Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

Editors
Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

SRI PAS IBS PAN

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Systems Research Institute
Polish Academy of Sciences

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 9788389475299

Stability conditions for generalized nets

Krassimir T. Atanassov[1], **Ivanka M. Stamova**[2] **and**
Vassia K. Atanassova[3]

[1] Centre for Biomedical Engineering - Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
and
International University College, Dobrich, Bulgaria
krat@bas.bg

[2] Centre for Engineering and Computer Sciences - Bourgas Free University,
62 San Stefano St., Bourgas-8000, Bulgaria
stamova@bfu.bg

[3] Institute of Information Technologies – Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 29A, Sofia-1113, Bulgaria
vassia.atanassova@gmail.com

1 Introduction

Generalized Nets (GNs) are defined as an extension of the ordinary Petri nets and their modifications, but in a way that is principally different from the ways of defining the other types of Petri nets. The additional components in the GN-definition give more and larger modelling possibilities and determine the place of the GNs among the separate types of Petri nets, similar to the place of the Turing machine among the finite automata. On the other hand, the definition of GN is more complex than the definitions of the rest Petri net modifications.

27 years since the introduction of GNs, a lot of open problems exist. One of them is related to the conditions for GN stability. With the present paper, we start a discussion on this theme.

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2009.*

2 Short remarks on generalized nets

Following [2, 3], we shall introduce the concepts of GN-transition and a Generalized Net (GN; for it see also [1, 4]).

Every GN-transition is described by a seven-tuple (Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are $L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [2, 3]):

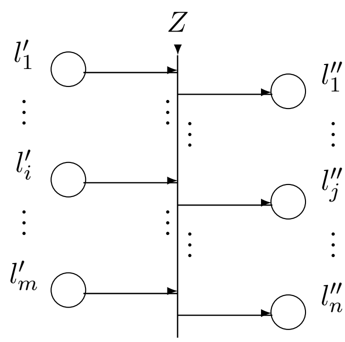

Fig. 1: GN-transition

$$r = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & r_{i,j} & & \\ l'_i & & & (r_{i,j} - \text{ predicate }) & & \\ \vdots & & & (1 \le i \le m, 1 \le j \le n) & & \\ l'_m & & & & & \end{array};$$

$r_{i,j}$ is the predicate which corresponds to the i-th input and j-th output places. When its truth value is "$true$", a token from i-th input place can be transferred to j-th output place; otherwise, this is not possible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ldots l''_j \ldots l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \geq 0 - \text{ natural number }) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array};$$

(f) $\Box$ is an object having a form similar to a Boolean expression. It may contain as variables the symbols which serve as labels for transition's input places, and is an expression built up of variables and the Boolean connectives $\wedge$ and $\vee$ whose semantics is defined as follows:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ - every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,

$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ - there must be at least one token in all places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "$true$", the transition can become active, otherwise it cannot.

The ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle,$$

$$\langle T, t^o, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

is called a *Generalized Net* (GN) if:

(a) A is a set of transitions;

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \rightarrow N$, where $N = \{0, 1, 2, \ldots\} \cup \{\infty\}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \rightarrow N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, L is the set of all GN-places);

(d) c is a function giving the capacities of the places, i.e., $c : L \rightarrow N$;

(e) f is a function, which calculates the truth values of the predicates of the transition's conditions (for the GN described here let the function f have the value "$false$" or "$true$", i.e., a value from the set $\{0, 1\}$;

(f) θ_1 is a function giving the next time-moment when a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i. e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \underset{l \in Q^I}{\cup} K_l,$$

where K_l is the set, of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \rightarrow N$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

(l) t^o is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) X is the set of all initial characteristics the tokens can receive when they enter the net;

(o) Φ is a characteristic function, which assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition.

(p) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \rightarrow N$.

A GN may lack some of the components, and such GNs give rise to special classes of GNs called *reduced GNs*. The omitted elements of the reduced GNs are marked by " $*$ ".

3 On the stability conditions for generalized nets

One of the hitherto unsolved problems from the GN theory is related to the conditions for stability of GN-functioning. Here we shall discuss some of these conditions and search for them on two levels of the level of GN-transitions and on the level of the whole GN.

