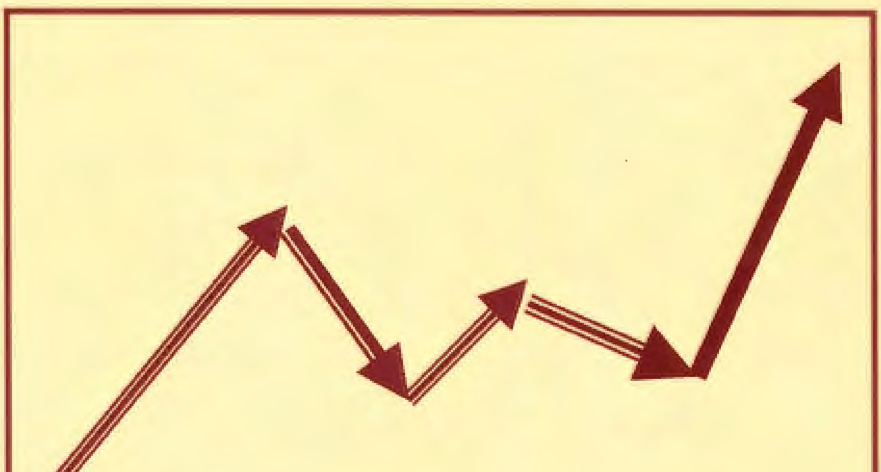
STANISŁAW PIASECKI

AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume II



Warsaw 2011

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INTRODUCTION

The purpose of the present book is to show the possibility of developing a quantitative description of the action of "invisible hand" on the market. This is why the text is full of mathematical expressions, even though they are kept purposefully at the possibly simple level.

At the same time, the book is a subsequent publication of results from the work on establishment of foundations for the theory of economic competition, limited, however, essentially to what is called price competition. Let us note at this point that the "competitors" here considered are the companies selling their produce on the common and limited market, and that price competition analysed takes place among the products of different companies, serving to satisfy the very same kind of demand from the side of the customers.

Price competition ought to be regarded as a dynamic market game, which takes place within the space of retail prices, i.e. in the "open", before the eyes of the consumers, or in the space of wholesale prices – behind the scenes. The strategies of the players consist in selection of prices, at which their products (services) are sold.

Under a close examination of the problem it turned out that most important for defining the market price game is determination of the "payoff function". The present book is devoted, therefore, mainly to this problem.

This volume constitutes a continuation of the considerations from the first volume of "*Introduction to a Theory of Market Competition*", in which territorial expansion strategy of companies has been analysed, this strategy allowing for expansion of sales and lowering of prices. Yet, sooner or later, the instant has to come when a company must enter an "alien" market, and, after a successful entry, face the problem of expanding its market share.

Hence, it is the last two issues that this book takes up and analyses.

The considerations forwarded therein are based on three fundamental assumptions:

- Increase of price of purchase entails a decrease of the number of products sold.
- Increase of the number of products turned out makes it possible to lower the cost of producing these products.
- The market secures the preservation of equilibrium between demand and supply. Customers are directed by reason when making purchasing decisions.

The first assumption results from the fact that each customer has limited financial capacities (of purchasing products and services).

The second assumption is justified by the commonly observed "production scale effect", which results from the continuous technological progress, taking place especially in the domain of production technologies. This fact finds its confirmation in the history of economic development – from handicraft through workshop production to the present-day mass (even if customised) production.

The third, double assumption is associated with the adoption of principles of free market.

In order to represent the "scale effect", the hyperbolic relation was used, resulting from the analysis of the constant and variable production costs.

To describe the dependence of demand upon the product price, stemming from the income structure of potential customers, the linear dependence was used, which is characteristic for the constant income density of customers.

Other adopted assumptions and simplifications are of technical character.

Many of the Readers shall certainly be disappointed, as they will not find in the book the statistical inquiries, based on what is called "real-life data", that would confirm the assumptions adopted and the results obtained. In order, though, for a theory to be subject to verification, it must first be formulated. It should be indicated that the precepts of this theory have been successfully implemented in economic reality, in the practice of quite a significant company in Poland.

Thus, the contents of this book ought to be regarded as an attempt of formulating a definite theory, by no means pretending to having exhausted the entire problem area. It should be added that the results contained in both volumes published so far result from the research done by the respective authors within the Systems Research Institute of the Polish Academy of Sciences. Separate thanks go to the NTT System S.A. company that supported financially the publication of both volumes.

The authors of both volumes hope that this modest contribution shall serve its purpose of providing to the Readers the very first insight into the possibility of representing and analysing in quantitative terms the processes we observe daily on the globalising markets. The authors would also like to announce the preparation of the subsequent volume, presenting the extension to the theory here expounded.

Warsaw, June 2011

Chapter I

DEMAND DYNAMICS AND MARKET DIVISION

This chapter starts with the definitions of some basic notions and a broader justification of the assumptions adopted in further course of the content.

1. Some basic definitions

We shall call **market** the **entirety of customer needs** for customers residing on a definite area.

These needs can be classified into **material** and non-material.

In what follows we shall be dealing with a fragment of the market, defined by the nature of the material needs of customers, these needs being satisfied through purchasing of goods, belonging to a concrete **group of products**. Such a fragment of the market shall be referred to as a **"sector" of the market**.

Products, belonging to the same group, satisfy the same needs of customers, but they usually differ significantly, in quantitative terms – by their purchase prices, and in qualitative terms – by their conditions of use. Consequently, a sector of the market undergoes further division into the customers requiring higher quality for satisfaction of their needs and purchasing better products, consenting to their higher prices. Products meant for more demanding customers are commonly referred to as products from the "higher shelves".

Sectors of the market may, therefore, be subdivided into "**market segments**", called sometimes "shelves". A market segment is thus determined by the nature of need and the level of requirements (and wealth) of customers.

Within the framework of a definite segment a group of products satisfying the same needs forms the **group of mutually competitive products**.

We shall define the group of competitive products as:

- the set of products meant to satisfy the same needs of the customers on a given market;
- the set of products featuring similar properties regarding their use and frequently of similar aspect;
- the set of products, whose conditions of purchase are similar (price of purchase and supply, as well as the value of "bonus" for having purchased the product).

The use of expression "similar" means, in the case of:

use-related properties – that the costs of using products originating from different companies vary at most by a couple of percentage points;
conditions of purchase – that the purchasing prices of products originating from various companies, with the potential "bonuses" deducted, differ at most by a couple of percentage points.

It is obvious that the notion of "similarity" is in this case not sharp, that it is a "fuzzy" notion.

A market is formed by the potential customers of the companies, who, by purchasing respective products, wish to satisfy their needs. A market is determined by two aspects:

= the nature of the need that the customers wish to have satisfied by purchasing the product;

= the set of potential customers with this kind of need,

while the demand for products, satisfying these needs depends, first of all, upon:

= the cost of owning (using) the product,

= the quality of the product – i.e. the degree of satisfaction of the need (satisfaction of the customer),

= the dimensions of the market (e.g. the number of potential customers).

As it is easily noticed, the cost of owning (and using) a product depends, in particular, upon the price (C) of the product.

When a customer can choose on the market a number of products that satisfy the same need, s/he will, of course, buy the one that is cheaper (in terms of price) or better (in terms of quality).

If a company wants to dominate on a market, pushing aside its competitors, it has to sell its products at a lower price, or these products have to be better than those of the competitors.

In the first case we deal with price competition of the products of comparable quality, while in the second case – with quality-based competition.

It should be kept in mind that producing a better competitive product is associated with a longer term undertaking and investment into turning out of a new – better – product. This is, as a rule, a costly, long-lasting undertaking, which, in addition, does not always give the expected result. It is lengthy because it usually requires conducting appropriate inquiry (often scientific research), developing a prototype and testing it, elaboration of a complete construction and production documentation, as well as adaptation of the existing production system and facilities to production of the new item. At the same time, all this does not guarantee achievement of the goals set, as during the undertaking our competitors may realise what our intentions are (as one can hardly hide them) and start turning out a new product earlier or lower the price of an own product so much, that our new and better product, but unavoidably more expensive, shall not find many buyers, as the one from a "higher shelf".

Thus, the entire investment undertaking may constitute a net loss instead of bringing expected profit.[1]

In distinction from the risk, associated with taking of strategic decision on starting production of a new product and problems with financing of such an undertaking, the goal of pushing away the competitors from the market might be attained immediately, by taking an operational decision of the maximum,

[1] A more detailed description of the process of putting in motion new production, along with determination of the forecast of future sales can be found in Piasecki (2003).

temporal lowering of the sales price of an own product, admitting a decrease of profit, or even agreeing to the complete lack of profit.

The sacrifice we thereby make is temporary, because after having pushed away the competitors from the market, we shall be able to raise the prices again, but also frequently gain a significant profit even without returning to the previous price, due to increase of production and sales volume. (The question of determination of optimum price was treated at length in volume 1.)

We shall consider in further course in a more detailed manner the price competition, and in particular – the significance of the sales price of products for the course of the process of competition.

As mentioned already before, demand for definite product on a given market (or a market segment) shall be measured with the expected number of products sold per unit of time. If we measure the volume of products sold in their sheer number, then demand shall be expressed in number of products sold per week, per year etc., depending upon the time unit adopted. If we measure the volume of products sold in tonnes, then demand shall be expressed in tonnes per week, per year, etc. In what follows we shall assume that we measure the volume of production in number of items, with year being the time unit, which, of course, does not in any way diminish the generality of considerations.

