

**Developments in Fuzzy Sets,  
Intuitionistic Fuzzy Sets,  
Generalized Nets and Related Topics.  
Volume I: Foundations**

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**Systems Research Institute  
Polish Academy of Sciences**

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# Knowledge from contradiction and inconsistency

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## Abstract

Contradiction and inconsistency often occur in reasoning, however they are also regarded as undesirable. This paper shows some areas where contradiction and inconsistency can result in new knowledge especially about the qualitative aspects of modeling and knowledge.

**Keywords:** contradiction, inconsistency, knowledge.

## 1 Introduction

The concept of the Intuitionistic Fuzzy Set (IFS, see Atanassov [2]) was introduced in 1983 as an extension of Zadeh's fuzzy set, and called intuitionistic based on Brouwer's intuitionism [18, 7]. The inspiration was to include a non-membership curve as well as membership curve. All operations, defined over fuzzy sets were transformed for the IFS case. There has been a discussion in the literature regarding the name intuitionistic for these sets [6], responded to by Atanassov [3] arguing precedence and widespread use. While Atanassov [1] has temporal precedence over Takeuti and Titani [16], together with a much wider following in the literature, the argument centres around the links to Brouwer's Intuitionistic Logic [18]. This paper follows Atanassov's line but to clarify the intentions the semantics of the sets are described. Contradictory sets are used to model contradictions between the membership curve and the non-membership

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curve. Atanassov clearly states that there can be no such contradiction in an Intuitionistic Fuzzy Set, as do various other authors [18, 7] and so we shall rename these sets incorporating contradiction, Contradictory Fuzzy Sets, intending to avoid any arguments with either point of view. Earlier work acknowledged the value of the two curves following Atanassov by referring to these sets as ‘Inconsistent Intuitionistic Fuzzy Sets’ [10, 11, 12, 13, 9, 14] and ‘Inconsistent Atanassov Intuitionistic Fuzzy Sets’ in others.

**Definition 1 (Atanassov Intuitionistic Fuzzy Set)**

$$A^* = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in \mathbb{U}\}$$

where

$$\mu_A : \mathbb{U} \rightarrow [0, 1]$$

$$\nu_A : \mathbb{U} \rightarrow [0, 1]$$

For each  $x$ , the numbers  $\mu_A(x)$  and  $\nu_A(x)$  are the degree of membership and degree of non-membership of  $x$  to  $A$  respectively. They are subject to the constraint:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad \forall x \in \mathbb{U}. \quad (1)$$

Where there is contradiction involved within the set, [10] or between sets [5] then this representation does not capture it, or worse results in breaking Constraint 1. If the measure of contradiction is included in the denotation of the set then we obtain the CFS as defined in Definition 2:

**Definition 2 (Contradictory Fuzzy Set)**

$$A^{t*} = \{\langle x, \mu_A(x), \nu_A(x), \iota_A(x) \rangle \mid x \in \mathbb{U}\}$$

where

$$\mu_A : \mathbb{U} \rightarrow [0, 1]$$

$$\nu_A : \mathbb{U} \rightarrow [0, 1]$$

$$\iota_A : \mathbb{U} \rightarrow [0, 1]$$

For each  $x$ , the functions  $\mu_A(x)$  and  $\nu_A(x)$  give the degree of membership and degree of non-membership of  $x$  to  $A$  respectively. The function  $\iota_A(x)$  gives the degree of contradiction in the evidence leading to  $\mu_A(x)$  and  $\nu_A(x)$ . They are subject to the constraint:

$$0 \leq \mu_A(x) + \nu_A(x) + \iota_A(x) \leq 1 \quad \forall x \in \mathbb{U}. \quad (2)$$

rather than Equation 1.

