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Volume I: Foundations

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Correctness-checking of uncertain-equation solutions on example of the interval-modal method

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Abstract

There exists a number of methods for solving of uncertain interval-equations (normal or fuzzy intervals). Particular methods deliver different solutions of one and the same problem. It causes confusion, pessimism and doubts as to value of proposed solution methods among scientists and engineers. To check correctness of particular solving-methods a testing-method for them is necessary. It seems, such method has not existed until now. The paper presents such method that was called testing-point method (TP-method). It was applied for correctness checking of solutions of fuzzy-interval equation $A + BX = C$ delivered by the interval-modal method of R.J. Bhivani and B.M. Patre [1]. It was shown that this method delivers generally incorrect results. The paper presents also a solution method of interval-equations based on multidimensional RDM interval-arithmetic. This method delivers a solution that satisfies all conditions and requirements imposed on solution of the investigated uncertain equation.

Keywords: uncertainty theory, interval arithmetic, fuzzy arithmetic, granular computing, soft computing.

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1 Introduction

Uncertainty theory [10] can be considered to be science of the future because it tries to find solutions of problems with uncertain inputs, parameters and uncertain model of the problem. In the reality uncertainty occurs very frequently. However, in scientific investigations it is mostly assumed that precise, crisp data and precise model is at disposal. Then conventional mathematics is used for the problem solving. Solutions achieved in this way are called in scientific literature "academic" or "laboratory" solutions because they mostly cannot be applied in the practice. Therefore mathematical methods are rarely used in firms and institutions. Uncertainty theory enables change of this situation. However, development of uncertainty theory is very rendered. The simplest method of uncertainty modeling are normal or fuzzy intervals. Arithmetic operations on normal intervals are formulated by interval arithmetic [13, 14] and on fuzzy intervals by fuzzy arithmetic [7, 8, 15]. If realization methods of simple arithmetic operations are known, then solution of more difficult and complicated interval problems is very difficult or sometimes even impossible. As Dymova writes in [6], difficulties and paradoxes occur even at solving the simplest uncertain equations as $A + X = B$ or $AX = B$, where A and B are usual or fuzzy intervals. Thus, there exists a great interest among scientists in solving of uncertain equations. A number of methods was elaborated. E.g.: Klir and Yuan proposed in [9] α-cut method. Mazarbhuiya et al in [12] proposed superimposition-method. Other methods proposed Buckley in [4, 5], Boukezzoula et al in [3]. Dymova and Sevastjanov in [6, 21] proposed extended-zero method and Propkopowicz in [20] a method based on OFN-numbers (ordered fuzzy numbers). The number of methods for uncertain equation solving is much higher and they all cannot be discussed here. Particular methods differ one from another and in the general case they deliver different solutions for one and the same problem. It causes confusion among users of these methods and doubts concerning their correctness. They ask themselves: Which method is correct?, Which solution is more precise?. Additionally, incessantly new methods are elaborated and proposed in the literature. Does there exist one correct solving-method of (fuzzy-) interval-equations? To answer above questions a testing method is necessary that would enable checking whether equation-solutions generated by given method are correct or not. It seems that such method has not been existed until now. If it has been existed then the number of various solving methods of interval equation would be minimal at present. This paper will present a method for correctness checking of solutions generated by various solving methods of fuzzy and not-fuzzy interval equations. This method was called testing-points' method (TP-method). Further on the method will be used for correctness checking of solvings delivered by the modal-interval method proposed by R.J. Bhiwani and B.M. Patre in [1].

2 Modal-interval method of solving first-order fuzzy-equations

Further on, this method will shortly be called MI-method. Its full description is given in [1] and in this chapter the method will be presented only shortly and mainly in form of citations. ”The MI-method is a natural extension of the classical interval analysis where the concept of interval is widened in following way. The modal interval $[a, b]$ is defined by (1).

$$X = [a, b] := \begin{cases} ([a, b]', \exists) & if \quad a \le b \\ ([a, b]', \forall) & if \quad a \ge b \end{cases} \tag{1}$$

The quantifiers $\exists$ and $\forall$ are called as proper and improper selection of modality, respectively. For example, the classical interval $[4, 7]$ corresponds to modal interval $([4, 7]', \exists)$ and $[8, 5]$ corresponds to $([5, 8]', \forall)$. If $a \le b$ we speak about an interval with ”proper” modality (or proper interval also called existential interval) and if $a \ge b$ we speak about interval with the ”improper” modality (or improper interval also called universal interval). Proper and improper intervals are related by the ”$dual$” operator (2).”

$$dual[(a, b)] = [b, a] \tag{2}$$

Authors of the MI-method described in [1] operations of addition, subtraction, multiplication and division of intervals and gave a method for solving fuzzy linear equation of type $A + BX = C$. This method will be presented in the sequence.