Demand for a definite product is a function of many variables (frequently non-measurable ones), and we shall not list them here. Yet, there are two quantities, which exert the biggest influence on the volume of demand, namely the dimensions of the market, determined by the number of buyers of the product, sold at price C (currency units per product unit) and the cost of using the product.

Before, however, we determine the magnitude of the market and the volume of demand, we should say a bit more on the notions of buyer and customer. These notions may, in particular, refer to natural persons, individuals of a definite age group, gender, denomination etc., to social or occupational groups (family, farmers, clerks, etc.), or to legal persons – enterprises, companies, corporations and so on – of a definite branch, scale etc.

Definition of the kind of customer is directly associated with the envisaged use of the product, established by the designer and the producer – for what kind of customer is this product meant? Identification of the character of customer target group ought, therefore, not pose any difficulty, since it should have been formulated at the stage of designing the product.

2. Demand function

Since demand is always linked to a definite product, we shall first of all distinguish the group of potential customers, formed by the (natural or legal) persons, as to whom we expect that they would like to own the product and for whom it is meant. The number of persons in this group of potential customers shall be denoted L_{mx}. In this group we will distinguish those customers, who could afford purchasing and using the product at the price of C. We shall refer to this sub-group of potential customers as buyers. The number of persons in this group shall be denoted L, with, of course, $L \le L_{mx}$.

Naturally, the value of L is a function, $L(C)$, of the price C, with

$$L(\infty) = 0 \le L(C) \le L(0) = L_{mx}\,.$$

In order to determine this function we must unambiguously establish when a given customer shall not be able to purchase the product in view of its too high price in relation to the income of the customer. Let us denote the annual income of the (potential) customer by d.

First, we ought to define the notion of “too high price”, which depends upon many factors and is in a natural manner a “fuzzy” notion. We shall resolve this problem in a simplified manner, assuming that the notion of “too high price” is the resultant of comparison of two quantities:

- the part γ of the annual income d can be devoted by the potential customer to the purchase and use of the product, with, of course, $0 < \gamma < 1$; and
- the cost (or benefit) resulting from the use of the product, depending upon the characteristics of the product.

In order to determine these quantities, we must distinguish two kinds of products: of one-time (consumable) and durable use. In both cases, though, it is necessary to determine the time period of use, T. If we deal with a one-time, consumable product, like a loaf of bread, the time period of use equals the time of consumption. If it is durable good, like a TV set, then the period of use adopted may be constituted by the technical or economic period of its exploitation. All these time periods are, of course, usually random variables, and so the symbol T shall denote further on the expected (mean) value of the respective random variable.

In the case of one-time use products, like the loaf of bread, mentioned before, the cost of use shall be defined by the value of the ratio C/T.

In other words, when we deal with a durable good, like the TV set, referred to above, then we should add to this ratio the cost (e) of its exploitation (for a TV set – the cost of electric energy consumed per time unit),i.e. of running it. Hence, total (unit) cost of use will take the form of $\frac{C}{T}+e$.

In the case of, e.g., a car, the running cost shall encompass the costs of fuel, engine oil, lubricants, tires etc., as well as repairs, periodical servicing, insurance, parking and garaging, etc.

If the product serves in conduct of business activity, then the cost of its exploitation should be decreased by the advantage (revenue) resulting from its use.[2] In order to stay with the simple and intuitive relations, we shall confine our analysis to the case of the exploitation costs in the form of the ratio C/T, leaving it to the Reader to develop more realistic expressions.

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[2] A more detailed account on the running costs (of exploitation) of durable goods can be found in Piasecki (2003) or in Piasecki (1972).

Let us add that similarly broad and fuzzy interpretation is associated with the quantity γd.

The simplest interpretation consists in treating this quantity as the limit value on the credit repayment installments of the credit used to buy the product, such that the customer can still afford. Conform with this interpretation, the limit value would have to account for the credit cost, associated with the banking service.

Such a situation illustrates well the issue of purchasing of a durable good by a natural or legal person.

In the case of consumables, the value of the expression γd corresponds to the willingness of a customer to spend (in every time unit) such an amount of money on purchasing this type of products.

Let us note that the quantity γ is widely used by persons managing household economy. It is namely common to divide up the monthly income of the family into the fractions (γ) meant for financing of particular kinds of expenditures, including the repetitive purchasing of consumable goods and services.

After these explanations we can define the notion of "too high price". And so, a price C of a product is too high for a potential customer when it does not fulfill the inequality

$$\gamma d \geq C/T.$$

In particular, using this inequality, we can determine the limit income of the potential customers, d_{lim}, such that if the income of a potential customer is lower than this value, then the customer is not capable of purchasing a product of price C with the period of use of T. This limit value is given by:

$$d_{lim} = (C/T)/\gamma.$$

Consequently, demand generated by an individual customer (buyer) is expressed as:

$$\lambda_0 = 1/T,$$

in units per year, tonnes per year, etc.

In order, therefore, to determine the overall demand for a product on a given market, we must yet establish the number of buyers L, i.e. the number of natural or legal persons, whose incomes are not less than d_{lim}. Hence, we should dispose of the cumulative distribution function of incomes (see Volume 1) of the potential customers, $F(d)$:

$$F(d) = \frac{1}{L_{mx}} \cdot L(d)$$

where the function $L(d)$ defines the number of potential customers, whose income is not higher than d. Then, the function $F(d)$ defines the proportion of the number of potential customers, whose income is not higher than d. These functions are, in general, nonnegative, and they satisfy the following conditions:

$$L(0) = 0,\ L(d_{mx}) = L_{mx};\ F(0) = 0,\ F(d_{mx}) = 1,$$

where d_{mx} is the highest income of a potential customer on a given market.

In the sociological studies it is most common to use another function, called "income distribution", $l(d)$, defined as follows:

$$l(d) = \lim_{\Delta d \to 0} \frac{F(d + \Delta d) - F(d)}{\Delta d}$$

The values of the income distribution function satisfy the following obvious conditions:

$$l(0) = 0,\ l(d) > 0 \text{ for } 0 < d < d_{mx},\ l(d_{mx}) = 0.$$

In particular, on the basis of knowledge of the income distribution function, we can determine, for instance, mean income of the potential customers:

$$\overline{d} = \int_{0}^{d_{mx}} x \cdot l(x) \cdot dx$$

and, most importantly, we can determine the number of buyers of the product, i.e. the number of those potential customers, whose incomes are not less than the respective d_{lim}.

The number of buyers equals $L_{mx} \cdot \int_{d_{\lim}}^{d_{mx}} l(x)dx = L_{mx} \cdot q(d_{\lim})$, where the function $q(d_{lim}) = \int_{d_{\lim}}^{d_{mx}} l(x)dx = 1 - F(d_{lim})$ defines the income group of customers.

When we know the intensity λ_0 (in, say, units per year) of demand of a single customer (buyer) for a given product, we can determine the magnitude of demand, Λ, from the following formula:

$$\Lambda(d_{lim}) = L_{mx}\lambda_0 q(d_{lim}) = \lambda_{mx} q(d_{lim}),$$

or, substituting the value for d_{lim}, we obtain:

$$\Lambda(C) = \lambda_{mx} q(C/\gamma T) = \lambda_{mx} q(C) = L_{mx}\lambda_0 q(C).$$

Note that the value of function $q(C)$ can be interpreted as the value of probability that a randomly selected potential customer has an income d sufficient for purchasing the product. Hence, we can write down that the expected (mean) intensity of demand for the product from a potential, statistical customer is expressed through the formula $\lambda = \lambda_0 q(C)$, while demand is expressed through $\Lambda = L_{mx}\lambda_0 q(C) = \Lambda_{mx} q(C)$.

This way of writing down the relations of interest allows for a distinct separation of the expression for demand from price C and the number of potential customers, i.e. the magnitude of the market.

So, in order to establish the formula relating demand (Λ) to price (C) of the product, we should know the income structure (q) of the potential customers or their income distribution (l), and the technical parameters of the product, which, in our simplified model, reduce to the knowledge of the period of using the product (T). It is, of course, also necessary to know the "intensity of will" (or "impatience") of purchasing the product, represented by the value of γ. On the other hand, the value of λ_0 is usually taken to be equal to the inverse of T.

Example of determination of demand as a function of price

Assume that the function of income density has the form as in Fig. 1.1, i.e. $l(d) = l_0 = \text{const.}$ for $0 < d < d_{\max}$, and 0 for remaining values of the argument.

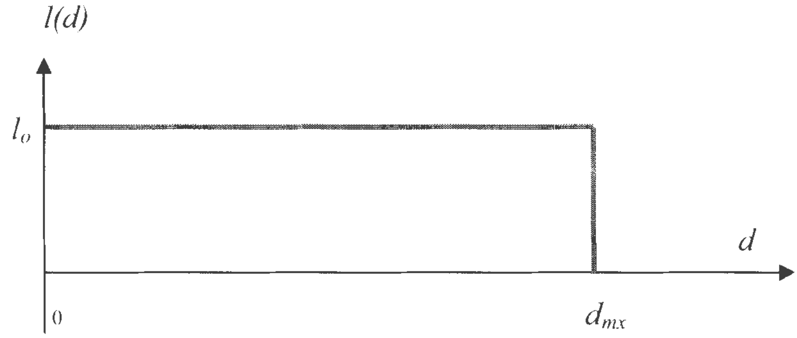

Figure 1.1. An example of income distribution function

Then:

$$q(d) = \frac{1}{l_o\, d_{\max}} \int_{d}^{d_{\max}} l_o dx = 1 - \frac{d}{d_{\max}}$$

We have already denoted with d_{lim} the limit income, below which a potential customer would not be able to afford the product that costs C and will last for T. The respective expression has been $d_{lim} = C/(\gamma T)$, in appropriate units, where γ is the coefficient defining the maximum fraction of the income that a potential customer is ready to devote for purchasing the product (coefficient of "impatience").

