Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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On the probability theory on the Kôpka D-posets

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Abstract

F. Kôpka [8] introduced the notion of a product on D-posets (see also [4], [9]) which is a generalization of MV algebras with product ([12], [14], [18]). The main results of the paper are the proof of the convergence theorem for the mean of the observables.

Keywords: D-poset, convergence theorem, Kôpka D-poset.

1 Introduction

First we introduce the algebraic structure, which is called D-poset. This concept was formulated by Kôpka end Chovanec in 1994 in the paper [10]. This structure is equivalent to the effect algebra defined by Foulis and Bennet.

Definition 1 *The structure $(D, \leq, -, 0, 1)$ is called a D-poset if the relation $\leq$ is a partial ordering on D, 0 is the smallest and 1 is the largest element on D and*

(i) *if* $b - a$ *is defined iff* $a \leq b$

(ii) *if* $a \leq b$ *then* $b - a \leq b$ *and* $b - (b - a) = a$,

(iii) $a \leq b \leq c \Longrightarrow c - b \leq c - a, (c - a) - (c - b) = b - a.$

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In this contribution we will work with probability, so we need to define a binary operation on D-poset, which is called the product.

Definition 2 *Let $(D, \leq, -, 1, 0)$ be a D-poset. It is called the Kôpka D-poset, if there is a binary operation $* : D \times D \to D$, which is commutative, associative, and has the following properties:*

(i) $a * 1 = a, \quad \forall a \in D$;

(ii) $a \leq b \Longrightarrow a * c \leq b * c, \quad \forall a, b, c, \in D$;

(iii) $a - (a * b) \leq 1 - b, \quad \forall a, b \in D$.

Now we define two important mappings, the state and the observable. These maps we need, because we will work with probability.

Definition 3 *A state on a D-poset D is any mapping $m : D \to [0, 1]$ satisfying the following properties:*

(i) $m(1) = 1, m(0) = 0$;

(ii) $a_n \nearrow a \Longrightarrow m(a_n) \nearrow m(a), \quad \forall a_n, a \in D$;

(iii) $a_n \searrow a \Longrightarrow m(a_n) \searrow m(a), \quad \forall a_n, a \in D$.

Definition 4 *Let $\mathcal{J} = \{(-\infty, t); t \in R\}$. An observable on D is any mapping $x : \mathcal{J} \to D$ satisfying the following conditions:*

(i) $A_n \nearrow R \Longrightarrow x(A_n) \nearrow 1$;

(ii) $A_n \searrow \emptyset \Longrightarrow x(A_n) \searrow 0$;

(iii) $A_n \nearrow A \Longrightarrow x(A_n) \nearrow x(A)$.

When we composite an observable and a state, then we get a function, which satisfy the properties of a distribution function.

Theorem 1 *Let $m : D \to [0, 1]$ be a state, $x : \mathcal{J} \to D$ be an observable. Define $F : R \to [0, 1]$ by the formula*

$$F(t) = m(x((-\infty, t))).$$

Then F has the following properties:

(i) *F is non-decreasing;*

(ii) F *is left continuous in any point* $t \in R$*;*

(iii) $\lim_{t\to\infty} F(t) = 1$*;*

(iv) $\lim_{t\to\infty} F(t) = 0$.

Proof: Let $t < s$, put $t_1 = t, t_n = s, \quad (n = 2, 3, ...)$. Then $t_n \nearrow s$, hence

$$F(t_n) = m(x((-\infty, t_n))) \nearrow m(x((-\infty, s))) = F(s).$$

Therefore $F(t) = F(t_1) \leq F(s)$, hence F is non-decreasing.
If $t_n \nearrow t$, then $x((-\infty, t_n)) \nearrow x((-\infty, t))$, hence

$$F(t_n) = m(x((-\infty, t_n))) \nearrow m(x((-\infty, t))) = F(t),$$

and therefore F is left continuous in t.
Similarly the equalities $F(\infty) = 1, F(-\infty) = 0$ can be proved.