”Let us assume A, B and C are fuzzy intervals and $0 \notin B$, which means B is either positive or negative. The fuzzy linear equation $A + BX = C$ can be expressed by the α-cuts of A, B, C and X, where $\alpha \in [0, 1]$, which leads to the following interval equation (3).

$$A_\alpha + B_\alpha X_\alpha = C_\alpha \tag{3}$$

Where $A_\alpha = [a_\alpha^-, a_\alpha^+]$, $B_\alpha = [b_\alpha^-, b_\alpha^+]$, $C_\alpha = [c_\alpha^-, c_\alpha^+]$, and $X_\alpha = [x_\alpha^-, x_\alpha^+]$ are α-cut intervals. The exact solution for the above equation (3) is not (4).

$$X_\alpha = [C_\alpha - A_\alpha] / B_\alpha \tag{4}$$

Although classical interval arithmetic gives a guaranteed enclosure of the solution, it is overestimated interval. Our objective is to find the exact solution of equation (3). Using modal interval arithmetic the solution can be found very easily. It is given by (5), (6), (7)."

$$A_\alpha + B_\alpha X_\alpha = C_\alpha \tag{5}$$

$$B_\alpha X_\alpha = C_\alpha - dual\,(A_\alpha) = D_\alpha \tag{6}$$

$$\therefore X_\alpha = \frac{D_\alpha}{dual\,(B_\alpha)} \tag{7}$$

Solving equation (7) for various α-levels, $\alpha \in [0,1]$ one achieves a fuzzy number X being solution of equation (3). Next, authors if the MI-method show its application in solving of a concrete equation. "Let A, B and C be triangular membership functions of equation (3).

$$\begin{array}{l} A = tri[0.5, 0.7, 0.9] \\ B = tri[0.3, 0.5, 0.7] \\ C = tri[1.0, 1.5, 2.0] \end{array} \tag{8}$$

The membership functions are plotted in Fig. 1.

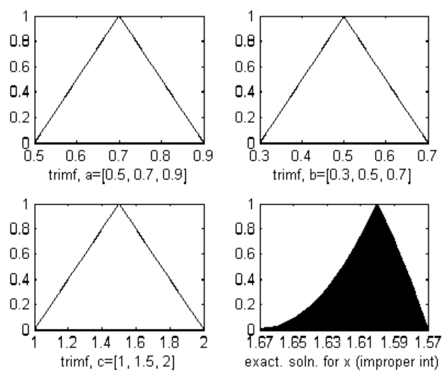

Figure 1: Membership functions of A, B, C and X.

The simulation examples are taken from [2], in which the author has proposed an existence condition for exact solution to exist. For the above example it is said the existence condition is not satisfied, hence there exists no result. Actually the result obtained is an improper interval. By using modal interval arithmetic approach proposed here we get the same results and they can be interpreted using semantics in the following way (9).

$$\begin{aligned}&\left(\forall\left[d_p, D'_p\right]\right)\left(\forall\left[b_p, B'_p\right]\right)\left(\exists\left[x_i, X'_i\right]\right) x_\alpha = f\left(d_\alpha, b_\alpha\right)\\&\left(\forall d_p \in \left[\underline{d_p}, \overline{d_p}\right]\right)\left(\forall b_p \in \left[\underline{b_p}, \overline{b_p}\right]\right)\left(\exists x_i \in \left[\underline{x_i}, \overline{x_i}\right]\right) x_\alpha = f\left(d_\alpha, b_\alpha\right)\end{aligned} \tag{9}$$

The plot of result with α varying from 0 to 1 in the steps of 0, 1 is given in Fig. 1. "Condition (9) can be semantically interpreted in the following way: "for each $d_p \in \left[\underline{d_p}, \overline{d_p}\right]$ and for each $b_p \in \left[\underline{b_p}, \overline{b_p}\right]$ there exists such $x_i \in \left[\underline{x_i}, \overline{x_i}\right]$ which satisfies condition $x_\alpha = f\left(d_\alpha, b_\alpha\right)$ concerning the cut on level α."