Hence, if within the zone of influence of a sales outlet there are L_{mx} potential customers, then the expression $q(d_{lim})L_{mx}$ defines the number of buyers of the product (i.e. potential customers, who would afford buying the product). Each product, after the time period T, will in principle have to be replaced by a new one (perhaps featuring already better parameters of use). Consequently, the process of replacement of the worn out or consumed products leads to determination of the intensity of purchase for a single customer (user), namely $\lambda_0 = 1/T$, with the forecasted intensity of product sale being equal

$$\Lambda = \lambda_0 q(d_{lim}) L_{mx}.$$

In the case of a uniform distribution of income, like in Fig. 1, we therefore obtain

$$\Lambda = \lambda_0(1 - d_{lim}/d_{mx})L_{mx} = \lambda_0(1 - C/(\gamma T d_{mx}))L_{mx}.$$

We can now introduce the notation

$$C_{mx} = \gamma T d_{mx}$$

for the maximum price of a product, at which the number of persons, who can afford it, falls down to zero. Then the previous expression becomes:

$$\Lambda = \lambda_{mx}(1 - C/C_{mx}) \text{ or } \Lambda = \lambda_{mx} - aC,$$

where $\lambda_{mx} = \lambda_0 L_{mx}$ and $a = \lambda_{mx}/C_{mx}$.

This is the simplest linear model of demand for definite products on a definite market, with a given structure of incomes of the population. We should not forget, though, that both the volume of the market and the needs of the users, as well as their incomes, change over time. There is more on that in Volume I.

3. On the influence of fashion and opinion on demand

We have not mentioned until now the influence of the opinion of the social environment nor the one of the changing fashions, exerted upon the decision of a customer of buying a product against the background of other competing products.

We have been assuming until now that a customer is guided solely by an own economic interest, meaning that if there is a choice of buying product A or B, these two products serving to satisfy the same need, for, respectively, the price C_A or C_B, the choice will be of the product, for which the ratio C_A/T_A or C_B/T_B is smaller, T_A, T_B being, as before, the respective duration (use) periods of the two products.

In particular, when the use of a product requires energy consumption, then the respective T can be represented by the time period of use of the products for the same cost of energy use. Thus, for instance, if the product is a passenger car, then the times T_A and T_B may correspond to the number of hours of driving (over the same kind of routes) on fuel purchased for the same money.

It turns out, though, that we sometimes act otherwise. Namely, we often give in to the pressure of the opinion of the environment we belong (or we wish to belong) to, and we purchase not the product that satisfies the above criterion of own interest, but the one that we ought or are supposed to own. A commonly known phenomenon is that a President or CEO of a company should own an adequately expensive car (*Mercedes, BMW,...*) in order to maintain appropriately high position in the community of company Presidents. And this - even if a much cheaper car would suffice, in functional terms.

By driving an expensive car, despite the (apparent) obvious violation of the principle of economic self-interest, we gain imponderable prestige, which, in turn, may generate quite definite economic advantages (like helping in getting more convenient credit conditions in a bank).

These "imponderable advantages" are sometimes gained in a stupefying, but successful manner by various frauds.

Another example of renouncing the criterion of own interest is provided by the influence of fashion, which is variable 'like a woman" (*la donna e mobile*). A distinct influence on the purchasing decisions can be observed on the example of women's garments. And so, for instance, a piece of knitwear is be sought and bought by female customers, even though of higher price and lower quality, only because its colour is currently fashionable.

Thus, while the opinion of a social environment has a character that is constant over longer periods of time, fashion usually changes from year to year. Besides, the opinion of the social environment tends to be concentrated on price characteristics of products, while fashion concerns the features often insignificant from the point of view of product utility (like colour). Yet, both kinds of influence drive the prices of the products purchased upwards, well beyond the cost of production, especially in the case of fashionable products.

Let us try to identify the mechanism of influence of the non-economic factors on the magnitude of demand, A. We shall consider this mechanism on the example of two products, A and B, satisfying the same need of customers, but differing distinctly as to their prices, $C_A > C_B$. Take as example sports footwear turned out under a known brand A (*Adidas*, *Nike*, …) and by some less known company B.

Assume that the price difference is perhaps justified by the difference of durability of the footwear, expressed through T_A and T_B. It may happen that the values

$$d^A{}_{lim} = C_A/(\gamma T_A) \text{ and } d^B{}_{lim} = C_B/(\gamma T_B),$$

defining the lower limits of the customer's income, at which s/he can afford the considered products of companies A and B get equal, so that

$$d^A{}_{lim} = d^B{}_{lim} = d_{lim}.$$

The products under consideration, even though physically different, but satisfying the same kind of need, become mutually competitive on a given market and may, consequently, be treated as "identical" (equivalent) from the point of view of competition.

It may also happen that the values of durability of the products considered are equal, so that $T_A = T_B = T$, and hence

$$d^A{}_{lim} > d^B{}_{lim}.$$

Now, the two products are from the point of view of competition different, even though they satisfy the very same kind of need of the customers. Of course, one of them, the more expensive one, ought normally to be eliminated from the market. This, though, shall not happen, when the essential role is played by the opinion of the snobbish society of the wealthier customers.

The initially considered group of potential customers for the sports footwear shall therefore get stratified into the sub-group of customers (acting against their economic interest) fulfilling the income condition $d > d^A{}_{lim} = C_A/(\gamma T)$, and the sub-group of customers of lower incomes.

Consequently, we deal here with a new phenomenon. Namely, demand for products B, featuring better economic properties, shall constitute only a fraction of the total demand for the sports footwear, with prices satisfying the condition:

$$0 < C_B < C_A < C_{mx}.$$

Accordingly, demand for products A shall be expressed as:

$$\Lambda_A = \lambda_0 L_{mx} q(d^A{}_{lim}) = \lambda_0 L_{mx}(1 - d^A{}_{lim}/d_{mx}) = \lambda_{mx}(1 - C_A/C_{mc}) = a(C_{mx} - CA),$$

while the total demand for the sports footwear of the two companies shall be equal

$$\Lambda_A + \Lambda_B = \lambda_{mx} q(d^A{}_{lim}) + \lambda_{mx}[q(d^B{}_{lim}) - q(d^A{}_{lim})] = \lambda_{mx}(1 - C_B/C_{mx}).$$

Note that such an irrational decision of the buyer, considering own economic interest, could be justified by the fact of gaining some additional, not defined more precisely, advantage M, which, in a way, decreases the cost of purchase down to the value of C_A-M, so that the following inequality holds:

$$\frac{C_A - M}{T} < \frac{C_B}{T}.$$

Alas, we do not know how to determine the value of M, and even how to strictly define this value.

Situation on the market may, though, get even more complicated, if we account for the influence of fashion. This may concern, for instance, colours of shoes, or the length of the shoe flap, or any other otherwise inessential detail of sports footwear.

If this inessential detail undergoes year-to-year change, then the period of use T' shrinks to just one year. Consequently, the values

$$d^A{}_{lim} = C_A/(\gamma T') \text{ and } d^B{}_{lim} = C_B/(\gamma T')$$

shall dramatically increase (usually the period of exploitation T for similar products is much longer than a year), and thus also the values of L_A and L_B shall significantly decrease.

Ultimately, the "prestige" group shall get further divided up into two sub-groups – the ones, who shall be buying every year the more expensive footwear, priced C_A, and the ones, who would not be able to afford such frequent purchase of new shoes and shall use the newly purchased shoes for more than one year. The income borders between the groups shall change once again.

Broader consideration of the influence of "prestige" and fashion on demand can be found in Volume I. The considerations herein are only meant to show the possibility of treating this kind of phenomena within the framework of the proposed theory of market competition. Further on, though, we shall be assuming rational economic behaviour of customers, which excludes from the scope of the considered application of the theory some products that depend strongly on, for instance, fashion.

4. Unit cost of production

As we know, production costs (e.g. over a year) are classified into constant (irrespective of the production scale) and variable costs, which increase proportionally to the scale (intensity) of production. The first group of costs includes, in particular, amortisation (repayment) of structures and technological apparatus, estate tax, costs of heating and protection, etc. The second group of costs encompasses direct costs of labour, of the mounted parts, of energy used during the production process, etc.

A Reader is certainly familiar with the commonly known diagram of dependence of production costs upon the intensity or scale of production, shown in Fig. 1.2. Such a diagram, side by side with the constant and variable costs, shows also the function of revenues from the sale of products turned out, allowing for the determination of the threshold (lower limit) of profitable scale or intensity of production. This diagram is complemented by the (heavy red) line of unit cost, arising from the division of production cost for a definite intensity by the number of products turned out. The diagram clearly illustrates the mechanism of the "production scale effect". At the same time, it indicates the character of the function describing the scale effect, which is adopted in further course of considerations.