The quantity  $\iota_A(x)$  is the contradiction between the membership and non-membership functions. Hinde [10] and Cubillo [5] differ in their treatment of contradiction. Whereas Cubillo computes the contradiction between the membership and non-membership values given those values, Hinde allows contradiction to occur before the membership functions are complete and so whereas the hesitation  $\pi_A(x)$  denoting the unknown between membership and non-membership must be zero in Cubillo's analysis, Hinde allows all 4 elements to be non-zero simultaneously giving rise to Equation 3. Trillas [17] examines the functions used in fuzzy logic concentrating on contradiction arising from the operator used rather than contradiction in the evidence; which is the focus of these sets.

$$\mu_A(x) + \nu_A(x) + \iota_A(x) + \pi_A(x) = 1 \quad \forall x \in \mathbb{U} \quad (3)$$

There may be contradiction between the membership and non-membership curves, however even when the two curves appear to be in agreement there may be contradiction involved. Let us use a voting model to derive a very simple set of curves for the fuzzy set *Tall*. The voters are asked to state:

1. whether they would allow a given height to be described as tall.
  - resulting in the membership curve
2. whether they would not allow a given height to be described as tall.
  - resulting in the non-membership curve

## 2 Contradiction

Further let the votes be cast as shown in Table 1.

Table 1: An incomplete table of votes where voter 3 casts votes for membership and non-membership

Height	160		170		180	
Interval	155-165		165-175		175-185	
Voter	$\mu$	$\nu$	$\mu$	$\nu$	$\mu$	$\nu$
1	-	+	+	-	+	-
2	-	+	-	-	+	-
3	-	+	+	+	+	-
4	-	+	+	-	+	-
5	-	+	-	-	+	-

So given the membership and non-membership votes the  $\mu$  and  $\nu$  values give no indication of contradiction.

$$\begin{aligned} Tall &= \{\langle x, \mu_{Tall}(x), \nu_{Tall}(x) \rangle \mid x \in \mathfrak{R}\} \\ Tall &= \{\langle 160, 0.0, 1.0 \rangle, \langle 170, 0.6, 0.2 \rangle, \langle 180, 1.0, 0.0 \rangle\} \end{aligned}$$

If we introduce  $\iota_{Tall}(x)$  to measure those votes which are contradictory then the following set results:

$$\begin{aligned} Tall &= \{\langle x, \mu_{Tall}(x), \nu_{Tall}(x), \iota_{Tall}(x) \rangle \mid x \in \mathfrak{R}\} \\ Tall &= \{\langle 160, 0.0, 1.0, 0.0 \rangle, \langle 170, 0.4, 0.0, 0.2 \rangle, \langle 180, 1.0, 0.0, 0.0 \rangle\} \end{aligned}$$

This tells us that there is contradiction.

If we examine the intervals associated with the membership and non-membership values of *Tall* we get the following:

$$\begin{aligned} Tall &= \{\langle 160, [0.0, 0.0], [1.0, 1.0] \rangle, \\ &\quad \langle 170, [0.6, 1.0], [0.2, 0.6] \rangle, \\ &\quad \langle 180, [1.0, 1.0], [0.0, 0.0] \rangle\} \end{aligned}$$

If we complete the table so voters 2 and 5 choose to vote for membership or non-membership we might get the Table 2.

Table 2: A possible completion of Table 1 which shows up the hitherto latent contradiction in the membership curves.

Height	160		170		180	
Interval	155-165		165-175		175-185	
Voter	$\mu$	$\nu$	$\mu$	$\nu$	$\mu$	$\nu$
1	-	+	+	-	+	-
2	-	+	-	+	+	-
3	-	+	+	+	+	-
4	-	+	+	-	+	-
5	-	+	+	-	+	-

Table 2 shows a contradiction between the membership and non-membership curves, in the manner of [5], although [5]'s work deals with contradiction between two separate sets we confine our work to contradiction within a set.

If we just count the votes for membership and non-membership as an A-IFS, then the following set arises:

$$Tall = \{\langle 160, 0.0, 1.0 \rangle, \langle 170, 0.8, 0.4 \rangle, \langle 180, 1.0, 0.0 \rangle\}$$

This breaks constraint1 and would give a contradiction of 0.2. But this only occurs after all votes have been cast.

If we do not count contradictory votes, and using an Atanassov Intuitionistic Fuzzy Set we obtain

$$Tall = \{\langle 160, 0.0, 1.0 \rangle, \langle 170, 0.6, 0.2 \rangle, \langle 180, 1.0, 0.0 \rangle\}$$

But in this case we have lost the evidence that voter 3 has made a contradictory statement and the votes become part of the hesitation.

