Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Intuitionistic fuzzy subtractions $-'^{\varepsilon,\eta}$ and $-''^{\varepsilon,\eta}$

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1 Introduction

In [6] the author introduced a set of new negations and implications over IFSs. They generalize the classical negation over IFSs, but on the other hand, they have some non-classical properties. The set has the form

$$\mathcal{N} = \{\neg^{\varepsilon,\eta} \mid 0 \leq \varepsilon < 1 \;\&\; 0 \leq \eta < 1\},$$

where for each IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$:

$$\neg^{\varepsilon,\eta} A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta)\rangle | x \in E\}.$$

For ε and η there are two cases.

• $\eta < \varepsilon$

As it is shown in [5], this case is impossible.

• $\eta \geq \varepsilon$

Let everywhere below $0 \leq \varepsilon \leq \eta < 1$ be fixed.

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In [6] we constructed the implication

$$\begin{aligned} & A \rightarrow^{\varepsilon,\eta} B \\ &= \{\langle x, \max(\mu_B(x), \min(1, \nu_A(x) + \varepsilon)), \\ &\min(\nu_B(x), \max(0, \mu_A(x) - \eta))\rangle | x \in E\} \\ &= \{\langle x, \min(1, \max(\mu_B(x), \nu_A(x) + \varepsilon)), \end{aligned}$$

For the needs of the discussion below we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see, [1, 2]) by:

$$x \text{ is an IFT if and only if (shortly: iff) } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$.

In [6] we checked the Georg Klir and Bo Yuan's axioms for implication (see [9]):

Axiom 1 $(\forall x, y)(x \leq y \rightarrow (\forall z)(I(x, z) \geq I(y, z))$.
Axiom 2 $(\forall x, y)(x \leq y \rightarrow (\forall z)(I(z, x) \leq I(z, y))$.
Axiom 3 $(\forall y)(I(0, y) = 1)$.
Axiom 4 $(\forall y)(I(1, y) = y)$.
Axiom 5 $(\forall x)(I(x, x) = 1)$.
Axiom 6 $(\forall x, y, z)(I(x, I(y, z)) = I(y, I(x, z)))$.
Axiom 7 $(\forall x, y)(I(x, y) = 1 \text{ iff } x \leq y)$.
Axiom 8 $(\forall x, y)(I(x, y) = I(N(y), N(x)))$, where N is an operation for a negation.
Axiom 9 I is a continuous function.

There, we proved

Theorem 1: Implication $\rightarrow^{\varepsilon,\eta}$ and negation $\neg^{\varepsilon,\eta}$
(a) satisfy Axioms 1,2,3,6 and 9;
(b) satisfy Axioms 4 and 5 as IFTs, but not as tautologies;
(c) satisfy Axiom 8 in the form

Axiom 8' $(\forall x, y)(I(x, y) \leq I(N(y), N(x)))$.

$$x \text{ is an IFT iff } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$.

In the same paper we discussed the axioms of intuitionistic logic (see, e.g., [10]). For arbitrary propositional forms A, B and C they are:

(a) $A \rightarrow A$,
(b) $A \rightarrow (B \rightarrow A)$,
(c) $A \rightarrow (B \rightarrow (A \& B))$,

(d) $(A \to (B \to C)) \to (B \to (A \to C))$,
(e) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$,
(f) $A \to \neg\neg A$,
(g) $\neg(A \& \neg A)$,
(h) $(\neg A \vee B) \to (A \to B)$,
(i) $\neg(A \vee B) \to (\neg A \& \neg B)$,
(j) $(\neg A \& \neg B) \to \neg(A \vee B)$,
(k) $(\neg A \vee \neg B) \to \neg(A \& B)$,
(l) $(A \to B) \to (\neg B \to \neg A)$,
(m) $(A \to \neg B) \to (B \to \neg A)$,
(n) $\neg\neg\neg A \to \neg A$,
(o) $\neg A \to \neg\neg\neg A$,
(p) $\neg\neg(A \to B) \to (A \to \neg\neg B)$,
(q) $(C \to A) \to ((C \to (A \to B)) \to (C \to B))$.

