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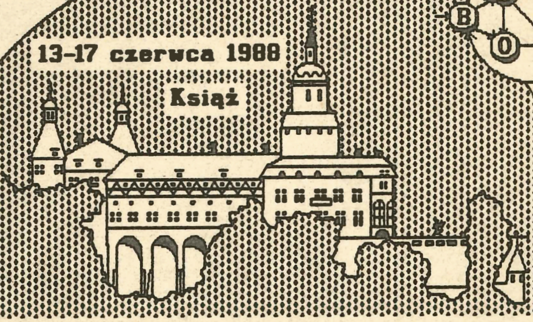

1. Krajowa Konferencja Badań Operacyjnych i Systemowych

Tom 2

BOS'88

POLSKIE TOWARZYSTWO BADAŃ OPERACYJNYCH I SYSTEMOWYCH

INSTYTUT BADAŃ SYSTEMOWYCH POLSKIEJ AKADEMII NAUK

POLSKIE TOWARZYSTWO BADAŃ OPERACYJNYCH I SYSTEMOWYCH

TOM 2

WSPOMAGANIE PODEJMOWANIA DECYZJI
MODELE I SYSTEMY

I KRAJOWA KONFERENCJA

BADAŃ OPERACYJNYCH i SYSTEMOWYCH

Książ, 13 - 17 czerwca 1988

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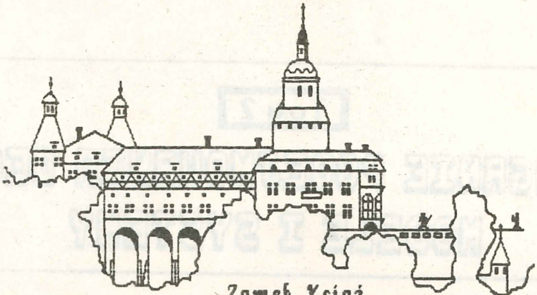

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6. Formalizacja modeli decyzyjnych

I Krajowa Konferencja
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ENTRY PROBLEM IN ONE DIMENSION UNDER APPROVAL VOTING FOR n=2+1 CANDIDATES

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The entry problem in a political race for a two-party system is considered in the paper. We assume that two parties are displayed along one ideological dimension and the approval voting is used. We discuss chances for winning of any new enterer. We formulated necessary and sufficient condition for win of any new enterer in terms of expected value of number of votes s/he receives and in terms of sets of voters with the same preferation order.

1. The basic assumption and properties of the model.

The entry problem in a political race is considered under the following assumptions about the model:

1. There is a single left - right ideological dimension along which the candidates L(=left) and C(=conservative) take positions.

2. Each voter has most-preferred position on this dimension.

3. Preferations are generated by distance metrics along ideological dimension.

4. Each voter casts sincerely his or her votes according to the rules of approval voting (Brams and Fishburn,1978).

5. The candidate with the most number of votes wins.

We assume that voters are distributed over the normalized interval [0,1] according to the distribution function F(x). It is obvious that for only two candidates (L and C) the approval voting is single plurality voting. The enter of a new candidate X changes this situation: for three candidates there is sincere approval voting.

We assume that according to the rules of approval voting choosing for three candidates one from two possibilities of voting system {1,2} (i.e. the decision of cast one vote or two votes made by every voter) is equally probable. Generaly, we assume that the every element of the voting system {1,2,...,n-1} may be uniformly choosen.

Our following considerations are based upon the definition of an elementary support.

Definition. Let { $E_1, E_2, ..., E_m$} be a partition of n voters in which E_k is a subset of all voters having the same order over the set of candidates, $E_i \cap E_k = \emptyset$ for $i \neq k$. We call E_k the k-th elementary support.

Let us divide interval [0,1] into disconnected intervals $I_1, I_2, ..., I_m$ such that $\bigcup_k I_k = [0,1]$. Those intervals arise between all subsequent midpoints found between every pair of candidates' positions. It is easy to see that positions of all voters from one elementary support belong to one and only one such constructed interval and there is no voter from any other elementary support whose position may be found inside this interval. If n is number of candidates we have $m = \binom{n}{2} + 1$ such intervals.