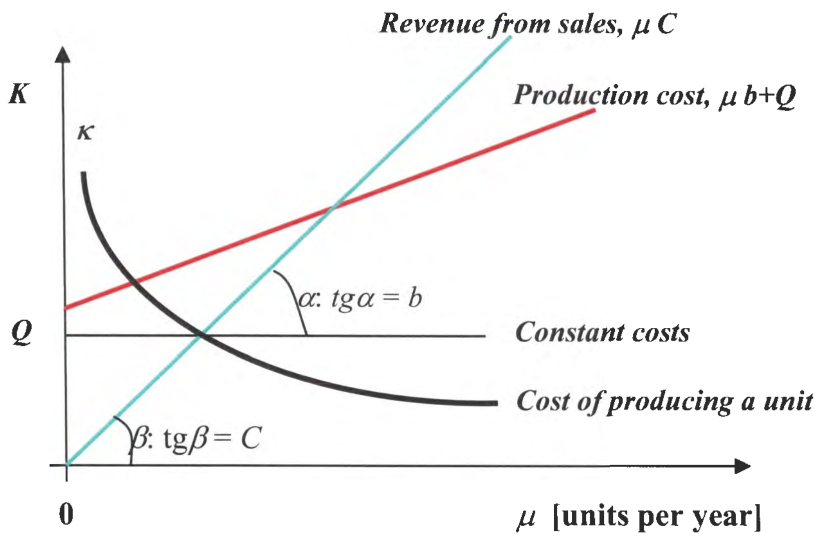

Figure 1.2. Dependence of costs upon production intensity

Until now, we have not taken into account the fact that the relation $K(\mu)=b+Q/\mu$ is a continuous function only for a limited scale of changes of the value of μ. For large values of changes in the scale of production changes take place in production technology, accompanying the changes in the value of Q.

Hence, the dependence $K(\mu)$ has the form shown in Fig. 1.3 for a broader scope of changes in the value of μ.

As can be seen in Fig. 1.3, particular ranges of the value of μ are covered by different production technologies. The figure shows also – thicker segments on the horizontal μ axis – these intervals of values of μ, which are not profitable for application in practice. One of the technologies, illustrated in the figure, marked with an asterisk (*), is entirely unprofitable.

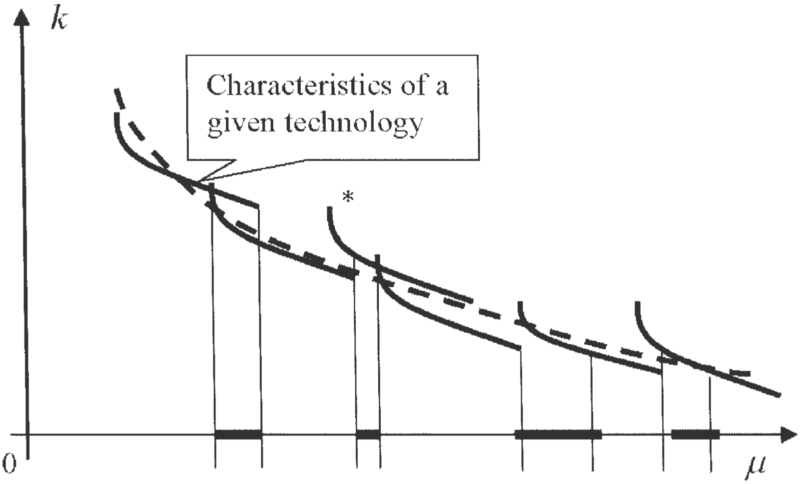

Figure 1.3. Cost function for a broader scope of changes in production intensity

The shape of dependence $K(\mu)$, adopted in this book, is shown in Fig. 1.3 with a broken line. It should be emphasized that the policy of granting rebates in case of purchase of a greater quantity of parts, serving to manufacture the final product, lowers the value of b, and hence of K, along with the increase of production scale.

The magnitude of the rebate is, alas, also a discontinuous function. Yet, the effect of rebate makes the nonlinear character of the dependence of unit cost of production upon the scale of production, μ, even more pronounced.

A broader justification of the above is provided in Volume 1.

5. Dynamics of market processes

Let us consider what would happen if we lower at a certain instant the price of a product from C down by some value ΔC. Doubtless, the number of buyers shall increase, since in the new situation a part of the potential customers, who could not afford purchasing the product, because of its price, shall become buyers.

Definitely, the new price, C-ΔC, shall still remain too high for a part of the potential customers. Let us denote the number of new buyers by ΔN.

When the density of incomes of potential customers in dependence upon the income levels is a constant function, then the demand function has the following form:

$$\Lambda(C) = \Lambda_{mx}(1\text{-}C/C_{mx}).$$

In case when two equivalent products appear on the market, total demand for these products shall be determined by the lower of the two prices. Thus, for instance, when a product priced C_B is being sold on the market, then demand is expressed as

$$\Lambda = \lambda_{mx}/C_{mx}\,(C_{mx}\text{-}C_B).$$

If, in this situation, an equivalent product appears, featuring a lower price C_A, then the number of customers, who would be able to purchase the (cheaper) products shall increase, and total demand shall increase to

$$\Lambda = \lambda_{mx}/C_{mx}\,(C_{mx}\text{-}C_A).$$

Consequently, the general formula, defining the limit total demand for two kinds of competitive products shall have the following form:

$$\Lambda = \lambda_{mx}/C_{mx}\,(C_{mx}\text{-}\min\{C_A,C_B\}) = \lambda_{mx}(1\text{-}\min\{C_A,C_B\}/C_{mx}).$$

And hence we shall get

$$\Delta\Lambda = \Lambda(C_A)\text{-}\Lambda(C_B) = \Delta C\cdot(\Lambda_{mx}/C_{mx}),\ \text{where}\ C_A = C\text{-}\Delta C,\ C_B = C.$$

It is also understandable that not everybody shall immediately switch over to buying the cheaper product. Some will receive the information on the lower price with a delay, others would start pondering whether they really can now afford, etc.

Thus, we will be observing a dynamic growth of the number of new customers over time, which can be expressed through a function $\Delta N(\tau)$, with $\Delta N(0) = 0$, where symbol $\tau = t\text{-}t_0 > 0$, denotes the time having passed since the decrease of price by ΔC.

Hence, it is interesting to track the course of function $N(t)$,not only as it depends upon price C, but also in time t.

After what time, assuming an initial value N_0 at the time instant t_0, shall the number of customers reach the value of $N_0+\Delta N$?

If we admit that the phenomenon we consider has a similar character as inertia in physics, then the course of the respective process shall be described with the exponential function $a^{\alpha\tau}$. In this context we shall assume that $0 < a < 1$ and $\alpha > 0$.

We then obtain:

$$N(t_0+ \tau) = N_0+ \Delta N\,(1- a^{\alpha\tau}),\ \text{where}\ \tau = t - t_0 > 0.$$

Note, further, that we might write down the exponential function $a^{\alpha\tau}$ in a different manner, namely as $a^{\alpha\tau} = e^{\tau\alpha\ln a}$.

This means that the speed, with which market reacts, is determined by the number, being the product of values of α and $\ln a$.

Observation of the market tells us that this speed depends upon two factors: the magnitude of price decrease, ΔC, and the nature of the product.

The value of a (depending upon the magnitude of price decrease) is decisive for the speed of growth of the group of new buyers. It should, therefore, depend upon the concrete values C_A and C_B, say, as $a = C_A/C_B$ (remembering that $C_A < C_B$).

If we assume that the new buyers do not differ in any way from the previous ones, the decrease of price by ΔC shall entail the increase of demand by ΔA, as considered before, with

$$\Delta A = A(C_A)\text{-}A(C_B) = \Delta N\cdot(\upsilon/T) = \Delta C\cdot(A_{mx}/C_{mx}),$$

where υ is the number of products, purchased in the framework of a single purchasing transaction, and T is the average time period between the successive purchasing transactions.

We will assume – like we have been doing implicitly until now – that $\upsilon = 1$, and then the time period T has the previously assumed interpretation of the time duration of product use. Let us note that purchasing of bigger numbers of products at a time may only be justified by the wish of minimizing transport costs, which is lower – per single item – when we transport bigger amounts of the product items.

Consequently, see Fig. 1.3, considering that $\Delta\Lambda = \Delta N / T$, we get

$$N(\tau) = N_0 + \Delta N \cdot \left\{ 1 - \left(\frac{C - \Delta C}{C} \right)^{\alpha\tau} \right\}$$

for $\tau \geq 0$, or, generally:

$$N(t) = \begin{Bmatrix} N_0, \; for \; t \leq t_0 \\ N_0 + \Delta N (1 - a^{\alpha\tau}) \; for \; \tau = t - t_0 \geq 0 \end{Bmatrix}.$$

As it can be inferred from this expression, for $\Delta C = 0$ we have $N(t) = N_0$, and for $\tau = t - t_0 = 0$ we also get $N(0) = N_0$. But the function $N(t)$ attains the value of $N_0 + \Delta N$ only in infinity, which is in contradiction with the observed reality.

This contradiction is being avoided in such a way that a certain value of tolerance is established, and it is admitted that the function actually attains the limit value of $N_0 + \Delta N$, when its calculated value $N(t)$ differs from the limit $N_0 + \Delta N$ by the value equal the assumed tolerance.

By multiplying both sides of the above formula for $N(t)$ by λ_0, we obtain

$$\Lambda(t) = \begin{Bmatrix} \Lambda_0 \; for \; t \leq t_0 \\ \Lambda_0 + \Delta\Lambda \cdot (1 - a^{t \;\; t_0}) \; for \; t \geq t_0 \end{Bmatrix}.$$

Knowing the values of ΔC and ΔN (or $\Delta\Lambda = \Delta N / T$), we can estimate the value of α, characterizing the speed of reaction of the market. This speed of reaction depends, though, not only upon the price decrease magnitude, but also on the characteristics of the product and the manner of selling it.