By the well known results of the measure theory it follows that there exists exactly one measure $\lambda_F : \mathcal{B}(R) \to [0, 1]$ such that $\lambda_F([a, b)) = F(b) - F(a)$ whenever $a \leq b$.

Sometimes we have to restrict to special kind of D-posets. Therefore we define some properties of D-posets.

Definition 5 *D-poset is called* σ*-complete iff every subset of countable elements has a supremum and an infimum.*

Definition 6 *D-Kôpka poset* $(D, -, *, 0, 1)$ *is called continuous iff the following holds:*

$$a_n \nearrow a \Rightarrow b * a_n \nearrow b * a, \forall a_n, a, b \in D$$

2 Sum of observables

Generally, if an observable is a morphism from $\mathcal{B}(R)$ to D, there is not problem to define the sum of observables. Namely, if

$$\xi, \eta : \Omega \to R$$

are two random variables, then

$$\xi + \eta = g(\xi, \eta) = g \circ T,$$

where $T = (\xi, \eta) : \Omega \to R^2$ is a random vector, and $g : R^2 \to R, g(u, v) = u+v$. Therefore

$$(\xi + \eta)^{-1}(A) = (g \circ T)^{-1}(A) = T^{-1}(g^{-1}(A)), A \in \mathcal{B}(R),$$

and it is natural to define the sum of two observables $x, y : \mathcal{B}(R) \to D$ by the following way:

$$(x + y)(A) = h(g^{-1}(A)),$$

where

$$h : \mathcal{B}(R^2) \to D$$

is the joint observable, i.e. a morphism satisfying the identities

$$h(A \times R) = x(A), h(R \times B) = y(B), A, B \in \mathcal{B}(R).$$

Of course, now x, y are mappings from $\mathcal{J} = \{(-\infty, t); t \in R\}$ to D and we are not able to construct the joint observable h. If $g(u, v) = u + v$, then

$$g^{-1}((-\infty, t)) = \{(u, v); u + v < t\} = \Delta_t \subset R^2.$$

therefore we shall construct a morphism

$$h : \mathcal{M} \to D,$$

where

$$\mathcal{M} = \{\Delta_t; t \in R\},$$

and then to define the sum $z = x + y : \mathcal{J} \to D$ by the formula

$$z((-\infty, t)) = h(\Delta_t).$$

Definition 7 *Let $\mathcal{M} = \{\Delta_t^2; t \in R\}$ be the set, where*

$$\Delta_t^2 = \{(u, v); u + v < t\} \text{ for } t \in R, n \in N$$

and by $\alpha_{t,n}^2$ is denoted the set

$$\alpha_{t,n}^2 = \left\{(i, j) ; \frac{1}{2^n} (i + j) < t\right\}.$$

Then we put a joint observable $h_2 : \mathcal{M} \to D$ by the formula

$$h_2(\Delta_t^2) = \bigvee_{n=1}^{\infty} \bigvee_{(i,j) \in \alpha_{t,n}^2} x\left(\left[\frac{i-1}{2^n}, \frac{i}{2^n}\right)\right) * y\left(\left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)\right)$$

(Here $* : D \times D \to D$ is the product on the σ-complete Kôpka D - poset and it holds $x\left([a, b)\right) = x\left((-\infty, b)\right) - x\left((-\infty, a)\right)$.)

Now we show that the mapping $h_2(\Delta_t^2)$ has the properties of an observable. We denote by $z : \mathcal{J} \to D$ the mapping, which is defined by the following equality:

$$z((-\infty, t)) = h_2(\Delta_t^2), t \in R.$$

The properties (i) and (ii) from Definition 4 imply from the inequalities:

$$x((-\infty, k)) * y((-\infty, k)) \leq z\left((-\infty, 2k)\right) \leq x((-\infty, 2k)) \vee y((-\infty, k)),$$

where $k \in Z$.
For the proof of property (iii) is used the associativity. Let $s_k \nearrow s$ is a sequence of real numbers, then holds:

$$\bigvee_{k=1}^{\infty} z((-\infty, s_k)) = \bigvee_{k=1}^{\infty} \bigvee_{n=1}^{\infty} \bigvee_{(i,j)\in\alpha^2_{s_k,n}} x\left(\left[\frac{i-1}{2^n}, \frac{i}{2^n}\right)\right) * y\left(\left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)\right) =$$

$$= \bigvee_{n=1}^{\infty} \bigvee_{k=1}^{\infty} \bigvee_{(i,j)\in\alpha^2_{s_k,n}} x\left(\left[\frac{i-1}{2^n}, \frac{i}{2^n}\right)\right) * y\left(\left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)\right) = z((-\infty, s)).$$

The definition 7 can be generalize by this way:

$$\Delta_t^k = \{(u_1, ..., u_k) \in R^k; u_1 + ... + u_k < t\},$$

for $k \in N$ and $t \in R$.
Analogy to the set $\alpha^2_{t,n}$ we can deduced the set $\alpha^k_{t,n}$ for the case of k observables by this way:

$$\alpha^k_{t,n} = \left\{(i_1, i_2, ..., i_k)\, ; \frac{1}{2^n}\sum_{l=1}^{k} i_l < t\right\},$$

for $k, n \in N$ and $t \in R$.
For the natural numbers $k = 1, 2, ...$ we have the sets: $M_k = \{\Delta_t^k; t \in R\}$.
Let $x_1, .., x_k$ be the observables, then their joint observable $h_k : M_k \to D$ is defined by the formula:

$$h_k(\Delta_t^k) = \bigvee_{n=1}^{\infty} \bigvee_{l=1}^{\infty} \bigvee_{(i_1,i_2,...,i_k)\in\alpha^k_{t,n}} \prod_{l=1}^{k} x_{i_l}\left(\left[\frac{i_l - 1}{2^n}, \frac{i_l}{2^n}\right)\right). \qquad (1)$$

In the following formula we show the property the sum of observables on σ-complete continuous Kôpka D-poset.

Theorem 2 *Let $\mathcal{D}$ be a σ-complete continuous Kôpka D-poset with σ-additive state $m : D \to [0,1]$. We denote by $x, y : \mathcal{J} \to D$ the independent observables and by F, G their distributive function. Then the following equality for all real numbers $t \in R$ holds:*

$$m(h_2(\Delta_t^2)) = \lambda_F \times \lambda_G(\Delta_t^2).$$

Proof: We use the assumptions of the theorem and the Definition 7:

$$m\left(h_2(\Delta_t^2)\right) = \lim_{n\to\infty} \sum_{(i,j)\in\alpha_{t,n}^2} \lambda_F\left(\left[\frac{i-1}{2^n}, \frac{i}{2^n}\right)\right) \lambda_G\left(\left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)\right) =$$

$$= \lambda_F \times \lambda_G\left(\bigcup_{n=1}^{\infty} \bigcup_{(i,j)\in\alpha_{t,n}^2} \left[\frac{i-1}{2^n}, \frac{i}{2^n}\right) \times \left[\frac{j-1}{2^n}, \frac{j}{2^n}\right)\right) =$$

$$= \lambda_F \times \lambda_G\left(\{(u,v); u+v<t\}\right) = \lambda_F \times \lambda_G(\Delta_t^2).$$

3 Convergence theorem

In this part of work is the Kolmogorov construction made. We define the convergence almost everywhere. Then we will have always for the proof of the convergence theorem on σ-complete continuous Kôpka D-poset.
The mapping $\pi_n : R^N \to R^n$, which is defined by this equality $\pi_n\left((u_i)_1^\infty\right) = (u_1, ..., u_n)$, is denoted as n-th coordinate random vector. The set of all cylinders is the set

$$C = \left\{\pi_n^{-1}(K) \subset R^N; K \in \mathcal{B}(R^n), n \in N\right\},$$