Let n_i for $i=1,2,...,m$ be a number of voters who belong to the elementary support E_i, $n_i = F(a_i) - F(a_{i-1})$ where $I_i = [a_i, a_{i-1})$.

We assume that for all i $n_i > 0$.

Let X be a position of a new enterer such that $L<X<C$. One may find in fig.1 a peculiar position of L, C and X fulfilling above condition together with elementary supports E_1, E_2, E_3, E_4 (in fact I_1, I_2, I_3 and I_4 respectively) connected with them. For those positions of L, C and X there are the following orderings given by voters. Let π_k^i - describes the position given by a voter from elementary support E_k for i-th candidate in this voter preferation order (Table 1).

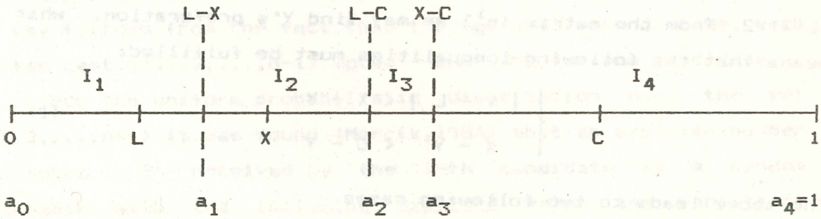

Fig.1. Elementary supports for L < X < C candidates' positions.

	E_1	E_2	E_3	E_4
L	1	2	3	3
C	3	3	2	1
X	2	1	1	2

Table 1. Matrix π_k^i of orders over L,C and X for voters from

elementary support E_i, $i=1,2,3,4$.

In our first theorem we show that for all position of X such that $0 \le L < X < C \le 1$ the above matrix $\{\pi_k^i\}$ is constant.

Theorem 1. Let L, C and X are positions of candidates. If $L<X<C$ then preferations of voters from particular elementary support do not depend from the position of X inside of $[L,C]$ and they are given by the above matrix $\{\pi_k^i\}$.

Proof. Constructing elementary supports one may notice that

$a_o=0$, $a_1=(L+X)/2$, $a_2=(L+C)/2$, $a_3=(X+C)/2$, $a_4=1$

and $a_o < a_1 < a_2 < a_3 < a_4$.

Let Y be position of a voter Y from E_1, i.e. $Y < a_1 \Leftrightarrow Y < (L+X)/2$. From the matrix $\{\pi_k^i\}$ we may find Y's preferation, what means that the following inequalities must be fulfilled:

$$\begin{cases} |Y-L| < X-Y \\ X-Y < C-Y \end{cases} \qquad (1)$$

The above leads to two following cases

$$\begin{cases} Y-L < X-Y \\ X-Y < C-Y \end{cases} \quad \text{for } Y-L \ge 0 \qquad (2)$$

and

$$\begin{cases} L-Y < X-Y \\ X-Y < C-Y \end{cases} \quad \text{for } Y-L < 0 \qquad (3)$$

or equivalently

$$\begin{cases} Y < (X+L)/2 \\ X < C \end{cases} \quad \text{for } Y-L \ge 0 \qquad (4)$$

and

$$\begin{cases} L < X \\ X < Y \end{cases} \quad \text{for } Y-L < 0 \qquad (5)$$

It is evidently noticeable that for $Y \in E_1$ there is no position X: L < X < C which may change any of inequalities (4) or (5) what means there is no position which may change the order (given by a voter from E_1) over the set of candidates. Analogical considerations may be done for the rest of elementary supports: E_2, E_3, and E_4. Q.E.D.

2. The conditions of win of new enterer X.

The conception of evaluation of new enterer X's chances for winning in political race with two other candidates is based on the expected value of number of X's votes. The necessity of such a way follows from the fact that the approval voting lets each voter cast $j \in \{1,2,\ldots,n-1\}$ votes - one vote for one candidate.