And so, for instance, lowering of the price of a commodity quoted on some commodity exchange entails an immediate reaction on all exchanges. For commodities not quoted on the exchanges the processes of evening out of prices are much slower.

In order to determine the value of α one can take advantage of the temporary price changes at the occasions of holidays, special promotions, company anniversaries etc., by registering shifts in demand at some established intervals of time.

Now, Fig. 1.4 shows an illustration for the courses of the function of increase in the number of buyers over time for various values of the expression $\alpha \ln a$.

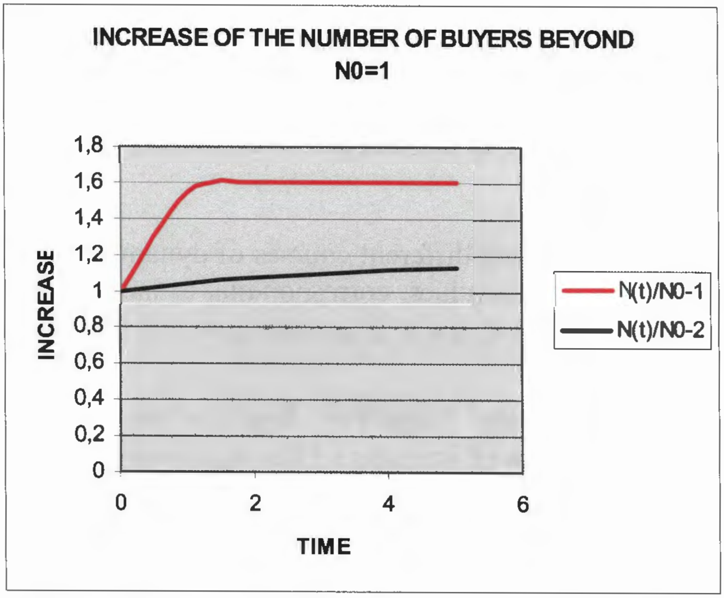

Figure 1.4. Two examples of the demand increase function

It is common in physics to characterize the inertia processes by the lag time τ_0. It is the value of the substitute time of delay of reaction of an inertia system to an external stimulus. The principle of determination of the lag time value is sketched in Fig. 1.5.

The area, contained between the segment $(0, \tau_0)$ of the axis t and the curve of $N(t)$ must be equal the area between the curve and the segment $(\tau_0, +\infty)$ of the straight line ΔN = const. For an exponential function this will define the point on the time axis, for which the function attains approximately 63% of its maximum value (i.e. 63% of the value of ΔN in Fig. 1.5).

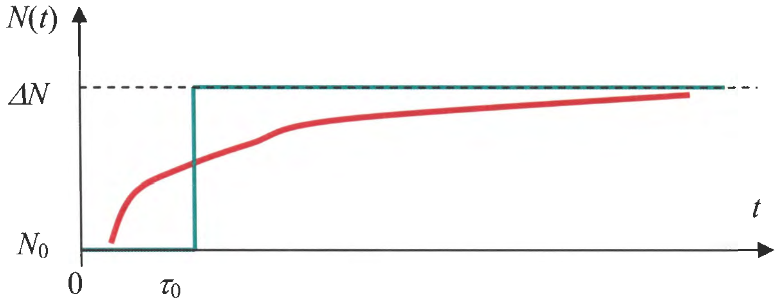

Figure 1.5. Illustration of the principle of determining the lag time

Fig. 1.6 shows four different courses of dynamics of demand increase. The parameter values, corresponding to individual curves are as follows: 1. $a = 0.5$, $\alpha = 4$; 2. $a = 0.5$, $\alpha = 1$; 3. $a = 0.667$, $\alpha = 1$; and 4. $a = 0.8$, $\alpha = 1$.

By comparing the respective diagrams we can see the difference of dynamics of increase of the additional ΔN buyers in relation to the dynamics of increase of the buyers of newly introduced products.

In addition, the diagrams provided allow for estimation of the substitute lag τ_0 of the above processes. If we draw in Fig. 1.6 a horizontal line at the level of 0.63, then the points, at which it will cross the curves shown, shall correspond to the values of the substitute time lag for the four processes illustrated, namely, respectively, 0.355, 1.44, 2.45 and 4.47.

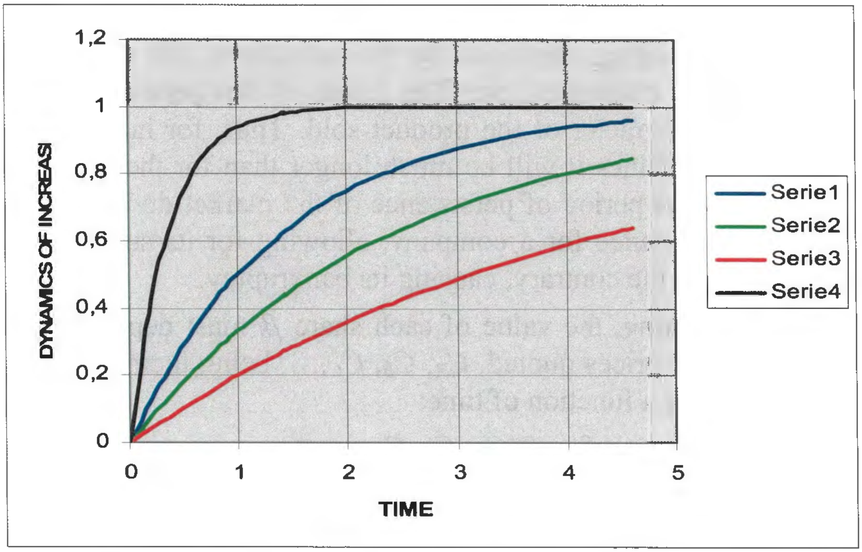

Figure 1.6. Four examples of demand increase dynamics

6. Division of the market among companies

In the case of a common market, on which products of many companies are sold, the individual companies, by selling their produce, take a share of this common market.

Denote by β_i the share of the market of company i, $i = A, B, C, D, \ldots$ The shares β_i take, of course, values from the open interval (0,1), and their sum must be equal 1.

We shall assume that products of all these companies satisfy a definite need of the customers and that there are no essential differences in the use of these products, and the sole property that differentiates the products is constituted by their sales prices C_i.

Of course, various sales prices can cause only temporary division of the market, described with the values of β_i, but after a period of time, the entire market would be taken over by the company selling its competitive product at the lowest price.

It is fully understandable that the time period, during which a definite market division persists, depends upon a number of factors, in particular – the speed of transmission of information on prices, the inertia in making decisions by the customers, the degree of conservatism of customers, etc. The length of this period depends also upon the character of the product sold. Thus, for instance, in the case of medicines it will be much longer than for the TV sets. Yet, even a short period of persistence of the market division may be of key importance for a company, allowing for its survival on the market or to the contrary, causing its bankruptcy.

In this setting, the value of each share β_i must depend upon the values of all prices quoted, C_A, C_B, C_C,..., being their function, along with being a function of time:

$$\beta_i = \beta_i(C_A, C_B, C_C, \ldots; t).$$

Note that the market share coefficients, when we consider sales of equivalent goods at different prices by different companies, must satisfy the following obvious conditions:

- for each pair of non-negative values, e.g. (C_A, C_B) the following relation must hold: if $C_A < C_B$ then $\beta_A > \beta_B$, for all other prices kept constant;
- for all $C_A = C_B$ there holds $\beta_A = \beta_B$, all other prices kept constant;
- for every set of non-negative prices (C_A, C_B, C_C, ...) the following equality must hold: $\Sigma_i\beta_i(t) = 1$ for all t.

An example of the market division function

The simplest system of two demand functions, fulfilling formal conditions, for two companies selling their produce on a common market, are the functions of the type

$$\beta_A = \frac{C_B^n}{C_A^n + C_B^n} \quad ; \quad \beta_B = \frac{C_A^n}{C_A^n + C_B^n}.$$

For this system, we obtain the following cases:

-- when n=0, then the division of the market does not depend upon the values of prices, is constant and equal:

$$\beta_A = \beta_B = \frac{1}{2}$$

-- when $n = ½$, then

$$\beta_A = \frac{(C_B)^{\frac{1}{2}}}{(C_A)^{\frac{1}{2}} + (C_B)^{\frac{1}{2}}} = \frac{1}{1 + \left(\frac{C_A}{C_B}\right)^{\frac{1}{2}}}$$

-- when n=1, then

$$\beta_A = \frac{C_B}{C_A + C_B} = \frac{1}{1 + \frac{C_A}{C_B}}$$

-- when n=2, then

$$\beta_A = \frac{C_B^2}{C_A^2 + C_B^2} = \frac{1}{1 + \left(\frac{C_A}{C_B}\right)^2}$$

-- when n=4, then

$$\beta_A = \frac{C_B^4}{C_A^4 + C_B^4} = \frac{1}{1 + \left(\frac{C_A}{C_B}\right)^4}$$

etc.

The functions, shown in the example, describe market split in a certain concrete time instant τ. Generally, though, the respective coefficients have to change in time – according to the present theory – conform to exponential functions.

Such functions are relatively easy to determine. The simplest of them are the functions of the following type:

$$\Lambda_A(\tau) = \Lambda_0 \cdot (C_B)^{\alpha\tau} / ((C_A)^{\alpha\tau} + (C_B)^{\alpha\tau})$$

$$\Lambda_B(\tau) = \Lambda_0 \cdot (C_A)^{\alpha\tau} / ((C_A)^{\alpha\tau} + (C_B)^{\alpha\tau}),$$

where: $\alpha = n/T$; $C_A = C - \Delta C$, $C_B = C$, $\Delta C \geq 0$.

