transitive if and only if $\rho\circ'\rho\geqslant\rho$ (cf. [7]).*

Proof. Now the proof for a connectedness will be presented. Other properties may be justified analogously. Let $\rho(x,y)=(R(x,y),R^d(x,y))$, $x,y\in X, x\neq y$ and $(\rho\vee\rho^{-1}\vee\iota)=\mathbf{1}$. Thus

$$\mathbf{1}(x,y)=(\rho\vee\rho^{-1}\vee\iota)(x,y)=$$

$$(\max(R(x,y),R^{-1}(x,y),I(x,y)),\min(R^d(x,y),(R^d)^{-1}(x,y),I^d(x,y))).$$

As a result, by the formula $R^{-1}(x,y)=R(y,x)$, one obtains

$$\mathbf{1}(x,y)=(\max(R(x,y),R(y,x),0),\min(R^d(x,y),(R^d)(y,x),1))=$$

$$(\max(R(x,y),R(y,x)),\min(R^d(x,y),(R^d)(y,x))).$$

This means that (15) is fulfilled, so ρ is connected. □

From definitions of fuzzy relation properties (cf. [11], p. 72) we obtain

Corollary 1. *Let $\rho=(R,R^d)\in AIFR(X)$. Intuitionistic fuzzy relation ρ is:*
• *reflexive if and only if fuzzy relation R is reflexive and fuzzy relation R^d is irreflexive,*

• *irreflexive if and only if fuzzy relation* R *is irreflexive and fuzzy relation* R^d *is reflexive,*
• *symmetric if and only if fuzzy relation* R *is symmetric and fuzzy relation* R^d *is symmetric,*
• *asymmetric if and only if fuzzy relation* R *is asymmetric and fuzzy relation* R^d *is totally connected,*
• *antisymmetric if and only if fuzzy relation* R *is antisymmetric and fuzzy relation* R^d *is connected,*
• *totally connected if and only if fuzzy relation* R *is totally connected and fuzzy relation* R^d *is asymmetric,*
• *connected if and only if fuzzy relation* R *is connected and fuzzy relation* R^d *is antisymmetric,*
• *transitive if and only if fuzzy relation* R *is transitive and fuzzy relation* R^d *is dually transitive,*
• *dually transitive if and only if fuzzy relation* R *is dually transitive and fuzzy relation* R^d *is transitive.*

Theorem 1 and Corollary 1 give the way of quick checking properties of the given relation $\rho \in AIFR(X)$. Below some examples are presented.

Example 2. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 1 & 0.3 & 0 \\ 0.5 & 1 & 0.1 \\ 0.1 & 0.4 & 1 \end{bmatrix}, \quad R^d = \begin{bmatrix} 0 & 0.7 & 1 \\ 0.5 & 0 & 0.8 \\ 0.8 & 0.6 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 0 & 0.4 & 0.3 \\ 0.3 & 0 & 0.5 \\ 0.2 & 0.2 & 0 \end{bmatrix}, \quad S^d = \begin{bmatrix} 1 & 0.6 & 0.7 \\ 0.6 & 1 & 0.5 \\ 0.8 & 0.8 & 1 \end{bmatrix},$$

where ρ is reflexive and σ is irreflexive.

Example 3. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 0 & 0 & 1 \\ 0.7 & 0 & 0 \\ 0 & 0.9 & 0 \end{bmatrix}, \quad R^d = \begin{bmatrix} 1 & 1 & 0 \\ 0.2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},$$

$$S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0.3 \\ 0.1 & 1 & 1 \end{bmatrix}, \quad S^d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.4 \\ 0.3 & 0 & 0 \end{bmatrix},$$

where ρ is asymmetric (antisymmetric) and σ is totally connected (connected).

Example 4. Let card $X = 2$, $\rho = (R, R^d)$ and card $X = 3$, $\sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}, \quad R^d = \begin{bmatrix} 0 & 0.6 \\ 0.7 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.3 & 0.7 & 0.2 \\ 0.5 & 0.8 & 0.5 \\ 0.1 & 0.4 & 0.1 \end{bmatrix}, \quad S^d = \begin{bmatrix} 0.6 & 0.1 & 0.8 \\ 0.2 & 0 & 0.4 \\ 0.6 & 0.2 & 0.7 \end{bmatrix}.$$

Relation ρ is transitive because $\rho \circ \rho \leqslant \rho$ (cf. [6]) and σ is dually transitive because $\sigma \circ' \sigma \geqslant \sigma$ (cf. [5]).

Now, some transformations of intuitionistic fuzzy relations will be discussed.