The set that arises using our contradictory notation, since we do not count contradictory votes in the membership or the non-membership, but count them as part of the contradiction is:

$$\begin{aligned} Tall = & \{\langle 160, 0.0, 1.0, 0.0 \rangle, \langle 170, 0.6, 0.2, 0.2 \rangle, \\ & \langle 180, 1.0, 0.0, 0.0 \rangle\} \end{aligned}$$

This contradiction was apparent in the earlier set where the votes were incomplete.

Table 3 isolates the contradictory votes in Table 1.

Table 3: The table of votes taken from Table1 where the contradictory votes are separated

Height	160			170			180		
Interval	155-165			165-175			175-185		
Voter	$\mu$	$\nu$	$\iota$	$\mu$	$\nu$	$\iota$	$\mu$	$\nu$	$\iota$
1	-	+	-	+	-	-	+	-	-
2	-	+	-	-	-	-	+	-	-
3	-	+	-	-	-	+	+	-	-
4	-	+	-	+	-	-	+	-	-
5	-	+	-	-	-	-	+	-	-

If we again examine the intervals associated with the memberships and non-memberships of *Tall* we get the following:

$$\begin{aligned} Tall = & \{\langle 160, [0.0, 0.0], [1.0, 1.0] \rangle, \\ & \langle 170, [0.8, 0.8], [0.4, 0, 4] \rangle, \\ & \langle 180, [1.0, 1.0], [0.0, 0.0] \rangle\} \end{aligned}$$

This also now shows the contradiction in the evidence for and against height 170 as the membership and non-membership sum to greater than 1.0.

## 2.1 The world is not contradictory

If we assume the world is not contradictory, then our perception of it may easily be contradictory. A possible circumstance where we obtain contradictory evidence is where the resolution in our measurements is not fine enough. Table 4 shows a possible distribution of votes that are non-contradictory but which result in a contradiction at a courser resolution.

Table 4: The new table of votes where voter 3 is now non-contradictory

Height	160			167.5			172.5			180		
Interval	155-165			165-170			170-175			175-185		
Voter	$\mu$	$\nu$	$\iota$									
1	-	+	-	-	-	-	+	-	-	+	-	-
2	-	+	-	-	-	-	-	-	-	+	-	-
3	-	+	-	-	+	-	+	-	-	+	-	-
4	-	+	-	-	-	-	+	-	-	+	-	-
5	-	+	-	-	-	-	-	-	-	+	-	-

The Contradictory Fuzzy Sets arising from Table 4 are shown in Equation 4

$$\begin{aligned} Tall = & \{\langle 160, 0.0, 1.0, 0.0 \rangle, \langle 167.5, 0.6, 0.2, 0.0 \rangle, \\ & \langle 172.5, 0.6, 0.0, 0.0 \rangle, \langle 180, 1.0, 0.0, 0.0 \rangle\} \end{aligned} \quad (4)$$

And we could make them to be a non contradictory Atanassov Intuitionistic Fuzzy Set, Equation 5.

$$\begin{aligned} Tall = & \{\langle 160, 0.0, 1.0 \rangle, \langle 167.5, 0.6, 0.2 \rangle, \\ & \langle 172.5, 0.6, 0.0 \rangle, \langle 180, 1.0, 0.0 \rangle\} \end{aligned} \quad (5)$$

In order that the reasoning about contradiction may be kept within the reasoning system and that it does not result in contradiction arising in places where it

shouldn't, [8] introduces an operator  $\diamond'$  that obtains a statement about the contradiction that is itself not contradictory.

So in conclusion to part 2 we see that detecting contradiction can enable us to refine our knowledge model in the sense of detecting where the resolution must be increased. There are other possible interpretations of contradiction arising, however this seems to be the most likely.

Further, if we just take either the membership curve or the non-membership curve in isolation it is not possible, from any of the tables of votes, to determine that the resolution must be increased. This is a clear indication that the dual membership and non-membership curves perform a useful practical role.