and $\sigma(C)$ is the smallest σ-algebra over the set C.
Put $p_n = \lambda_{F_1} \times \lambda_{F_2} \times ... \times \lambda_{F_n} : \mathcal{B}(R^n) \to [0,1]$. Then the mappings p_n make a consistency system:

$$p_{n+1}(A \times R) = p_n(A), A \in \mathcal{B}(R^n), n \in N.$$

Then by Kolmogorov theorem there exists probability measure $P : \sigma(C) \to [0,1]$ ant the following holds:

$$P \circ \pi_n^{-1} = p_n = \lambda_{F_1} \times \lambda_{F_2} \times ... \times \lambda_{F_n} : \mathcal{B}(R^n) \to [0,1],$$

for every $n \in N$. The system $\left(R^N, \sigma(C), P\right)$ is probability space.

For every natural number $n \in N$ is defined the function $\xi_n : R^N \to R$ by this equality

$$\xi_n\left((u_i)_{i=1}^{\infty}\right) = u_n.$$

This mapping is random variable according to the probability space $\left(R^N, \sigma(C), P\right)$.

Now we define the probability almost everywhere. In classical probability theory with the space $(\Omega, \mathcal{S}, P)$, we say that the sequence of the random variables ξ_n converges to P-almost everywhere, if

$$P\left(\{\omega; \xi_n(\omega) \to 0\}\right) = 1. \tag{2}$$

Let X be the set $\{\omega; \xi_n(\omega) \to 0\}$, then by using previous theorem we get:

$$(\forall l \in N)(\exists k \in N)(\forall n \geq k)\left(\xi_n^{-1}\left(\left(-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right) \subset X.$$

This type of convergence for a sequence of observables $(\xi_n)_{n=1}^{\infty}$ can be define by the following way:

$$\lim_{l\to\infty}\lim_{k\to\infty}\lim_{i\to\infty} P\left(\bigcap_{n=k}^{k+i} \xi_n^{-1}\left(\left(-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right) = 1.$$

Similarly we define a convergence m-almost everywhere for the sequence of observables on D-poset.

Definition 8 *Let $(x_i)_{i=1}^{\infty}$ be a sequence of observables on σ-complete Kôpka D-poset $\mathcal{D}$ with σ-additive state m. We say, that this sequence converges m-almost everywhere to zero, if the following equality holds:*

$$\lim_{l\to\infty}\lim_{k\to\infty}\lim_{i\to\infty} m\left(\bigwedge_{n=k}^{k+i} x_n\left(\left[-\frac{1}{l}, \frac{1}{l}\right)\right)\right) = 1.$$

Now we go to the main aim of this paper, to the formulation and the proof of the convergence theorem.

Theorem 3 *Let $\mathcal{D}$ is σ-complete continuous Kôpka D-poset with σ-additive state $m : D \to [0,1]$. Let we have a sequence of independent observables $(x_n)_{n=1}^{\infty}$, h_n is sequence of observables defined by the equality (1). Let the sequence $(\eta_n)_{n=1}^{\infty}$ is defined by the following equality: $\eta_n = \frac{1}{n}\sum_{i=1}^{n}\xi_i$, where $\xi_n : R^N \to R$ is n-th coordinate random vector. If the sequence $(\eta_n)_{n=1}^{\infty}$ converges P-almost everywhere to zero, then the sequence $y_n = \frac{1}{n}\sum_{i=1}^{n} x_i$ converges m-almost everywhere to zero.*