Theorem 2. *Let $\rho \in AIFR(X)$.*
- *ρ is reflexive if and only if ρ' is irreflexive ([7]).*
- *ρ is symmetric if and only if ρ' is symmetric ([7]).*
- *ρ is asymmetric if and only if ρ' is totally connected.*
- *ρ is antisymmetric if and only if ρ' is connected.*
- *ρ is transitive if and only if ρ' is dually transitive ([7]).*

Proof. We will prove only one of the given properties. Let $x, y \in X$, $\rho \in$ *AIFR*(X). Relation ρ is asymmetric if and only if $\rho(x, y) \wedge \rho(y, x) = \mathbf{0}$. This means that $R(x, y) \wedge R(y, x) = 0$ and $R^d(x, y) \vee R^d(y, x) = 1$. By definition of ρ' this is equivalent to the total connectedness of ρ'. □

In virtue of the fact that $(\rho')' = \rho$ and by the results of the previous theorem one obtains

Theorem 3. *Let $\rho \in AIFR(X)$.*
- *ρ is irreflexive if and only if ρ' is reflexive.*
- *ρ is totally connected if and only if ρ' is asymmetric.*
- *ρ is connected if and only if ρ' is antisymmetric.*
- *ρ is dually transitive if and only if ρ' is transitive.*

Theorem 4. *Let $\rho \in AIFR(X)$.*
- *ρ is reflexive if and only if ρ^{-1} is reflexive (cf. [7]).*
- *ρ is irreflexive if and only if ρ^{-1} is irreflexive (cf. [7]).*
- *ρ is symmetric if and only if ρ^{-1} is symmetric.*
- *ρ is asymmetric if and only if ρ^{-1} is asymmetric.*
- *ρ is antisymmetric if and only if ρ^{-1} is antisymmetric.*
- *ρ is connected if and only if ρ^{-1} is connected.*

• *ρ is totally connected if and only if ρ^{-1} is totally connected.*
• *ρ is transitive if and only if ρ^{-1} is transitive.*
• *ρ is dually transitive if and only if ρ^{-1} is dually transitive.*

Proof. We will prove only the transitivity case. Let $x, y, z \in X$ and $\rho \in AIFR(X)$ be transitive. Thus $R(z,y) \wedge R(y,x) \leqslant R(z,x)$ and $R^d(z,y) \vee R^d(y,x) \geqslant R^d(z,x)$. As a result $R^{-1}(x,y) \wedge R^{-1}(y,z) \leqslant R^{-1}(x,z)$ and $(R^d)^{-1}(x,y) \vee (R^d)^{-1}(y,z) \geqslant (R^d)^{-1}(x,z)$ so ρ^{-1} is transitive. The converse implication is due to the fact that $(\rho^{-1})^{-1} = \rho$. □

Now preservation of the basic properties by the lattice operations will be discussed.

Theorem 5. *Let $\rho, \sigma \in AIFR(X)$.*
• *If ρ, σ are reflexive, then $\rho \wedge \sigma$ is reflexive ([7]).*
• *If ρ is irreflexive, then $\rho \wedge \sigma$ is irreflexive ([7]).*
• *If ρ, σ are symmetric, then $\rho \wedge \sigma$ is symmetric.*
• *If ρ, σ are asymmetric, then $\rho \wedge \sigma$ is asymmetric.*
• *If ρ, σ are antisymmetric, then $\rho \wedge \sigma$ is antisymmetric.*
• *If ρ, σ are transitive, then $\rho \wedge \sigma$ is transitive.*

Proof. We will prove only the transitivity case. Let $x, y, z \in X$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be transitive. Then by associativity, commutativity and monotonicity of minimum it follows that

$$(R \wedge S)(x,y) \wedge (R \wedge S)(y,z) =$$
$$(\min(\min(R(x,y), S(x,y)), \min(R(y,z), S(y,z))) =$$
$$\min(\min(R(x,y), R(y,z)), \min(S(x,y), S(y,z)))$$
$$\leqslant \min(R(x,z), S(x,z)) = (R \wedge S)(x,z).$$

Similarly it can be proven that

$$(R^d \vee S^d)(x,y) \vee (R^d \vee S^d)(y,z) \geqslant (R^d \vee S^d)(x,z).$$

As a result $(\rho \wedge \sigma)(x,y) \wedge (\rho \wedge \sigma)(y,z) \leqslant (\rho \wedge \sigma)(x,z)$, so $\rho \wedge \sigma$ is transitive. □

Example 5. Let card $X = 2$, $\rho = (R, R^d), \sigma = (S, S^d)$ be presented by matrices:

$$R = \begin{bmatrix} 1 & 0.2 \\ 1 & 1 \end{bmatrix}, \quad R^d = \begin{bmatrix} 0 & 0.6 \\ 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 1 \\ 0.1 & 1 \end{bmatrix}, \quad S^d = \begin{bmatrix} 0 & 0 \\ 0.7 & 0 \end{bmatrix},$$

$$R \wedge S = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 1 \end{bmatrix}, \quad R^d \vee S^d = \begin{bmatrix} 0 & 0.6 \\ 0.7 & 0 \end{bmatrix}.$$

Relations ρ and σ are totally connected while $\rho \wedge \sigma = (R \wedge S, R^d \vee S^d)$ does not have this property.