Theorem 2: Implications $\to^{\varepsilon,\eta}$ and negation $\neg^{\varepsilon,\eta}$ satisfy all axioms as IFTs, but not as tautologies.

Now, we will introduce a new operation on IFSs and will study its properties.

2 On operations "subtraction"

During the last four years more than 130 different versions of operation "implication" and more than 35 different versions of operation "negation" were introduced over the Intuitionistic Fuzzy Sets (IFS, see [2]). The definitions of the negation operations are introduced in [8] (see, also [11]).

In [7] a series of new versions of operation "subtraction" was introduced. As a basis of the new versions of operation "subtraction" from [7], the well-known formula from set theory:

$$A - B = A \cap \neg B$$

was used, where A and B are given sets. In the IFS-case, if the sets

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

are given (for the description of their components see [2]) we can define the following versions of operation "subtraction":

$$A -'_i B = A \cap \neg_i B, \tag{1}$$

where $i = 1, 2, ..., 36$. On the other hand, as we discussed in [3], the Law for Excluded Middle is not always valid in IFS theory. By this reason, we can introduce a new series of "subtraction" operations, that will have the form:

$$A -''_i B = \neg_i \neg_i A \cap \neg_i B, \qquad (2)$$

where $i = 1, 2, ..., 36$.

Of course, for every two IFSs A and B it will be valid that:

$$A -'_1 B = A -''_1 B,$$

because the first negation will satisfy the Law for Excluded Middle, but in the rest cases this equality will not be valid.

In [7] and [11] the properties of negation $\neg_2$ and $\neg_{11}$ and the generated by them four IF-subtractions were studied.

Below we will make a next step of the research on the new IF-operations, discussing the properties of two new IF-subtractions: $-'^{\varepsilon,\eta}$ and $-''^{\varepsilon,\eta}$.

3 Basic properties of operation $-'^{\varepsilon,\eta}$

Using (1), we obtain the following form of the operation $-'^{\varepsilon,\eta}$:

$$A -'^{\varepsilon,\eta} B = A \cap \neg^{\varepsilon,\eta} B$$
$$= \{\langle x, \min(\mu_A(x), 1, \nu_B(x) + \varepsilon), \max(\nu_A(x), 0, \mu_B(x) - \eta)\rangle | x \in E\}$$
$$= \{\langle x, \min(\mu_A(x), \nu_B(x) + \varepsilon), \max(\nu_A(x), \mu_B(x) - \eta)\rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for two given IFSs A and B and for each $x \in E$ we obtain that:
(a) if $\nu_A(x) \leq \mu_B(x) - \eta$, then

$$0 \leq \min(\mu_A(x), \nu_B(x) + \varepsilon) + \max(\nu_A(x), \mu_B(x) - \eta)$$
$$= \min(\mu_A(x), \nu_B(x) + \varepsilon) + \mu_B(x) - \eta$$
$$= \min(\mu_A(x) + \mu_B(x) - \eta), \nu_B(x) + \varepsilon + \mu_B(x) - \eta) \leq 1,$$

because $\mu_B(x) + \mu_B(x) + \varepsilon - \eta \leq 1$;
(b) if $\nu_A(x) > \mu_B(x) - \eta$, then

$$0 \leq \min(\mu_A(x), \nu_B(x) + \varepsilon) + \max(\nu_A(x), \mu_B(x) - \eta)$$
$$= \min(\mu_A(x), \nu_B(x) + \varepsilon) + \nu_A(x)$$
$$= \min(\mu_A(x) + \nu_A(x), \nu_B(x) + \varepsilon + \nu_A(x)) \leq 1$$

because $\mu_A(x) + \nu_B(x) \leq 1$.

Let us define the *empty IFS,* the *totally uncertain IFS, unit IFS, ε-uncertain IFS,* and the *η-empty IFS* (see [2]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$
$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$
$$E^* = \{\langle x, 1, 0\rangle | x \in E\},$$
$$\varepsilon^* = \{\langle x, \varepsilon, 0\rangle | x \in E\},$$
$$\eta^* = \{\langle x, 0, 1 - \eta\rangle | x \in E\},$$

and also two other special sets:

$$(\varepsilon, \eta)^* = \{\langle x, \varepsilon, 1 - \eta\rangle | x \in E\},$$
$$(\varepsilon, \eta)^{**} = \{\langle x, 1 - \eta + \varepsilon, 0\rangle | x \in E\}.$$

By analogy, we can prove the following assertions.