The temporal courses of functions Λ/Λ_0 are shown in Fig. 1.7.

Fig. 1.7 shows the case, when in the initial time instant, $\tau = 0$, both companies have equal (50%) shares of the market. The curves numbered odd refer to Company A (selling its products at a significantly lower price) and they are ordered according to the increasing value of α ($\alpha_1 = 1$, $\alpha_3 = 2$, $\alpha_5 = 3$). The curves numbered even concern Company B and are ordered analogously.

The exponent α of the function considered characterises the speed of reaction of the market, which depends upon the mobility of the opinion of buyers concerning the appearance of a cheaper (and not worse!) product. Hence, obviously, the speed of change of market shares depends upon the value of α. As can be easily seen, as time tends to infinity the cheaper product shall wipe out completely from the market the more expensive one.

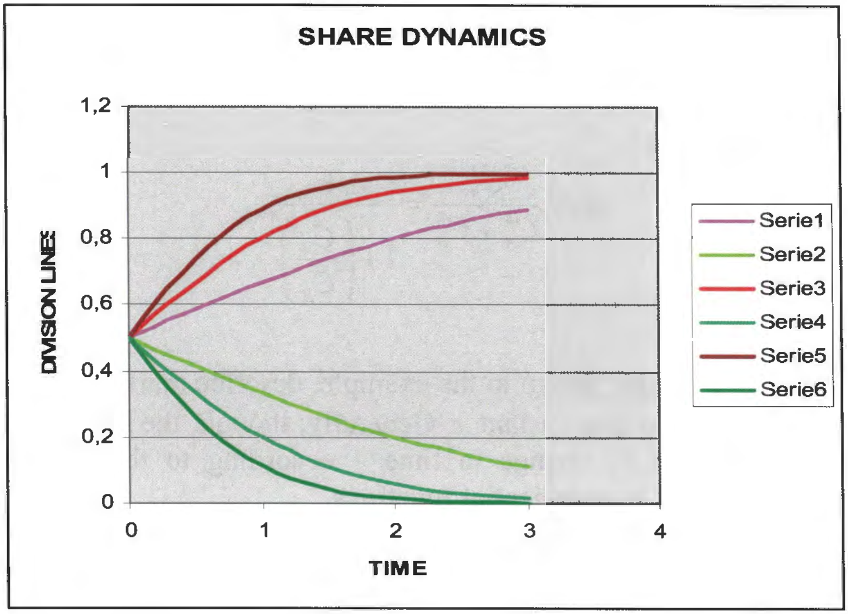

Figure 1.7. Examples of temporal courses of market share functions

So, the properties of the function describing market shares correspond in qualitative terms to the real market processes, with one reservation: in reality the complete removal of the more expensive product (of the same quality) from the market takes place not in infinity, but in a finite time period.

In order to deal away with this discrepancy we can safely assume that with low sales, the company that has been supplying half of the market before, shall be liquidated by the banking system, under the weight of the debts, arising from the maintenance of the unnecessarily vast fixed assets. Hence, it is reasonable to assume that if the sales fall below, say, 40% of the initial volume, the company shall go bankrupt and its sales shall abruptly dwindle to zero. In this manner we can define the limit value h and the interval $(0, h)$, within which the respective functions are valid.

So, in order to preserve the properties of the market split functions from the example and, at the same time, to account for the change in the character of dependence of the market shares for large value of time τ, we can modify these functions, assuming that it is valid only within a predefined interval of values of τ, between zero and h.

We then have

$\Lambda_A(\tau) = \Lambda_0 \cdot (C_B)^{\alpha\tau} / ((C_A)^{\alpha\tau} + (C_B)^{\alpha\tau})$ for $\tau < h$ and $\Lambda_A(\tau) = 1$ for $\tau \geq h$

$\Lambda_B(\tau) = \Lambda_0 \cdot (C_A)^{\alpha\tau} / ((C_A)^{\alpha\tau} + (C_B)^{\alpha\tau})$ for $\tau < h$ and $\Lambda_B(\tau) = 0$ for $\tau \geq h$.

It is more convenient to use another form of the function. Namely, if we divide the functions by Λ_0, and the nominators and denominators by $(C_B)^{\alpha\tau}$, then we obtain the following expressions for the coefficients $\beta(\tau)$ of market shares:

$$\beta_A(\tau) = 1 / (1 + (C_A/C_B)^{\alpha\tau}) = 1 / (1 + a^{\alpha\tau}),$$

and

$$\beta_B(\tau) = (C_A/C_B)^{\alpha\tau} / (1 + (C_A/C_B)^{\alpha\tau}) = a^{\alpha\tau} / (1 + a^{\alpha\tau}).$$

It can easily be noticed that in time instants $\tau = 0$ and $\tau = \infty$ we get:

$\Lambda_A(0)/\Lambda = ½$, $\Lambda_B(0)/\Lambda = ½$, and $\lim_{\tau\to\infty} \Lambda_A(\tau)/\Lambda = 1$, $\lim_{\tau\to\infty} \Lambda_B(\tau)/\Lambda = 0$.

An attentive Reader must certainly have noticed by now that the problems of price competition are being considered in this book on the examples concerning two competing firms, but there is no obstacle to broadening the application of the method presented to more than two firms.

Authors wished to avoid the situation of a significant extension of the book with a high mathematical load that might frighten away all those that do not deal in mathematics. There is certainly enough of it already now.

Namely, in the case of competition of many companies, before solving the problem of the most advantageous strategies of their behaviour, we would have to face the problem of determination of the coalitions of some companies against the other ones. The latter, difficult problem, let us hope, shall be taken up in the future by other researchers.

The dynamic processes of changes of market shares of companies, induced by the decreases of prices of some of the competitive products, shall be considered for two cases, namely when:

- Case 1: on a market, monopolized by the Company B (100% share), a company A enters with its product;
- Case 2: on a balanced ("equilibrated") market, on which two companies function, A and B, each with 50% share, competing with similar products, having the same sales prices, company A decides at some time instant to start selling its products for a lower price, wishing to increase its market share, or even drive company B entirely from the market.

Let us note that the case of increase of profits by the dominating company (having 100% market share) was considered in Volume I. The strategy in such a case consists in the change of price of the product to the optimum price. The manner of determining the optimum price was also provided in Volume I.

It is necessary to explain at this point the notion of "equilibrated" (balanced) market.

This very name originates from the notion of market equilibrium or of a stable (equilibrated) situation. Thus, namely, if we admit existence of two (or more) competing companies on a common, limited market, then their persistent coexistence is conditioned by the selling of their competitive products at the same price, and, in addition, by them having similar market shares. The latter condition results from the "production scale effect". If some company had a significantly bigger market share, it would be able to produce at a cost lower than the competitors. Having achieved higher profits, it could use them for a more intensive advertising or it could lower the product sales price. In both cases this would lead to a further increase in the market share.

Thus, only satisfaction of both conditions ensures a stable situation. Such a stable situation shall be referred to as equilibrated market or market equilibrium. It is easy to see that this equilibrium is actually unstable, for reasons mentioned above (i.e. if situation departs from this equilibrium, it will unavoidably tend to a qualitatively different situation, e.g. disappearance of a competitor from the market). The equilibrium, like the one outlined, might have arisen also under different circumstances, when the companies enter into collusion and become a cartel, whose existence is secured by factors outside of our consideration here.

In conditions of competition on a free market, with equal access to production and sales technology, the sole stable equilibrium is constituted by the situation, when one, victorious company remains on the market, with its product.

Yet, even then this company cannot feel completely assured.

Let us, namely, consider the first of the cases outlined before – when Company A wants to enter the market with its competitive products. For this purpose it lowers the price of its products by ΔC with respect to the current sales price C on the market of similar products.

The simplest function, describing the dynamic reaction of the market is constituted by the already invoked exponential function, for which the market share coefficients take the forms of:

$$\beta_A(\tau) = 1 - a^{\alpha\tau} \qquad\qquad \beta_B(\tau) = a^{\alpha\tau},$$

where $a = (C - \Delta C)/C$, fulfilling the obvious conditions

$$\beta_A(0) = 0,\ \beta_B(0) = 1,$$
$$\beta_A(\infty) = 1,\ \beta_B(\infty) = 0,$$
$$\beta_A(1/\alpha) = \Delta C/C,\ \beta_B(1/\alpha) = (C - \Delta C)/C,$$
$$\beta_A(\tau) + \beta_B(\tau) = 1 \text{ for } \tau > 0.$$

The here described process of market sharing shall divide the group of "old" buyers, those from before the time instant $\tau = 0$, who have been purchasing until then only the products of Company B, into two groups: the ones purchasing the cheaper products of Company A, and the ones, who continue to purchase the products of Company B.

This, however, is not everything that happens. Namely, the number of buyers shall increase. Appearance on the market of a new, cheaper product shall cause that those potential customers, who have not been able, until now, to afford the purchase of product too expensive for them, shall now become able to afford buying it, as we have indicated this before. The increase of the number of new buyers shall proceed according to the function

$$\Delta\Lambda(\tau) = \Delta\Lambda \cdot \left(1 - a^{\alpha_2 \tau}\right).$$

Consequently, demand for products of Company A shall attain the value

$$\Lambda_A(\tau) = \Lambda_0(1 - a^{\alpha_1 \tau}) + \Delta\Lambda(1 - a^{\alpha_2 \tau}), \text{ with } \alpha_1 < \alpha_2,$$

where Λ_0 is the magnitude of demand from before the instant $\tau = 0$.