Example 6. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 0.2 & 1 & 1 \\ 0.1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad R^d = \begin{bmatrix} 0.7 & 0 & 0 \\ 0.8 & 0.9 & 0 \\ 0.8 & 0.9 & 0.8 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad S^d = \begin{bmatrix} 0.7 & 0.8 & 0.8 \\ 0 & 0.9 & 0.9 \\ 0 & 0 & 0.8 \end{bmatrix}.$$

Relations ρ and σ are dually transitive but $\rho \wedge \sigma = (T, T^d)$ is not dually transitive because

$$T = R \wedge S = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad T^d = R^d \vee S^d = \begin{bmatrix} 0.7 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \\ 0.8 & 0.9 & 0.8 \end{bmatrix},$$

where $T \circ' T = [t_{ij}]$ and $t_{ij} = 0$ for all $i, j = 1, ..., n$. As a result $T \circ' T < T$ and $T = R \wedge S$ is not dually transitive so $\rho \wedge \sigma$ is not dually transitive.

Theorem 6. *Let $\rho, \sigma \in AIFR(X)$.*
- *If ρ is reflexive, then $\rho \vee \sigma$ is reflexive ([7]).*
- *If ρ and σ are irreflexive, then $\rho \vee \sigma$ is irreflexive ([7]).*
- *If ρ, σ are symmetric, then $\rho \vee \sigma$ is symmetric.*
- *If ρ, σ are connected, then $\rho \vee \sigma$ is connected.*
- *If ρ, σ are totally connected, then $\rho \vee \sigma$ is totally connected.*
- *If ρ, σ are dually transitive, then $\rho \vee \sigma$ is dually transitive.*

Proof. We will prove only the dual transitivity case. Let $x, y, z \in X$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be dually transitive. Then by associativity, commutativity and monotonicity of maximum we get

$$\begin{gathered} (R \vee S)(x, y) \vee (R \vee S)(y, z) = \\ = \max(\max(R(x, y), S(x, y)), \max(R(y, z), S(y, z))) = \\ \max(\max(R(x, y), R(y, z)), \max(S(x, y), S(y, z))) \geqslant \\ \max(R(x, z), S(x, z)) = (R \vee S)(x, z). \end{gathered}$$

Similarly it can be proven that

$$(R^d \wedge S^d)(x,y) \wedge (R^d \wedge S^d)(y,z) \leqslant (R^d \wedge S^d)(x,z).$$

As a result $(\rho \vee \sigma)(x,y) \vee (\rho \vee \sigma)(y,z) \geqslant (\rho \vee \sigma)(x,z)$, so $\rho \vee \sigma$ is dually transitive. □

Example 7. Let card $X = 2$, $\rho = (R, R^d), \sigma = (S, S^d)$ be presented by matrices:

$$R = \begin{bmatrix} 0 & 0.5 \\ 0 & 0 \end{bmatrix}, \quad R^d = \begin{bmatrix} 1 & 0.2 \\ 1 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 \\ 0.6 & 0 \end{bmatrix}, \quad S^d = \begin{bmatrix} 1 & 1 \\ 0.1 & 1 \end{bmatrix},$$

$$R \vee S = \begin{bmatrix} 0 & 0.5 \\ 0.6 & 0 \end{bmatrix}, \quad R^d \wedge S^d = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 1 \end{bmatrix}.$$

Relations ρ and σ are asymmetric but $\rho \vee \sigma = (R \vee S, R^d \wedge S^d)$ does not have this property.

Example 8. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 0.7 & 0 & 0 \\ 0.8 & 0.9 & 0 \\ 0.8 & 0.9 & 0.8 \end{bmatrix}, \quad R^d = \begin{bmatrix} 0.2 & 1 & 1 \\ 0.1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.7 & 0.8 & 0.8 \\ 0 & 0.9 & 0.9 \\ 0 & 0 & 0.8 \end{bmatrix}, \quad S^d = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

Relations ρ and σ are transitive because $\rho \circ \rho = \rho$ and $\sigma \circ \sigma = \sigma$. However, $\rho \vee \sigma = (T, T^d)$ is not transitive, where $T = R \vee S$, $T^d = R^d \wedge S^d$ and

$$T = \begin{bmatrix} 0.7 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \\ 0.8 & 0.9 & 0.8 \end{bmatrix}, T^d = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, T \circ T = \begin{bmatrix} 0.8 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \\ 0.8 & 0.9 & 0.9 \end{bmatrix}.$$

We see that $T \circ T > T$, so T is not transitive and $\rho \vee \sigma$ is not transitive.

Now the composition and dual composition of intuitionistic fuzzy relations with the given property will be examined. We will concentrate mainly on the compositions of the relation $\rho \in AIFR(X)$ by itself.