Theorem 1: For every two IFSs A and B:

(a) $A -'^{\varepsilon,\eta} E^* = O^*$,

(b) $A -'^{\varepsilon,\eta} O^* = A$,

(c) $E^* -'^{\varepsilon,\eta} A = \neg^{\varepsilon,\eta} A$,

(d) $O^* -'^{\varepsilon,\eta} A = O^*$,

(e) $(A -'^{\varepsilon,\eta} B) \cap C = (A \cap C) -'^{\varepsilon,\eta} B = A \cap (C -'^{\varepsilon,\eta} B)$,

(f) $(A \cap B) -'^{\varepsilon,\eta} C = (A -'^{\varepsilon,\eta} C) \cap (B -'^{\varepsilon,\eta} C)$,

(g) $(A \cup B) -'^{\varepsilon,\eta} C = (A -'^{\varepsilon,\eta} C) \cup (B -'^{\varepsilon,\eta} C)$,

(h) $(A -'^{\varepsilon,\eta} B) -'^{\varepsilon,\eta} C = (A -'^{\varepsilon,\eta} C) -'^{\varepsilon,\eta} B$.

Obviously

$$O^* -'^{\varepsilon,\eta} U^* = O^*,$$
$$O^* -'^{\varepsilon,\eta} E^* = O^*,$$
$$O^* -'^{\varepsilon,\eta} O^* = O^*,$$
$$U^* -'^{\varepsilon,\eta} U^* = O^*,$$
$$U^* -'^{\varepsilon,\eta} O^* = U^*,$$
$$U^* -'^{\varepsilon,\eta} E^* = \eta^*,$$
$$E^* -'^{\varepsilon,\eta} O^* = E^*,$$
$$E^* -'^{\varepsilon,\eta} U^* = \varepsilon^*,$$
$$E^* -'^{\varepsilon,\eta} E^* = (\varepsilon, \eta)^*.$$

4 Basic properties of operation $-''^{\varepsilon,\eta}$

Now, using (2) and having in mind that

$$\neg^{\varepsilon,\eta}\neg^{\varepsilon,\eta}A = \neg^{\varepsilon,\eta}\{\langle x, \min(1, \nu_A(x)+\varepsilon), \max(0, \mu_A(x)-\eta)\rangle x, |x \in E\}$$
$$= \{\langle x, \min(1, \max(0, \mu_A(x)-\eta)+\varepsilon),$$
$$\max(0, \min(1, \nu_A(x)+\varepsilon)-\eta)\rangle x, |x \in E\}$$
$$= \{\langle x, \min(1, \max(\varepsilon, \mu_A(x)-\eta+\varepsilon)),$$
$$\max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta))\rangle x, |x \in E\}$$
$$= \{\langle x, \max(\varepsilon, \mu_A(x)-\eta+\varepsilon),$$
$$\max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta))\rangle x, |x \in E\}$$

we obtain the following form of the operation $-''^{\varepsilon,\eta}$:

$$A -''^{\varepsilon,\eta} B = \neg^{\varepsilon,\eta}\neg^{\varepsilon,\eta}A \cap \neg^{\varepsilon,\eta}B$$
$$= \{\langle x, \max(\varepsilon, \mu_A(x)-\eta+\varepsilon), \max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta))\rangle x, |x \in E\}$$
$$\cap\{\langle x, \min(1, \nu_B(x)+\varepsilon), \max(0, \mu_B(x)-\eta)\rangle x, |x \in E\}$$
$$= \{\langle x, \min(\max(\varepsilon, \mu_A(x)-\eta+\varepsilon), 1, \nu_B(x)+\varepsilon),$$
$$\max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta), \mu_B(x)-\eta)\rangle x, |x \in E\}$$
$$= \{\langle x, \min(\max(\varepsilon, \mu_A(x)-\eta+\varepsilon), \nu_B(x)+\varepsilon),$$
$$\max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta), \mu_B(x)-\eta)\rangle x, |x \in E\}. \qquad (3)$$

The check that in a result of the operation we obtain an IFS and the proofs of the next assertions are similar to above ones.