For the uniform probabilistic distribution over the set $\{1,2,\ldots,n-1\}$ it was found (Mercik,1986) that an expected number of votes - S^i - received by the i-th candidate is a random variable with the following expected value and variation, respectively:

$$E(S^i) = 1/(n-1)(n - \sum_k n_k \pi_k^i) \quad (6)$$

and

$$Var(S^i) = \sum_k (n-\pi_k^i)(\pi_k^i - 1)n_k \; / \; (n-1)^2 \quad (7)$$

where n denotes number of candidates and number of voters is normalized, i.e. $\sum_k n_k = 1$.

X entering into political race with other two candidates may take the following positions: (a) more left then L, (b) more right then C, and (c) between L and C. It was shown (Mercik, 1988) that for X only the position between L and C may give him or her the largest possible number of votes. So, in our further consideration we will omit the cases (a) and (b).

Let S^L, S^C and S^X are random variables describing a number of votes received under approval voting by L, C and X, respectively.

Let n_i are defined for all i as on page 2.

Theorem 2. If X is expected to be a winner then number of voters giving him or her the first position in their order over the set of candidates must be greater then zero.

Proof. Expression "If X is expected to be winner" is equivalent to the following inequalities:

$$E(S^L) < E(S^X) \tag{8}$$

$$E(S^C) < E(S^X) \tag{9}$$

From (6) and the matrix $\{\pi_k^i\}$ given in Table 1 one may receive the following

$$n_1 - n_2 - 2n_3 - n_4 < 0 \tag{10}$$

$$-n_1 - 2n_2 - n_3 + n_4 < 0 \tag{11}$$

and then

$$n_2 + n_3 > 0 \tag{12}$$

From the matrix $\{\pi_k^i\}$ we find that X takes first position for voters from E_2 and E_3. In view of (12) the theorem is proved.

The above condition (12) is not sufficient for X to expect him or her to be winner. We try to solve this problem in the following theorem.

Theorem 3. If X is winning under plurality voting then X is also expected to be winner under approval voting.

Proof. Let (8) and (9) not be held. Then from (10) and (11)

$$n_1 - n_2 - 2n_3 - n_4 \geq 0 \tag{13}$$

or

$$-n_1 - 2n_2 - n_3 + n_4 \geq 0 \tag{14}$$

From (13) and for $\sum_k n_k = 1$ one may receive

$$n_1 \geq 1/2 + n_3/2 \tag{15}$$

or from (14)

$$n_4 \geq 1/2 + n_2/2 \tag{16}$$

The win of X under plurality voting is equivalent to the following inequalities

$$n_2 + n_3 > 1/2 \tag{17}$$

or, because $\sum_k n_k = 1$

$$n_1 + n_4 < 1/2 \tag{18}$$

Hence (15) and (16) are contraditions to (18). Q.E.D.

3. Conclusions.

From theorem 3 one may obtain also, so called "1/2 separation opportunity" because the condition (18) $n_2 + n_3 > 1/2$ means also that (in the sense of number of votes) the minimal distance (between L and C) giving for X the opportunity to be expected winner under approval voting is equal to 1/2. This is necessary but not sufficient condition.

It is also noticeable that "1/2 separation opportunity" under approval voting is slitly less rigorus then "2/3 separation opportunity" of Brams and Straffin (1982) under plurality voting.

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W ramach analizy pozrawczej zaprezentowano nowy terenowy system pomocy doraźnej i dołączono do niego bogaty zestaw charakterystyk działania. W badaniach prognostycznych uwzględniono wpływ możliwości przeprowadzania różnorodnych eksperymentów. W części dotyczącej wspomagania procesów decyzyjnych zapewniono użytkownikowi bezpośredni udział w symulowanej rzeczywistości. Student podejmuje decyzje dyspozytorskie i obserwuje w jaki sposób wpływają one na efektywność pracy systemu. Dodatkowo, dokonuje oceny medycznej każdego zgłoszenia i klasyfikuje każde zgłoszenie wg stopnia pilności obsługi.

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