Theorem 7. *Let $\rho \in AIFR(X)$.*
• *If ρ is reflexive, then $\rho \circ \rho$ is reflexive ([5]).*
• *If ρ is irreflexive, then $\rho \circ' \rho$ is irreflexive ([5]).*
• *If ρ is symmetric, then $\rho \circ \rho$ is symmetric (cf. [5]).*
• *If ρ is symmetric, then $\rho \circ' \rho$ is symmetric (cf. [5]).*
• *If ρ is transitive, then $\rho \circ \rho$ is transitive.*
• *If ρ is dually transitive, then $\rho \circ' \rho$ is dually transitive.*

Proof. If ρ is transitive, then $\rho \circ \rho \leqslant \rho$. By the monotonicity of the composition $\circ$ (cf. [5], Theorem 5) it follows that $(\rho \circ \rho) \circ (\rho \circ \rho) \leqslant \rho \circ \rho$, which means that $\rho \circ \rho$ is transitive. Similarly, by the monotonicity of the dual composition (cf. [5], Theorem 5) it follows that $\rho \circ' \rho$ is dually transitive. □

It can be easily proven that composition of arbitrary two reflexive relations is also reflexive. However, the composition (dual composition) of two arbitrary symmetric relations is not always symmetric ([5]).

Example 9. Composition of asymmetric, antisymmetric, connected, totally connected relation $\rho \in AIFR(X)$ by itself need not be asymmetric, antisymmetric, connected, totally connected, respectively. It follows from definition of composition of intuitionistic fuzzy relations and the fact that fuzzy relation - representing the membership value of the given intuitionistic fuzzy relation - which is asymmetric, antisymmetric, connected, totally connected, respectively need not have the adequate property (cf. [11], p. 78-79). Dual composition of asymmetric, antisymmetric, connected, totally connected relation $\rho \in AIFR(X)$ by itself need not asymmetric, antisymmetric, connected, totally connected, respectively. It follows from definition of dual composition of intuitionistic fuzzy relations and the fact that fuzzy relation - representing the non-membership value of the given intuitionistic fuzzy relation - which is totally connected, connected, antisymmetric, asymmetric, respectively need not have the adequate property (cf. [11], p. 78-79).

4 Preservation of semi-properties

Now, we define parameterized versions of intuitionistic fuzzy relation properties. We follow the concept of such properties given by Drewniak [11] for fuzzy relations but we restrict ourselves only to the parameter $\alpha = 0.5$. This is why we will call these properties *semi-properties*.

Definition 5 ([12])**.** Relation $\rho = (R, R^d) \in AIFR(X)$ is called:
• semi-reflexive if

$$\underset{x \in X}{\forall} \rho(x, x) \geqslant (0.5, 0.5), \tag{18}$$

• semi-irreflexive if

$$\underset{x \in X}{\forall} \rho(x,x) \leqslant (0.5, 0.5), \tag{19}$$

• semi-symmetric if

$$\underset{x,y \in X}{\forall} \rho(x,y) \geqslant (0.5, 0.5) \Rightarrow \rho(y,x) = \rho(x,y), \tag{20}$$

• semi-asymmetric if

$$\underset{x,y \in X}{\forall} \rho(x,y) \wedge \rho(y,x) \leqslant (0.5, 0.5), \tag{21}$$

• semi-antisymmetric if

$$\underset{x,y \in X, x \neq y}{\forall} \rho(x,y) \wedge \rho(y,x) \leqslant (0.5, 0.5), \tag{22}$$

• totally semi-connected if

$$\underset{x,y \in X}{\forall} \rho(x,y) \vee \rho(y,x) \geqslant (0.5, 0.5), \tag{23}$$

• semi-connected if

$$\underset{x,y \in X, x \neq y}{\forall} \rho(x,y) \vee \rho(y,x) \geqslant (0.5, 0.5), \tag{24}$$

• semi-transitive if

$$\underset{x,y,z \in X}{\forall} \rho(x,y) \wedge \rho(y,z) \geqslant (0.5, 0.5) \Rightarrow \rho(x,z) \geqslant \rho(x,y) \wedge \rho(y,z). \tag{25}$$

From Definition 4 and Definition 5 it follows

Corollary 2. *If intuitionistic fuzzy relation $\rho = (R, R^d) \in AIFR(X)$ is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, totally connected, connected, transitive, then it is semi-reflexive, semi-irreflexive, semi-symmetric, semi-asymmetric, semi-antisymmetric, totally semi-connected, semi-connected, semi-transitive, respectively.*

From definition of semi-transitivity and definition of the composition of intuitionistic fuzzy relations it follows

Corollary 3 ([12])**.** *Let $\rho = (R, R^d) \in AIFR(X)$ be an intuitionistic fuzzy relation. Relation ρ is semi-transitive if and only if*

$$\underset{x,z \in X}{\forall} (\rho \circ \rho)(x,z) \geqslant (0.5, 0.5) \Rightarrow \rho(x,z) \geqslant (\rho \circ \rho)(x,z), \tag{26}$$

which means that

$$\underset{x,z\in X}{\forall} (R \circ R)(x,z) \geqslant 0.5 \Rightarrow R(x,z) \geqslant (R \circ R)(x,z),$$

$$\underset{x,z\in X}{\forall} (R^d \circ' R^d)(x,z) \leqslant 0.5 \Rightarrow R^d(x,z) \leqslant (R^d \circ' R^d)(x,z).$$

These properties may be especially useful for intuitionistic fuzzy preference relations.