It is important to note that some GN-tokens can unite, while other – can split during the GN-functioning, i.e., their number may change. In the present paper we will discuss the case when tokens neither merge, nor split (this can be guaranteed if we use some dynamical operators (see [2, 3]). In the next authors' research, the more complex cases will be discussed, as well.

Let a fixed transition Z be given. Therefore, we know the characteristic functions $\Psi_1, \Psi_2, ..., \Psi_m$ of each of the m output places of Z. Let us fix one of its input places (l) and let us know the tokens that will enter it – they generate the set

$$T_l = \{\alpha_1, \alpha_2, ..., \alpha_s\}.$$

Let us know the current characteristic $x_{cu}^{\alpha_i}$ of the i-th α-token. Without restriction we can assume that this characteristic can be estimated by some function V. Of course, if $x_{cu}^{\alpha_i}$ is some number (natural, integer, real, etc.) V can be an identity, but when $x_{cu}^{\alpha_i}$ does not have numerical nature, e.g., if $x_{cu}^{\alpha_i}$ is some element of a data base, string, object, etc., we will assume that V assigns some number to the characteristic.

Let the set of all possible characteristics of token α in place l be $S(\alpha, l)$. Now, we can determine some element of $S(\alpha, l)$ that according to a given criterion possesses an average value. For example, it can be the element x with estimation $V(x)$ that is the one closest to the arithmetic mean

$$V_{arithm}(\alpha, l) = \frac{1}{|S(\alpha, l)|} \sum_{x \in S(\alpha, l)} V(x_{cu}^{\alpha_i}),$$

where $|X|$ is the cardinality of set X.

First, we formulate a criterion for the stability of functioning of transition Z with respect to a fixed input place l, an output place m and token α. This criterion can have the form:

$$C_1(\alpha, l, m) = (\forall \alpha \in T_l)(\forall \varepsilon > 0)(\exists \delta = \delta(\alpha, \varepsilon) > 0)$$
$$(\forall x_{cu}^{\alpha} \in S(\alpha, x) : \rho(V(x_{cu}^{\alpha}), V_{arithm}(\alpha, l)) < \delta$$
$$\rightarrow \rho(V(\Psi_m(\alpha)), V_{arithm}(\alpha, m)) < \varepsilon)).$$

Let us assume that

$$C_1(\alpha, l, m) = \begin{cases} 1, & \text{the criterion is valid} \\ 0, & \text{otherwise} \end{cases}.$$

Let

$$T_{l,m} = \{\alpha | (\alpha \in T_l) \& (\text{ from place } l \text{ token } \alpha \text{ enters place } m)\}.$$

Second, we construct a criterion for the stability of functioning of transition Z with respect to a fixed input place l, an output place m and set $T_{l,m}$:

$$C_2(l,m) = \text{“} \bigwedge_{\alpha \in T_{l,m}} C_1(\alpha, l, m) = 1\text{”}.$$

Third, we construct a criterion for the stability of functioning of transition Z:

$$C_3(Z) = \text{“} \bigwedge_{l \in pr_1 Z, m \in pr_2 Z} C_2(l,m) = 1\text{”}.$$

Finally, we construct a criterion for the stability of functioning of a given GN E:

$$C_4(E) = \text{“} \bigwedge_{Z \in pr_1 pr_1 E} C_3(Z) = 1\text{”}.$$

In all these cases, the respective criterion $C_i(*)$ will be valid if its truth-value is 1 (or $true$).

When the GN has to be trained with particular values of its token characteristics, then we can determine the intervals I_{T_l} and I_m in which all values $V(x_{cu}^{\alpha})$ and $V(\Psi_m(\alpha))$ will be collected, for each token $\alpha \in T_l$ and for each output place m of transition Z. So, we can change the first condition to

$$C_1'(\alpha, l, m) = (\forall \alpha \in T_l)(\forall x_{cu}^{\alpha} \in S(\alpha, x))$$

$$(V(x_{cu}^{\alpha}) \in I(T_l) \rightarrow V(\Psi_m(\alpha)) \in I_m).$$

The three other conditions will keep their previous forms.