Proof: Let $g_n : R^n \to R$ be the function defined for every $n \in N$ by this way:

$$g_n(u_1, u_2, ..., u_n) = \frac{1}{n}\sum_{i=1}^{n} u_i,$$

and $\pi_n : R^N \to R^n$ is n-th coordinate random vector.
For the mapping η_n for every $n \in N$ and function g_n holds:

$$\eta_n = g_n(\xi_1, \xi_2, ..., \xi_n) = g_n \circ \pi_n.$$

Then the mapping $\eta_n^{-1} : \mathcal{B}(R) \to \mathcal{B}(R^n)$ satisfy:

$$\eta_n^{-1}(A) = \pi_n^{-1}\left(g_n^{-1}(A)\right)$$

for each $n \in N$ and for every borel set $A \in \mathcal{B}(R)$.
For the composite mapping $m \circ y_n : \mathcal{J} \to [0, 1]$ we get with using previous equalities and the Theorem 2:

$$m \circ y_n = m \circ h_n \circ g_n^{-1} = P \circ \pi_n^{-1} \circ g_n^{-1} = P \circ \eta_n^{-1}$$

for every natural number n.
The assumptions of the theorem implies that the sequence $(g_n \circ \pi_n)_{n=1}^{\infty}$ converge P-almost everywhere to zero:

$$\lim_{l\to\infty}\lim_{k\to\infty}\lim_{i\to\infty} P\left(\bigcap_{n=k}^{k+i}(g_n \circ \pi_n)^{-1}\left(\left[-\frac{1}{l}, \frac{1}{l}\right)\right)\right) = 1.$$

We denote $\pi_{k+i,n}$ for $\forall k, i, n \in N$ $(k+i > n)$ the projection from the space R^{k+i} to the space R^n:

$$\pi_{k+i,n}(u_1, u_2, ..., u_{k+i}) = (u_1, u_2, ..., u_n).$$

So the following holds:

$$P\left(\bigcap_{n=k}^{k+i}(g_n \circ \pi_n)^{-1}\left(\left[-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right) = P\left(\bigcap_{n=k}^{k+i}\left(\pi_n^{-1} \circ g_n^{-1}\right)\left(\left[-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right) =$$

$$= P\left(\bigcap_{n=k}^{k+i}\pi_n^{-1}\left(g_n^{-1}\left(\left[-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right)\right) =$$

$$P\left(\pi_{k+i}^{-1}\left(\bigcap_{n=k}^{k+i}\pi_{k+i,n}^{-1}\left(g_n^{-1}\left(\left[-\tfrac{1}{l}, \tfrac{1}{l}\right)\right)\right)\right)\right) =$$

$$= m\left(h_{k+i}\left(\bigcap_{n=k}^{k+i}\pi_{k+i,n}^{-1}\left(g_n^{-1}\left(\left[-\tfrac{1}{l},\tfrac{1}{l}\right)\right)\right)\right)\right) \leq$$

$$m\left(\bigwedge_{n=k}^{k+i}\left(h_n\left(\Delta^n_{\frac{1}{l}}\right) - h_n\left(\Delta^n_{-\frac{1}{l}}\right)\right)\right) =$$

$$= m\left(\bigwedge_{n=k}^{k+i}\left(y_n\left(\left(-\infty,\tfrac{1}{l}\right)\right) - y_n\left(\left(-\infty,-\tfrac{1}{l}\right)\right)\right)\right) = m\left(\bigwedge_{n=k}^{k+i} y_n\left(\left[-\tfrac{1}{l},\tfrac{1}{l}\right)\right)\right).$$

We get the inequality:

$$P\left(\bigcap_{n=k}^{k+i}(g_n\circ\pi_n)^{-1}\left(\left[-\frac{1}{l},\frac{1}{l}\right)\right)\right) \leq m\left(\bigwedge_{n=k}^{k+i} y_n\left(\left[-\frac{1}{l},\frac{1}{l}\right)\right)\right).$$

And because the following holds:

$$\lim_{l\to\infty}\lim_{k\to\infty}\lim_{i\to\infty} P\left(\bigcap_{n=k}^{k+i}(g_n\circ\pi_n)^{-1}\left(\left[-\frac{1}{l},\frac{1}{l}\right)\right)\right) = 1,$$

we can finish the proof

$$\lim_{l\to\infty}\lim_{k\to\infty}\lim_{i\to\infty} m\left(\bigwedge_{n=k}^{k+i} y_n\left(\left[-\frac{1}{l},\frac{1}{l}\right)\right)\right) = 1.$$

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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