Definition 6 ([18], cf. [17])**.** Let $\overline{\overline{X}} = n$. An intuitionistic fuzzy preference relation ρ on the set X is represented by a matrix $\rho = (\rho_{ij})_{n\times n}$ with $\rho_{ij} = (r_{ij}, r_{ij}^d)$, for all $i, j = 1, ..., n$, where ρ_{ij} is an intuitionistic fuzzy value, composed by the degree r_{ij} to which x_i is preferred to x_j, the degree r_{ij}^d to which x_i is non-preferred to x_j, and the uncertainty degree π_{ij} to which x_i is preferred to x_j. Furthermore, r_{ij}, r_{ij}^d satisfy the following properties for all $i, j = 1, ..., n$:

$$0 \leq r_{ij} + r_{ij}^d \leq 1,$$

$$r_{ij} = r_{ji}^d, \;\; r_{ii} = r_{ii}^d = 0.5.$$

Directly from Definition 6 it follows that $\pi_{ij} = \pi_{ji}$ for all $i, j = 1, ..., n$.

Corollary 4 ([12])**.** *Each intuitionistic fuzzy preference relation is semi-reflexive and semi-irreflexive.*

Other results connected with intuitionistic preference relations and semi-properties are presented in [12].

Example 10. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be presented by matrices:

$$R = \begin{bmatrix} 0.5 & 0.2 & 0.9 \\ 0.7 & 0.5 & 0 \\ 0 & 0.9 & 0.5 \end{bmatrix}, \;\; R^d = \begin{bmatrix} 0.5 & 0.6 & 0 \\ 0.3 & 0.5 & 1 \\ 1 & 0 & 0.5 \end{bmatrix},$$

$$S = \begin{bmatrix} 0.6 & 0.7 & 0.3 \\ 0.7 & 0.5 & 0.3 \\ 0.1 & 0.4 & 1 \end{bmatrix}, \;\; S^d = \begin{bmatrix} 0.3 & 0.2 & 0.7 \\ 0.2 & 0.4 & 0.6 \\ 0.6 & 0.6 & 0 \end{bmatrix}.$$

ρ is semi-reflexive, semi-irreflexive, semi-asymmetric, semi-antisymmetric, totally semi-connected, semi-connected and σ is semi-symmetric.

Example 11. Let us consider relations from Example 8. Relations ρ and σ are semi-transitive because they are transitive.

Now, some transformations of intuitionistic fuzzy relations having semi-properties will be considered.

Theorem 8. *Let $\rho \in AIFR(X)$.*
- *ρ is semi-reflexive if and only if $\rho^{'}$ is semi-irreflexive.*
- *ρ is semi-irreflexive if and only if $\rho^{'}$ is semi-reflexive.*
- *ρ is semi-asymmetric if and only if $\rho^{'}$ is totally semi-connected.*
- *ρ is semi-antisymmetric if and only if $\rho^{'}$ is semi-connected.*
- *ρ is totally semi-connected if and only if $\rho^{'}$ is semi-asymmetric.*
- *ρ is semi-connected if and only if $\rho^{'}$ is semi-antisymmetric.*

Proof. We will prove only one of the given properties. Let $x, y \in X$, $\rho \in AIFR(X)$. Relation ρ is semi-asymmetric if and only if $\rho(x,y) \wedge \rho(y,x) \leqslant (0.5, 0.5)$. This means that $R(x,y) \wedge R(y,x) \leqslant 0.5$ and $R^d(x,y) \vee R^d(y,x) \geqslant 0.5$. From definition of $\rho^{'}$ this is equivalent to the total semi-connectedness of $\rho^{'}$. □

Example 12. Let card $X = 2$. We consider relation ρ from Example 4. This relation is semi-transitive but $\rho^{'} = (W, W^d)$, where

$$W = \begin{bmatrix} 0 & 0.6 \\ 0.7 & 0 \end{bmatrix}, \quad W^d = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}$$

is not semi-transitive because

$$W \circ W = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad W^d \circ' W^d = \begin{bmatrix} 0.1 & 1 \\ 1 & 0.1 \end{bmatrix},$$

so condition (26) is not fulfilled and $\rho^{'} = (W, W^d)$ is not semi-transitive. Now, let card $X = 3$. We consider semi-symmetric relation σ from Example 10. Relation $\sigma^{'} = (T, T^d)$, where

$$T = \begin{bmatrix} 0.3 & 0.2 & 0.7 \\ 0.2 & 0.4 & 0.6 \\ 0.6 & 0.6 & 0 \end{bmatrix}, T^d = \begin{bmatrix} 0.6 & 0.7 & 0.3 \\ 0.7 & 0.5 & 0.3 \\ 0.1 & 0.4 & 1 \end{bmatrix}$$

is not semi-symmetric because $t_{13} \geqslant 0.5$ but $t_{13} \neq t_{31}$.