On the other hand, demand for products of Company B shall decrease according to $\Lambda_B(\tau) = \Lambda_0\, \beta_B(\tau) = \Lambda_0\, a^{\alpha_1 \tau}$.

Fig. 1.8 shows the situation described with the above formulae.

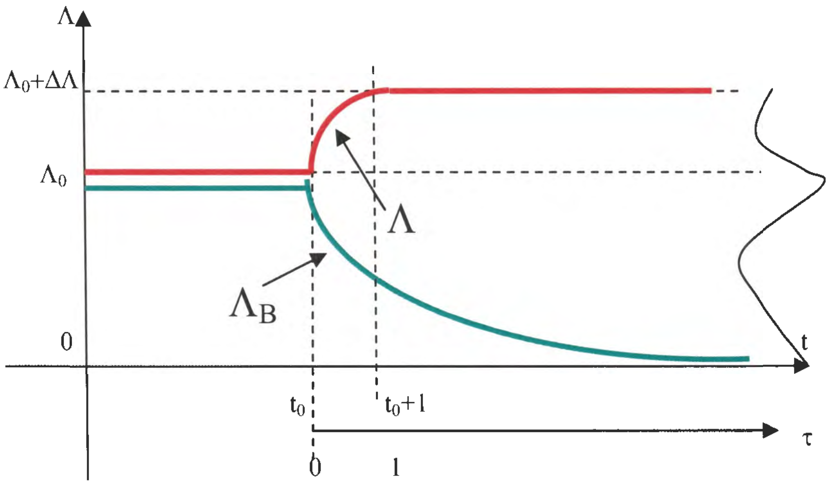

Figure 1.8. Illustration for case one of market share dynamics

Let us now consider Case 2: division of the market between two companies. Assume, as before, that on an equilibrated market two companies function, A and B, selling for the same price C similar products (not differing as to the possibility of satisfying definite needs of customers).

It is sensible to assume for this case that demand for products of each of the companies is the same, i.e.

$$\Lambda_A = \Lambda_B \text{ and } \Lambda_A + \Lambda_B = \Lambda_0.$$

The problem we consider now is to determine the proportion, in which this total demand Λ_0 is split when one of the two companies, say, Company A, lowers the price of its product by the value of $\Delta C > 0$, down to the level of $C - \Delta C$?

The functions, defining the division of the magnitude Λ_0 into two parts must, naturally, satisfy the following conditions:

- the sum of the two parts must always equal Λ_0;
- when the value of ΔC increases, then the value of Λ_A must increase, as well;
- no part may take negative value at any time;
- for $\Delta C = 0$ the equality $\Lambda_A = \Lambda_B = ½\, \Lambda_0$ must hold.

The following pair of functions fulfils these conditions:

$$\Lambda_A^0(\tau) = \Lambda_0 \frac{1}{a^{\alpha_1\tau}+1}, \qquad \Lambda_B^0(\tau) = \frac{a^{\alpha_1\tau}}{a^{\alpha_1\tau}+1}, \qquad a = \frac{C-\Delta C}{C}.$$

Conform to these functions the split takes place of the pre-existing demand (and associated buyers) Λ_0 into two parts, $\Lambda^0{}_A$ and $\Lambda^0{}_B$, of (an increasing number of) those, who purchase the cheaper products of Company A, and those, who continue to purchase the product of Company B.

Besides, demand Λ_0 shall increase by a definite fraction, due to the lowered price by Company A:

$$\Delta\Lambda = (\Lambda_{mx}/C_{mx})\cdot\Delta C.$$

Consequently, the entire demand for the product of Company A shall be given by

$$\Lambda_A(\tau) = \Lambda_A^0(\tau) + \Delta\Lambda(\tau) = \Lambda_0 \frac{1}{a^{\alpha_1\tau}+1} + \Delta\Lambda\cdot\left(1-a^{\alpha_2\tau}\right),$$

while demand for the product of Company B shall be given by

$$\Lambda_B(\tau) = \Lambda_B^0(\tau) = \Lambda_0 \cdot \frac{a^{\alpha_2\tau}}{a^{\alpha_2\tau}+1}.$$

The respective scheme of the dynamics of market shares is shown in Fig. 1.9.

Yet, it must be remembered that such function describes the dynamics of market shares for a definite starting point at $\tau = 0$. The shape of the market share functions can be easily generalised by introducing a parameter u, defining the initial market split (u being equal 1 for the split of 50%/50%). The shares take then the form

$$\beta_A(\tau) = \frac{u}{a^{\alpha_1\tau}+u} \qquad \beta_B(\tau) = \frac{a^{\alpha_1\tau}}{a^{\alpha_1\tau}+u},$$ where $a = C_A/C_B$, i.e. the ratio of prices after and before the decrease.

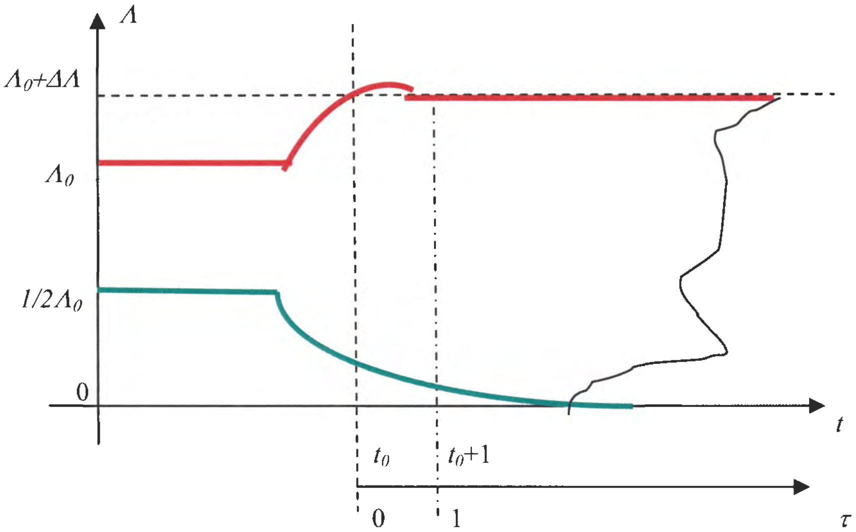

Figure 1.9. Dynamics of the initially equal market shares after unilateral price decrease

The initial values, at $\tau = 0$, of the two functions, are:

$$\Lambda_A(0) = u/(1+u) \text{ and } \Lambda_B(0) = 1/(1+u).$$

The values of parameters of the exponential functions, that is – a and α are either known (a), or can be identified (α) on the basis of measurements of the demand processes for a concrete market segment and a given group of products. The values of these parameters can also be estimated on the basis of historical data.

We yet should explain where the differences of values of parameters $\alpha_1 \neq \alpha_2$ come from.

In fact, we assumed tacitly different velocities, α_1 and α_2, of changes in the numbers of buyers of both the "new" ones (α_2), who appeared only at the time instant $\tau = 0$, and of the "old" ones (α_1), who had been buying before the products of the other company.

This distinction is justified by some qualitatively different circumstances. We ought to consider, namely, *two situations*.

The first one can be characterised as follows. There exists a definite group of "old" customers, using since some time ("since long") the kind of product under consideration and generating a stable demand for this product, when purchased for the price C. At a certain time instant t_0 ($\tau = 0$) a new, competitive product appears on the market, sold for a lower price, $C\text{-}\Delta C$. In this situation, such customers, after having used previously purchased products, shall replace them with the new, cheaper ones, and generate demand for the new product, of the same magnitude (provided the two products have the same duration of use). Theoretically, all the products purchased to date shall be replaced by the new ones after the time period T (for $\alpha = 1/T$ and $n = 1$), i.e. after the end of use of the products purchased to date. During this period of time the market share coefficient β of the new products shall increase from zero to one, while the share of the old product shall drop from one to zero. In reality, though, the process of replacement of the old, used products by the new ones shall be prolonged in time, owing to the random nature of the length of time period T. This is described by an exponential function.

The second situation is characterised by the fact that it concerns "new" customers, who shall appear on the market at the instant of appearance of the "new", cheaper products. They are those potential customers, for whom purchasing of the needed product for the price C was not possible, but when the price dropped, purchasing became possible. If so, theoretically, this group shall generate a new demand instantaneously at time t_0 of the maximum feasible intensity, limited only, in terms of its realisation, by the production capacities of the respective producing companies. Lines would form in front of the sales outlets, if the price decrease were sufficiently attractive. This is the second situation.

The demand function in question could theoretically have the shape as shown in Figs. 1.10a or 1.10b.

In reality, the courses of the respective actual processes shall be flattened, as not everybody would at once go shopping, for a variety of reasons that are beyond our interest here.

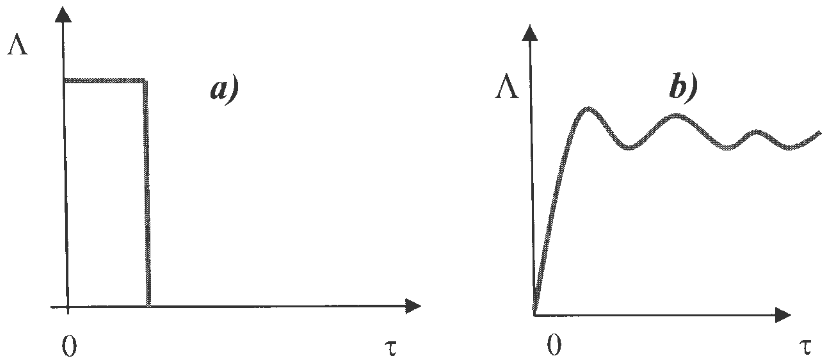

Figure 1.10. Theoretically possible courses of the demand functions for the change conditions considered

And this is exactly why two significantly different values of the exponents, α_1 and α_2, have been assumed here, as illustrated in Fig. 1.11.