Theorem 2: For every IFS A:
(a) $A -''^{\varepsilon,\eta} E^* = (\varepsilon, \eta)^*$,
(b) $A -''^{\varepsilon,\eta} O^* = \neg^{\varepsilon,\eta}\neg^{\varepsilon,\eta}A$,
(c) $O^* -''^{\varepsilon,\eta} A = (\varepsilon, \eta)^*$,
(d) $(A \cap B) -'^{\varepsilon,\eta} C = (A -'^{\varepsilon,\eta} C) \cap (B -'^{\varepsilon,\eta} C)$,
(e) $(A \cup B) -'^{\varepsilon,\eta} C = (A -'^{\varepsilon,\eta} C) \cup (B -'^{\varepsilon,\eta} C)$,
(f) $(A -''^{\varepsilon,\eta} B) \cap \neg^{\varepsilon,\eta}\neg^{\varepsilon,\eta}C = (C -''^{\varepsilon,\eta} B) \cap \neg^{\varepsilon,\eta}\neg^{\varepsilon,\eta}A$.

Obviously,

$$\begin{aligned}
O^* -'^{\varepsilon,\eta} U^* &= (\varepsilon,\eta)^*,\\
O^* -'^{\varepsilon,\eta} E^* &= (\varepsilon,\eta)^*,\\
O^* -'^{\varepsilon,\eta} O^* &= (\varepsilon,\eta)^*,\\
U^* -'^{\varepsilon,\eta} U^* &= \varepsilon^*,\\
U^* -'^{\varepsilon,\eta} E^* &= (\varepsilon,\eta)^*,\\
U^* -'^{\varepsilon,\eta} O^* &= \eta^*,\\
E^* -'^{\varepsilon,\eta} O^* &= (\varepsilon,\eta)^{**},\\
E^* -'^{\varepsilon,\eta} U^* &= \varepsilon^*,\\
E^* -'^{\varepsilon,\eta} E^* &= (\varepsilon,\eta)^*.
\end{aligned}$$

5 Conclusion of again for De Morgan's Laws

We shall finish with the following observation. In classical logic

$$\neg(X \to Y) = \neg(\neg X \vee Y) = \neg\neg X \wedge \neg Y = X \wedge \neg Y = X - Y,$$

where the last operation "$-$" is conditionally marked by symbol "$-$", because there is an analogous with set-theoretical operation

$$X - Y = X \cap \neg Y.$$

Now, we will mention that in [3] it is proved that for some intuitionistic fuzzy negations the De Morgan's Laws are not valid in the forms

$$\begin{aligned}
X\neg(\neg X \vee \neg Y) &= X \wedge Y\\
X\neg(\neg X \wedge \neg Y) &= X \vee Y
\end{aligned}$$

but they are valid in the forms

$$\begin{aligned}
X\neg(\neg X \vee \neg Y) &= \neg\neg X \wedge \neg\neg Y\\
X\neg(\neg X \wedge \neg Y) &= \neg\neg X \vee \neg\neg Y.
\end{aligned}$$

In the case of intuitionistic fuzzy form of Kleene-Dienes implication (that in [4] is marked by $\to_4$) and the corresponding to it negation (marked by $\neg_1$, there), for every two IFSs A and B, we have

$$\neg_1(A \to_4 B) = \neg_1\{\langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | x \in E\}$$
$$= \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle | x \in E\} = A \cap \neg_1 B = A -_1 B.$$

Also, we can check that it is valid that

$$\neg_1(A -_1 B) = A \rightarrow_4 B.$$

It can be checked that for two IFSs A and B:

$$\neg^{\varepsilon,\eta}(A \rightarrow^{\varepsilon,\eta} B) \neq A -'^{\varepsilon,\eta} B,$$
$$\neg^{\varepsilon,\eta}(A -'^{\varepsilon,\eta} B) \neq A \rightarrow^{\varepsilon,\eta} B,$$
$$\neg^{\varepsilon,\eta}(A -''^{\varepsilon,\eta} B) \neq A \rightarrow^{\varepsilon,\eta} B,$$

but

$$\neg^{\varepsilon,\eta}(A \rightarrow^{\varepsilon,\eta} B) = A -''^{\varepsilon,\eta} B.$$