Theorem 9. *Let $\rho \in AIFR(X)$.*
- *ρ is semi-reflexive if and only if ρ^{-1} is semi-reflexive.*
- *ρ is semi-irreflexive if and only if ρ^{-1} is semi-irreflexive.*
- *ρ is semi-symmetric if and only if ρ^{-1} is semi-symmetric.*
- *ρ is semi-asymmetric if and only if ρ^{-1} is semi-asymmetric.*
- *ρ is semi-antisymmetric if and only if ρ^{-1} is semi-antisymmetric.*
- *ρ is semi-connected if and only if ρ^{-1} is semi-connected.*
- *ρ is totally semi-connected if and only if ρ^{-1} is totally semi-connected.*
- *ρ is semi-transitive if and only if ρ^{-1} is semi-transitive.*

Proof. We will prove only the semi-transitivity case. Let $x, y, z \in X$, $\rho \in AIFR(X)$ be semi-transitive and $\rho^{-1}(x,y) \wedge \rho^{-1}(y,z) \geqslant (0.5, 0.5)$. As a result $R^{-1}(x,y) \wedge R^{-1}(y,z) \geqslant 0.5$ and $(R^d)^{-1}(x,y) \vee (R^d)^{-1}(y,z) \leqslant 0.5$. Thus $R(z,y) \wedge R(y,x) \geqslant 0.5$ and $R^d(z,y) \vee R^d(y,x) \leqslant 0.5$. As a result $\rho(z,y) \wedge \rho(y,x) \geqslant (0.5, 0.5)$ so by semi-transitivity of ρ it follows that $\rho(z,x) \geqslant \rho(z,y) \wedge \rho(y,x)$. Finally, $\rho^{-1}(x,z) \geqslant \rho^{-1}(x,y) \wedge \rho^{-1}(y,z)$ and ρ^{-1} is semi-transitive. The converse implication follows from the fact that $(\rho^{-1})^{-1} = \rho$. □

Now preservation of the semi-properties of intuitionistic fuzzy relations by the lattice operations will be discussed.

Theorem 10. *Let $\rho, \sigma \in AIFR(X)$.*
- *If ρ, σ are semi-reflexive, then $\rho \wedge \sigma$ is semi-reflexive.*
- *If ρ is semi-irreflexive, then $\rho \wedge \sigma$ is semi-irreflexive.*
- *If ρ, σ are semi-symmetric, then $\rho \wedge \sigma$ is semi-symmetric.*
- *If ρ, σ are semi-asymmetric, then $\rho \wedge \sigma$ is semi-asymmetric.*
- *If ρ, σ are semi-antisymmetric, then $\rho \wedge \sigma$ is semi-antisymmetric.*
- *If ρ, σ are semi-transitive, then $\rho \wedge \sigma$ is semi-transitive.*

Proof. Let $x, y \in X$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$. If ρ is semi-irreflexive then

$$(R \wedge S)(x,x) = \min(R(x,x), S(x,x)) \leqslant \min(0.5, S(x,x)) \leqslant 0.5$$

regardless of the value $S(x,x)$. Similarly

$$(R^d \vee S^d)(x,x) = \max(R(x,x), S(x,x)) \geqslant \max(0.5, S^d(x,x)) \geqslant 0.5$$

for any value $S^d(x,x)$. As a result $(\rho \wedge \sigma)(x,x) \leqslant (0.5, 0.5)$, so $\rho \wedge \sigma$ is semi-irreflexive.

Now we will prove the property for semi-symmetry. If ρ and σ are semi-symmetric and $(\rho \wedge \sigma)(x,y) \geqslant (0.5, 0.5)$ then $\min(R(x,y), S(x,y)) \geqslant 0.5$ and

$\max(R^d(x,y), S^d(x,y)) \leqslant 0.5$. Thus $R(x,y) \geqslant 0.5$, $S(x,y) \geqslant 0.5$, $R^d(x,y) \leqslant 0.5$ and $S^d(x,y) \leqslant 0.5$, so $\rho(x,y) \geqslant (0.5, 0.5)$ and $\sigma(x,y) \geqslant (0.5, 0.5)$. As a result $\rho(x,y) = \rho(y,x)$, $\sigma(x,y) = \sigma(y,x)$ and $(\rho \wedge \sigma)(x,y) = (\rho \wedge \sigma)(y,x)$, so $\rho \wedge \sigma$ is semi-symmetric.

Other properties may be justified in a similar way. □

Example 13. Intersection of arbitrary two semi-connected and totally semi-connected intuitionistic fuzzy relations need not be semi-connected, totally semi-connected, respectively. It follows from the fact that fuzzy relations representing the membership values of the given intuitionistic fuzzy relation which is semi-connected, totally semi-connected need not be semi-connected, totally semi-connected, respectively ([11], p. 78-79).