### 3 Inconsistency

Inconsistency arises where the fuzzy set membership function does not reach 1.0, or where the non-membership function does not reach 0.0 at any point. Some situations where this may arise and the consequences of that are detailed in the following sections.

#### 3.1 Inconsistency arising in definitions

Often the increased granularity requires an increase in the number of fuzzy sets at the point where the increase is required. For example the 3 conventional fuzzy sets shown in Figure 1 is a possible representation of the membership curve arising from our earlier example.

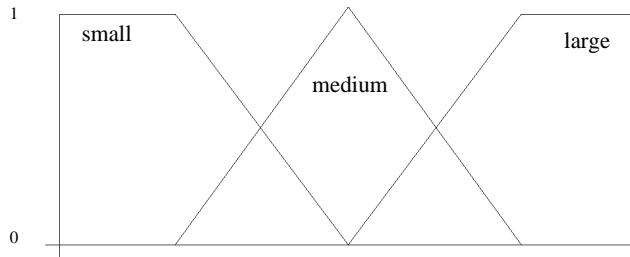


Figure 1: Three fuzzy sets

We may need to increase the resolution of these sets. Generating  $small \cap medium$  and  $medium \cap large$  results in the shaded sets shown in Figure 2.

In the case of the shaded sets in Figure 2 the sets required are the normalised sets resulting from the shaded sets. The fact that the shaded sets are unnormalised

is no surprise and they would generally be modified to become a normalised set as shown in Figure 3.

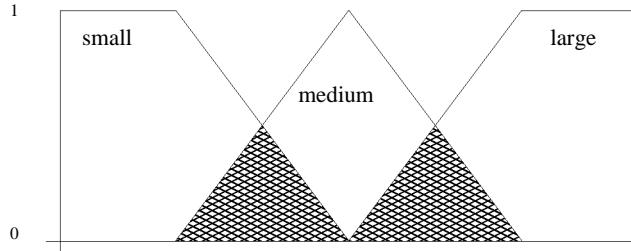


Figure 2: The shaded sets are the intersections of the others and are inconsistent.

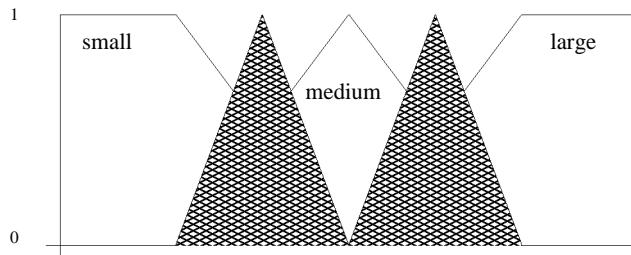


Figure 3: The shaded sets are the normalised versions of the ones shown in Figure 2.

We may further wish to ensure that the new sets crossover and delete the centre set. But in order to do that we do need to be aware of the inconsistency. This is well known and common practice.

### 3.2 Inconsistency in frames

We may wish to establish a frame for a concept, where certain values are appropriate and therefore certain descriptors may be relevant. We propose that there are two objects:

1. Father William
2. Father Thames

In the first case Father William [4] is an old man and part of the concept of Father William is the variable height, and so it is reasonable to argue that a descriptor such as “Father William is Tall” may be applied. So in answer to the question:

Would you allow “Father William is Tall” to be True?

We might obtain the membership in the set  $\{0.7 \mid \text{True}\}$

Because of the way the question is phrased we would expect the result to be a possibility distribution. If we had asked the question:

Would you state “Father William is Tall” is True?

We might obtain more restrictive results and the level of truth of the statement could drop. This in turn depends on the interpretation of the question.

However, this is sufficient to establish that the statement “Father William is Tall” is a reasonable question but if we had the result for membership to be  $\{0.0 \mid \text{True}\}$  this does not establish that the question is reasonable, but neither does it establish that the question is unreasonable.

If we subsequently ask:

Would you allow “Father William is Tall” to be False?

meaning that Father William’s height belongs to some other set.