We shall prove the fourth assertion (the rest are check analogously):

$$\begin{gathered}\neg^{\varepsilon,\eta}(A \rightarrow^{\varepsilon,\eta} B)\\ = \neg^{\varepsilon,\eta}\{\langle x, \min(1, \max(\mu_B(x), \nu_A(x) + \varepsilon)),\\ \max(0, \min(\nu_B(x), \mu_A(x) - \eta)))\rangle | x \in E\}\\ = \{\langle x, \min(1, \max(\varepsilon, \min(\nu_B(x) + \varepsilon, \mu_A(x) - \eta + \varepsilon))),\\ \max(0, \min(1 - \eta, \max(\mu_B(x) - \eta, \nu_A(x) + \varepsilon - \eta))))\rangle | x \in E\}.\end{gathered}$$

Now, from (3) we obtain the following equalities. If

$$\begin{gathered}Z = \min(1, \max(\varepsilon, \min(\nu_B(x) + \varepsilon, \mu_A(x) - \eta + \varepsilon)))\\ - \min(\max(\varepsilon, \mu_A(x) - \eta + \varepsilon), \nu_B(x) + \varepsilon),\end{gathered}$$

then we have two cases.

If $\nu_B(x) \geq \mu_A(x) - \eta$, then

$$\begin{aligned}Z &= \min(1, \max(\varepsilon, \mu_A(x) - \eta + \varepsilon)) - \min(\max(\varepsilon, \mu_A(x) - \eta + \varepsilon), \nu_B(x) + \varepsilon),\\ &= \max(\varepsilon, \mu_A(x) - \eta + \varepsilon) - \min(\max(\varepsilon, \mu_A(x) - \eta + \varepsilon), \nu_B(x) + \varepsilon) = 0,\end{aligned}$$

because $\max(\varepsilon, \mu_A(x) - \eta + \varepsilon) \leq \nu_B(x) + \varepsilon)$.

If $0 \leq \nu_B(x) < \mu_A(x) - \eta$, then

$$\begin{aligned}Z &= \min(1, \max(\varepsilon, \nu_B(x) + \varepsilon)) - \min(\mu_A(x) - \eta + \varepsilon, \nu_B(x) + \varepsilon)\\ &= \min(1, \nu_B(x) + \varepsilon) - (\nu_B(x) + \varepsilon) = 0,\end{aligned}$$

because $\nu_B(x) + \varepsilon < \mu_A(x) - \eta + \varepsilon \leq \mu_A(x) \leq 1$.

If

$$Y = \max(0, \min(1-\eta, \max(\mu_B(x)-\eta, \nu_A(x)+\varepsilon-\eta))) \\ - \max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta), \mu_B(x)-\eta),$$

then we again have two cases.

If $\mu_B(x) \geq \nu_A(x)+\varepsilon$, then

$$Y = \max(0, \min(1-\eta, \mu_B(x)-\eta)) - \max(0, \nu_A(x)+\varepsilon-\eta, \mu_B(x)-\eta) \\ = \max(0, \mu_B(x)-\eta) - \max(0, \mu_B(x)-\eta) = 0.$$

If $\mu_B(x) < \nu_A(x)+\varepsilon$, then

$$Y = \max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta)) - \max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta)) = 0.$$

Hence,

$$\neg^{\varepsilon,\eta}(A \to^{\varepsilon,\eta} B) \\ = \{\langle x, \min(\max(\varepsilon, \mu_A(x)-\eta+\varepsilon), \nu_B(x)+\varepsilon), \\ \max(0, \min(1-\eta, \nu_A(x)+\varepsilon-\eta), \mu_B(x)-\eta)\rangle x, |x \in E\} \\ = A -''^{\varepsilon,\eta} B.$$

Therefore, the new operations have different properties and behaviour.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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