Theorem 11. *Let $\rho, \sigma \in AIFR(X)$.*
- *If ρ is semi-reflexive, then $\rho \vee \sigma$ is semi-reflexive.*
- *If ρ and σ are semi-irreflexive, then $\rho \vee \sigma$ is semi-irreflexive.*
- *If ρ is semi-symmetric, then $\rho \vee \sigma$ is semi-symmetric.*
- *If ρ, σ are semi-connected, then $\rho \vee \sigma$ is semi-connected.*
- *If ρ, σ are totally semi-connected, then $\rho \vee \sigma$ is totally semi-connected.*

Proof. We will prove only the property for semi-symmetry. Let $x, y \in X$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be semi-symmetric and $(\rho \vee \sigma)(x,y) \geqslant (0.5, 0.5)$. Thus $\max(R(x,y), S(x,y)) \geqslant 0.5$ and $\min(R^d(x,y), S^d(x,y)) \leqslant 0.5$. There are four possible cases:

$1^0)$ $R(x,y) \geqslant S(x,y)$ and $R^d(x,y) \geqslant S^d(x,y)$,

$2^0)$ $R(x,y) \geqslant S(x,y)$ and $R^d(x,y) \leqslant S^d(x,y)$,

$3^0)$ $R(x,y) \leqslant S(x,y)$ and $R^d(x,y) \geqslant S^d(x,y)$,

$4^0)$ $R(x,y) \leqslant S(x,y)$ and $R^d(x,y) \leqslant S^d(x,y)$.

We will consider the first case and proof for the rest is analogous. From semi-symmetry of ρ and σ it follows that $R(x,y) \geqslant 0.5 \Rightarrow R(x,y) = R(y,x)$, $R^d(x,y) \leqslant 0.5 \Rightarrow R^d(x,y) = R^d(y,x)$, $S(x,y) \geqslant 0.5 \Rightarrow S(x,y) = S(y,x)$, $S^d(x,y) \leqslant 0.5 \Rightarrow S^d(x,y) = S^d(y,x)$. Thus form the first case it follows that $R(x,y) \geqslant 0.5$ and $S^d(y,x) \leqslant 0.5$, so $R(x,y) = R(y,x)$ and $S^d(x,y) = S^d(y,x)$. We will show that $R(x,y) \geqslant S(y,x)$. Suppose that $R(x,y) < S(y,x)$. Then from assumptions of the first case we obtain $0.5 \leqslant R(x,y) < S(y,x)$ so from semi-symmetry of σ we have $S(x,y) = S(y,x)$. As a result $R(x,y) < S(x,y)$ which contradicts to assumptions of the first case. So $\max(R(x,y), S(x,y)) = R(x,y)$, $\max(R(y,x), S(y,x)) = \max(R(x,y), S(y,x)) = R(x,y)$ and this implies

$(R \vee S)(x,y) = (R \vee S)(y,x)$. Similarly we can prove that $(R^d \wedge S^d)(x,y) = (R^d \wedge S^d)(y,x)$. As a result $(\rho \vee \sigma)(x,y) = (\rho \vee \sigma)(y,x)$, so $\rho \vee \sigma$ is symmetric. □

Example 14. Let card $X = 3$, $\rho = (R, R^d), \sigma = (S, S^d) \in AIFR(X)$ be the ones form Example 8. Relations ρ and σ are semi-transitive. Relation $\rho \vee \sigma = (T, T^d)$ is presented by the following matrices:

$$T = R \vee S = \begin{bmatrix} 0.7 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \\ 0.8 & 0.9 & 0.8 \end{bmatrix}, \quad T^d = R^d \wedge S^d = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and it is not semi-transitive. We can check it with the use of Corollary 3 where

$$W = T \circ T = \begin{bmatrix} 0.8 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \\ 0.8 & 0.8 & 0.9 \end{bmatrix}, \quad T^d \circ' T^d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Relation $\rho \vee \sigma$ is not semi-transitive because $t_{11} \geqslant 0.5$ and $t_{11} < w_{11}$.

Example 15. Sum of any two semi-asymmetric and semi-antisymmetric intuitionistic fuzzy relations need not be semi-asymmetric,
semi-antisymmetric, respectively. It is the consequence of the fact that fuzzy relations representing the membership values of the given intuitionistic fuzzy relation which is semi-asymmetric, semi-antisymmetric need not be semi-asymmetric, semi-antisymmetric, respectively ([11], p. 78-79).

Now the composition and dual composition of the intuitionistic fuzzy relations with the given property from Definition 5 will be examined. We will concentrate only on the compositions of the relation $\rho \in AIFR(X)$ by itself.