4 Generalized net registering satisfying of the stability conditions for generalized nets

Let a GN E be given and let us be interested to understand whether it works in a stable way. Then, we can add to it special components that will register the moments when the given GN will satisfy the stability conditions.

First, to each GN-transition Z of E (see Fig. 2) we add a new place l_Z that is input and output of the transition and in which one new token (different from all other tokens of the net) stays. In this place no other tokens enter. The special token (let us mark it by σ_Z) obtains as a current characteristic

$$x_{cu}^{\sigma_Z} = \text{“}x_{cu-1}^{\sigma_Z}.C_3(Z)\text{”},$$

where $x_{cu-1}^{\sigma_Z}$ is the previous characteristic of σ_Z. If the final characteristic of σ_Z is equal to 0, then in some time-moment of the GN-functioning the transition Z has not been stable. If the final characteristic of σ_Z is equal to 1, then the transition has been stable all the time.

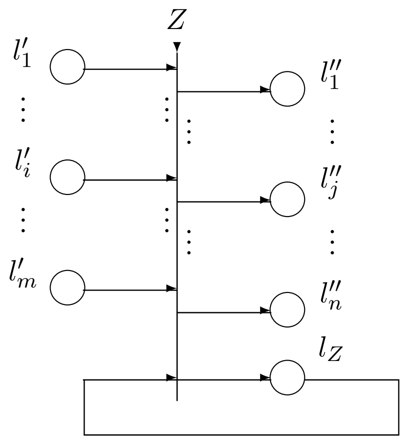

Fig. 2: The modified GN-transition

If we would like to have more information about the process, for instance, the time-moments when the GN has been unstable, we can modify σ_Z's current characteristic to the form:

$$x_{cu}^{\sigma_Z} = \text{“}x_{cu-1}^{\sigma_Z}.C_3(Z), t\text{”},$$

where t is the time-moment when the characteristic has been obtained.

Now, we will extend the GN E adding one new transition Z_E (see Fig. 3) that contains only one (simultaneously input and output) place l_E in which permanently stays only one token σ_E that on each step obtains as a current characteristic

$$x_{cu}^{\sigma_E} = \text{“}x_{cu-1}^{\sigma_E}.C_4(E)\text{”}$$

or (if we like to have more detailed information) a current characteristic

$$x_{cu}^{\sigma_E} = \text{“}x_{cu-1}^{\sigma_E}.C_4(E), t\text{”}.$$

As we mentioned above, if the final σ_E's characteristic is equal to 1, then the GN has permanently functioned in a stable way; while in the opposite case, there has been at least one moment when the GN had not worked stably.

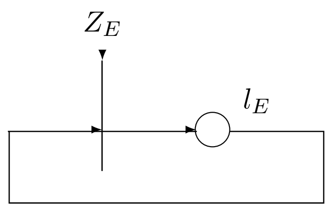

Fig. 3: Transition Z_E

5 Conclusion

The so-constructed criteria give information about the behaviour of a given GN whose tokens do not unite or split. In next papers we will discuss some more complex cases and will formulate other criteria for GN stability.

Acknowledgements

The third author of the present work is supported by project 010077 “Research of Decision Making Models in Fuzzy Environments”.

References

[1] Alexieva, J., E. Choy, E. Koycheva. Review and bibliography on generalized nets theory and applications. – In: A Survey of Generalized Nets (E. Choy, M. Krawczak, A. Shannon and E. Szmidt, Eds.), Raffles KvB Monograph No 10, 2007, 207-301.

[2] Atanassov, K. Generalized Nets, World Scientific. Singapore, New Jersey, London, 1991.

[3] Atanassov, K. On Generalized Nets Theory, “Prof. M. Drinov” Academic Publishing House, Sofia, 2007.

[4] Radeva, V., M. Krawczak, E. Choy. Review and bibliography on generalized nets theory and applications. – Advanced Studies in Contemporary Mathematics, Vol. 4, 2002, No 2, 173-199.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

http://www.ibspan.waw.pl/ifs2009

The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475299
ISBN 838947529-4