The two outlined situations clearly differ as to the speed of increase of demand for the new product, appearing in the time instant $\tau = 0$. In the first of these situations demand increases with a moderate speed, attaining a significant value after time $T = 1/\alpha_1$, while in the second situation demand increases dramatically, attaining its full dimension in a much shorter time $1/\alpha_2$.

If in the first case the increase of sales of the new products can be described with the exponential function

$$\Lambda_A(\tau) = \Lambda_0 \cdot \frac{u}{a^{\alpha_1 \tau} + u}$$

then in the second case, when new customers appear, the increase of sales must be described with a different function, for instance:

$$\Delta\Lambda(\tau) = \Delta\Lambda \cdot \left(1 - a^{\alpha_2 \tau}\right)$$

where $\alpha_1 << \alpha_2$, and $a = C_A/C_B$. In more general terms, for two players on the market, the change considered of the sales volume would be described by the functions

$$\Lambda_A(\tau) = \Lambda_0 \cdot \frac{u}{a^{\alpha_1 \cdot \tau} + u}$$

$$\Delta_A(\tau) = \Delta\Lambda \cdot \left(1 - a^{\alpha_2 \cdot \tau}\right)$$

$$\Lambda_B(\tau) = \Lambda_0 \cdot \frac{a^{\alpha_1 \cdot \tau}}{a^{\alpha_1 \cdot \tau} + 1}$$

where $\alpha_1 << \alpha_2$.

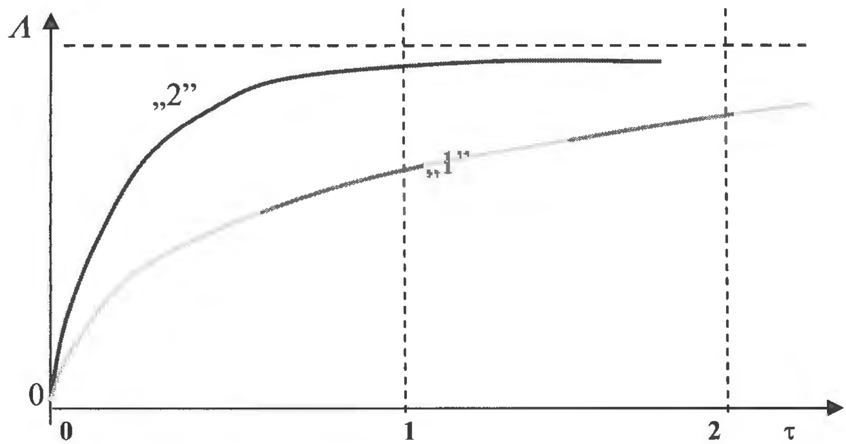

Figure 1.11. Illustration for the different speeds of demand increase

If $\alpha_2 \geq 2\alpha_1$, $3\alpha_1$, $4\alpha_1$ ... etc., then we shall accept the approximation that at time $\tau = 1$, which corresponds to the unit of time τ, having the length of $1/\alpha_1$, the value of demand increase shall be equal $\Delta\Lambda(\tau_\Delta) \approx \Delta\Lambda$, for $\tau_\Delta = (1/\alpha_1)/(1/\alpha_2) = \alpha_2/\alpha_1 = 2, 3,$ 4, etc. This approximation may produce an error of less than 5%.

In distinction from the above, the value of demand generated by the "old" customers shall attain at time $\tau = 1$ only some 63% of the value it should theoretically attain in infinity.

These essential differences between the processes described must be accounted for both in the case, when a Company A enters on the market, monopolised by Company B, with its competitive and cheaper product, and in the case, when two companies exist and compete on a common market, and one of them, say – Company A, lowered the sales price of its product.

Using the exponential functions, whose course depends on time, in the assessment of the interrelations of profits of two competing companies, is quite cumbersome in terms of calculations as it would ultimately involve the use of variational calculus. That is why in the further parts of the book we shall analyse the situation of the two competing companies only in selected instants of time. We shall be, namely, taking the values of τ corresponding to definite value of the expression $\alpha\tau$. Thus, as the first such time instant we shall take $\tau_1 = 1/\alpha$, as the second - $\tau_2 = 2/\alpha$, etc.

We shall devote most attention to the analysis of the relations concerning company profits after the first time interval, as depending upon the adopted strategy in the domain of lowering of the price C_A of own product.

In particular, by lowering the price:

-- can we increase our market share (which appears obvious) and our profits (which appears, at first sight, much less probable)?

-- can we increase our profits in comparative terms, that is – beyond those of our competitors?

Next chapter of the book shall be devoted to consideration of these issues.

CONCLUDING REMARKS

This book, like the previous one, constituting Volume I of the introduction to a theory of market competition, contains considerations that involve a number of approximations and simplifications. We, that is – the authors of both these volumes – would like to draw the attention of the Reader to them.

In general, we do not explicitly consider the vague, uncertain or "fuzzy" character of some of the quantities we refer to. This concerns, in particular, such quantities as the limit value of d in the determination of the demand function, $\lambda(C)$.

Likewise, the uncertain, or specific character of some relations has not been treated in an explicit manner. Thus, for instance, we state in the book that the situation when the market shares of two competitors are equal, 50% each, constitutes indeed a kind of equilibrium, but this is an unstable equilibrium point, for any disturbance to this situation shall drive it far away from the equilibrium (assuming, of course, that this disturbance, due to behaviour of one of the competitors, does not find any "appropriate" reaction from the side of the other competitor).

In reality, though, this equilibrium point is not that unstable, i.e. it is not that sensitive to the very small disturbances. Actually, an interval of insensitivity always exists, due to various reasons, such as delays, information shortage, lack of reaction of customers to very small price changes etc. It is even possible that the "hysteresis" effect may appear. In terms of the notions introduced in this book, the magnitude of the zone of insensitivity depends upon the slope of the production characteristics (the value of the derivative $dk/d\mu$).

Independently of the above remarks the considerations here presented neglect the effect of the change in the number of potential customers due to the change in product price. Namely, along with the change in product price, there is also change in the value of the difference *C-b*, exerting the decisive influence on the magnitude of the optimum radius R^* of the area, over which the company effectively caters to its potential customers. This radius R^* defines, in turn, the number of such potential customers, i.e. the ones, to whom the products are effectively supplied. This number, in turn, together with the income structure of the customers, defines the value of Λ_{mx} (see also Volume I). Yet, in the book, for both companies selling their products for different prices, the very same value of Λ_{mx} was adopted.

Neglecting this particular aspect is justified by the following circumstances:

- a company that just enters the market (as well as the one, which tries to expand its market share) can hardly afford the advertising saying that its product is not worse than the one of the competitor, even though it is cheaper – and this not for all the potential customers, exception being constituted by the farthest ones;
- on the other hand, the company defending its market share and for this purpose decreasing the sales price of its products, ought not get rid of its more distant customers, since this would make a very disadvantageous effect on the remaining customers and would actually accelerate elimination of such a company from the market.

Of course, the fact that we neglected the influence, exerted by the changes in the reach, *R*, was also largely due to the wish of simplifying the complicated interrelations, constituting the description of the market process, the mechanism of functioning of the "invisible hand of the market".

Considerations, contained in the book, do not account, either, for the influence of advertising, although certain remarks on this subject are forwarded in Chapter I.

Likewise, we did not forward the estimates for the cost of entry onto an alien market, which could be formulated with the use of the formulae for the sales magnitudes (Λ_A and Λ_B).

When considering the (initial) shares of two competing companies, we analysed the case, when they start from equal market shares. For modelling and analysing other possible situations, we could use the coefficient u.

In the case when more than two companies have (non-negligible) shares in a market, the struggle for the market share ought to be started with the weakest company, avoiding the appearance of a hostile coalition of the remaining companies on the market. Otherwise, it would become necessary to establish an own coalition that would be able to withstand the competition of the other coalition. In such a case the struggle for the market shares would reduce to the case of two competitors, that is – to the situation described in the book.

In view of these and, indeed, many other aspects that remain to be accounted for, it is obvious that the description of the mechanism behind the functioning of the “invisible hand” is far from complete.

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This book is the second part of an exposition of a coherent and far-reaching theory of market competition. The theory is based on simple precepts, does not require deep knowledge of either economics or mathematics, and is therefore aimed primarily at undergraduate students and all those trying to put in order their vision of how the essential market mechanisms might work. The present Volume II constitutes a complement to the considerations, contained in Volume I.

The logic of the presentation is straightforward; it associates the easily grasped microeconomic elements of quantitative character in order to arrive at both more general conclusions and at concrete formulae defining the way the market mechanisms work under definite assumed conditions.

Some may consider this exposition too simplistic. In fact, it is deliberately kept very simple, for heuristic purposes, as well as in order to make the conclusions more clear. Adding a lot of details that make theory more realistic – these details, indeed, changing from country to country, and from sector to sector – is mainly left to the Reader, who is supposed to be able to design the more accurate image on the basis of the foundations, provided in the book.

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