We might obtain the membership in the set  $\{0.3 \mid \text{False}\}$

Giving  $\{0.7 \mid \text{True}, 0.3 \mid \text{False}\}$ , or expressed as an Atanassov Intuitionistic Fuzzy Set as:

Father William is Tall =  $\{\langle \text{True}, 0.7, 0.0 \rangle, \langle \text{False}, 0.3, 0.0 \rangle\}$ .

Note that in this case we have not established a non-membership curve. We obtain the non-membership by asking:

Would you not allow “Father William is Tall” to be True?

We would expect the answer to be  $\{0.3 \mid \text{True}\}$  as the non-membership values for *True* would typically be the same as the membership values for *False* giving:

Father William is Tall =  $\{\langle \text{True}, 0.7, 0.3 \rangle, \langle \text{False}, 0.3, 0.0 \rangle\}$ .

But it may not be as there could be hesitation or contradiction.

Given that Father William may be medium sized the result of the question:

Would you allow “Father William is Tall” to be True?

We might obtain the membership in the set  $\{0.0 \mid \text{True}\}$

and:

Would you allow “Father William is Short” to be True?

We might obtain the membership in the set  $\{0.0 \mid \text{True}\}$

This does not provide evidence that the underlying slot is inappropriate as there may be other descriptors, such as “medium” that are appropriate.

The question:

Would you allow “Father William is Tall” to be False? meaning that Father William’s height belongs to some other set, might result in the membership of the set  $\{1.0 \mid \text{False}\}$ , which indicates that Father William’s height belongs to some other descriptor. These results indicate that the underlying variable is reasonable for the object under consideration.

If we ask the same questions of “Father Thames”, “Father Thames” is a name given to the river running through London England. Our autonomous system might connect “Father William” and “Father Thames” and initially assign them the same frame. The following questions are meaningless in the context of “Father Thames”. In answer to the question:

Would you allow “Father Thames is Tall” to be True?

We might obtain the membership in the set  $\{0.0 \mid \text{True}\}$

and subsequently

Would you allow “Father Thames is Tall” to be False?

We might obtain the membership in the set  $\{0.0 \mid \text{False}\}$

Giving  $\{0.0 \mid \text{True}, 0.0 \mid \text{False}\}$ , or expressed as an Atanassov Intuitionistic Fuzzy Set as:

Father Thames is Tall =  $\{\langle \text{True}, 0.0, 0.0 \rangle, \langle \text{False}, 0.0, 0.0 \rangle\}$ .

This is a strong indication, but not proof, that the underlying measure associated with Tall is inappropriate. Taking the disjunction of the consistency of the descriptors over a domain gives us a measure of the appropriateness of the underlying measure; and so gives us evidence about the frame associated with the concept. In this case the result tells us that the membership of the underlying variable height in the set of frame slots is zero.

In this last case we have not asked the question:

Would you not allow “Father Thames is Tall” to be True? but have the result that the non-membership value is zero. So even at this stage there is still information missing about the state of the questioning.

[15] describes operators that allows reasoning about inconsistency without requiring an inconsistency management system. These operators allow the inconsistency to be extracted from a statement so the truth about inconsistency can be part of a chain of reasoning. [8] describes modal like operators “contradict  $\perp$ ” that extracts a statement about contradiction from a potentially contradictory statement and “decontradict  $\perp$ ” which allows the contradiction values to be restored; that allows a similar stance to be taken about contradiction.

### 3.3 Fuzzy Frames

Section 3 has shown that the level of inconsistency associated with a variable slot may itself be a fuzzy set. In the example given we take a crisp interpretation of the membership and suggest that the variable height is inappropriate as a descriptor for the frame associated with “Father Thames”. In this case it is a reasonable response; however if this is the basis for an evidence gathering exercise then we could use the appropriateness level as a guide to what questions to pose or what experiments to perform.

## 4 Conclusion

This work has shown that an ability to reason about inconsistency and contradiction has allowed a reasoning system to potentially remove inconsistency and contradiction from a knowledge base. If the premiss that the world is not contradictory is true then removal of contradiction and inconsistency allows the knowledge base to be qualitatively restructured rather than just tuned.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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