Theorem 12. *Let $\rho \in AIFR(X)$.*
- *If ρ is semi-reflexive, then $\rho \circ \rho$ is semi-reflexive.*
- *If ρ is semi-irreflexive, then $\rho \circ' \rho$ is semi-irreflexive.*

Proof. Let $x \in X$, $\rho \in AIFR(X)$ be semi-reflexive. Thus $R(x,x) \geqslant 0.5$ and $R^d(x,x) \leqslant 0.5$, so

$$(R \circ R)(x,x) = \sup_{y \in X} \min(R(x,y), R(y,x)) \geqslant$$

$$\sup_{y=x} \min(R(x,y), R(y,x)) \geqslant \min(0.5, 0.5) = 0.5$$

and

$$(R^d \circ' R^d)(x,x) = \inf_{y \in X} \max(R(x,y), R(y,x)) \leqslant$$
$$\inf_{y=x} \max(R(x,y), R(y,x)) \leqslant \max(0.5, 0.5) = 0.5.$$

This means that $(\rho \circ \rho)(x,x) \geqslant (0.5, 0.5)$ and $\rho \circ \rho$ is semi-reflexive. Similarly we can prove the case of irreflexivity. □

5 Conclusions

In the paper the problem of preservation of some intuitionistic fuzzy relation properties by transformations was discussed. The complement, the converse, composition and dual composition of a relation were considered as the basic operations on relations of the type $\mathfrak{F} : AIFR(X)^n \to AIFR(X)$, $n \in \mathbb{N}$. In addition, preservation of properties by lattice operations was checked. Among others, semi-properties of intuitionistic fuzzy relations were examined, namely: semi-reflexivity, semi-irreflexivity, semi-symmetry, semi-connectedness, semi-asymmetry, semi-transitivity. As a result theorems or examples presented obtained results were provided. However, there is an open problem whether the composition or dual composition of semi-asymmetric, semi-antisymmetric, semi-connected, totally semi-connected, semi-symmetric and semi-transitive relation $\rho \in \mathit{AIFR}(X)$ by itself is semi-asymmetric, semi-antisymmetric, semi-connected, totally semi-connected, semi-symmetric, semi-transitive, respectively. For the further considerations other operations on intuitionistic fuzzy relations and other types of composition may be considered as transformations and examined whether they preserve given properties or do not.

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References

[1] Atanassov K. (1984) Intuitionistic fuzzy relations. Proc. of the Third Intern. Symp. Automation and Sci. Instrumentation, Varna, October 1984, Part II, 1984, 56–57.

[2] Atanassov K. (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96.

[3] Atanassov K. (1999) Intuitionistic Fuzzy Sets. Theory and Applications. Springer-Verlag. Heidelberg.

[4] Birkhoff G. (1967) Lattice theory. American Mathematical Society. Providence.

[5] Burillo P., Bustince H. (1995) Intuitionistic fuzzy relations. (Part I). Mathware Soft Comput. 2, 5–38.

[6] Burillo P., Bustince H. (1995) Intuitionistic fuzzy relations. (Part II) Effect of Atanassov's operators on the properties of the intuitionistic fuzzy relations. Mathware Soft Comput. 2, 117–148.

[7] Burillo P., Bustince H. (1996) Structures on intuitionistic fuzzy relations. Fuzzy Sets Syst. 78, 293–303.

[8] Bustince H. (2000) Construction of intuitionistic fuzzy relations with predetermined properties. Fuzzy Sets Syst. 109, 379–403.

[9] Deschrijver G., Kerre E.E. (2002) A generalisation of operators on intuitionistic fuzzy sets using triangular norms and conorms. Notes on IFS 8, 19–27.

[10] Deschrijver G., Kerre E.E. (2003) On the composition of intuitionistic fuzzy relations. Fuzzy Sets Syst. 136 (3), 333–361.

[11] Drewniak J. (1989) Fuzzy Relation Calculus. Silesian University. Katowice.

[12] Dudziak U., Pękala B. (2011) Intuitionistic fuzzy preference relations, in: Proceedings of the 7th conference of the European Society for Fuzzy Logic and Technology (EUSFLAT-2011) and LFA-2011, S. Galichet et al. (eds.) Atlantis Press, 529–536.

[13] Gerstenkorn T., Tepavčević A. (2004) Lattice valued intuitionistic fuzzy relations and applications, in: K.T.Atanassov et al. (eds.), Soft Computing Foundations and Theoretical Aspects. EXIT. Warszawa, 221–234.

[14] Gong Z-W., Li L-S., Zhou F-X., Yao T-X. (2009) Goal programming approaches to obtain the priority vectors from the intuitionistic fuzzy preference relations, Computers and Industrial Engineering 57, 1187-1193.

[15] Gong Z-W., Li L-S., Forrest J., Zhao Y. (2011) The optimal priority models of the intuitionistic fuzzy preference relation and their application in selecting industries with higher meteorological sensitivity. Expert Systems with Applications 38, 4394-4402.

[16] Lin L., Yuan X-H., Xia Z-Q. (2007) Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets. Journal of Computer and System Sciences 73, 84–88.

[17] Szmidt E., Kacprzyk J. (2002) Using intuitionistic fuzzy sets in group decision making, Control and Cybernetics 31, 1037–1053.

[18] Xu Z. (2007) Intuitionistic preference relations and their application in group decision making, Information Sciences 177, 2363-2379.

[19] Zadeh L.A. (1965) Fuzzy sets. Information and Control 8, 338–353.

[20] Zadeh L.A. (1971) Similarity relations and fuzzy orderings. Inform. Sci. 3, 177–200.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2011

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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