3 Checking the solution delivered by the model-interval method

Authors of the MI-method show its applications to solving few examples. In Chapter 2 example 1 was presented. Authors' solution of this example is shown in Fig. 1. The authors called it "exact solution". On the level of supports of fuzzy numbers ($\alpha = 0$) the solution has character of improper interval $x \in [5/3, 11/7] = [1.67, 1.57]$. On this level fuzzy equation $A + BX = C$ can be formulated as (10).

$$\begin{aligned}&A_0 + C_0 X_0 = C_0\\&[0.5, 0.9] + [0.3, 0.7]X_0 = [1.0, 2.0]\end{aligned} \tag{10}$$

According authors of the MI-method the exact solution is the improper interval $X_0 = [1.67, 1.57]$. To check correctness of this solution the test-point method can be used. The test-point method can be characterized as follows:

1. Determine test-points $TP_i(a_{0i} \in A_0, b_{0i} \in B_0, x_{0i} = (c_{0i} - a_{0i})/b_{0i})$, $i = 1, 2, \ldots$, that satisfy all interval conditions and dependences concerning the problem, and then check whether solutions delivered by the examined interval-equation solving-method also delivers for these test-points results satisfying all conditions and dependences imposed on the problem.

2. Determine test-points TP_j, $j = 1, 2, \ldots$, that do not satisfy at least one of interval- and dependence conditions imposed on the problem. Results delivered by examined solving-method of interval-equations also should not satisfy the same conditions.

Let us apply the test-point $TP_1(a_0, b_0, c_0, x_0) = TP_1(0.51, 0.31, 1.11, x_0)$. Particular coordinates of these test-point satisfy the problem conditions: $a_0 =$

$0.51 \in [0.5, 0.9] = A_0$, $b_0 = 0.31 \in [0.3, 0.7] = B_0$, $c_0 = 1.11 \in [1.0, 2.0] = C_0$, $x_0 = (c_0 - a_0)/b_0$.

The test-point TP_1 is corresponded by value x_0 given by (12) that was calculated from the crisp equation (11). The analyzed interval-equation (10) $A_0 + B_0X_0 = C_0$ is the interval extension of equation (11) in which the crisp parameters a_0, b_0, c_0 were replaced by uncertain, interval-parameters A_0, B_0, C_0.

$$a_0 + b_0x_0 = c_0 \tag{11}$$

$$x_0 = (c_0 - a_0)/b_0 = (1.11 - 0.51)/0.31 = 1.935 \tag{12}$$

The achieved value $x_0 = 1.935$ is not contained in the interval $X_0 = [1.67, 1.57]$ suggested by authors of the MI-method as the "exact" solution of equation (10).

Now, let us investigate the test-point $TP_2(a_0, b_0, c_0, x_0) = TP_2(0.70, 0.69, 1.90, x_0)$. All coordinates of this point satisfy conditions of the problem, i.e.: $a_0 = 0.70 \in [0.5, 0.9] = A_0$, $b_0 = 0.69 \in [0.3, 0.7] = B_0$, $c_0 = 1.90 \in [1.0, 2.0] = C_0$, $x_0 = (c_0 - a_0)/b_0$.

The test-point TP_2 is corresponded by value $x_0 = 1.739$ calculated (13) from equation (11) being basis for the intervally extended equation (3) $A_0 + B_0X_0 = C_0$.

$$x_0 = (c_0 - a_0)/b_0 = (1.90 - 0.70)/0.69 = 1.739 \tag{13}$$

The achieved value $x_0 = 1.739$ also is not contained in the interval $X_0 = [1.67, 1.57]$ given by the MI-method as the "exact" solution of the support-equation. The "exact" solution $x_0 = [1.67, 1.57]$ is exact only in the sense of semantic interpretation (9) of the solution assumed by the authors. However, this interpretation is far insufficient for practical aims and it gives solutions that are strongly underestimated, which will be shown in Chapter 4.

4 Solution of the example with multidimensional RDM interval-arithmetic

Multidimensional RDM interval-arithmetic, which idea had been conceived by A. Piegat, was presented in [16, 17, 18, 19]. In the analyzed example interval-equation (14) is to be solved.

$$\begin{aligned} &A_0 + C_0X_0 = C_0 \\ &[0.5, 0.9] + [0.3, 0.7]X_0 = [1.0, 2.0] \end{aligned} \tag{14}$$

Intervals A_0, B_0, C_0 are modeled with use of RDM-variables (Relative-Distance-Measure) $\alpha_{a0} \in [0,1]$, $\alpha_{b_0} \in [0,1]$, $\alpha_{c_0} \in [0,1]$. Fig. 2 explains sense of RDM-variable α_{a_0}.

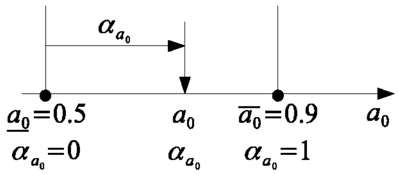

Figure 2: RDM-variable as measure of the relative distance of a_0-value from the lower interval-limit $\underline{a_0}$.

With use of RDM-variables intervals A_0, B_0, C_0 can be described in form of equations (15), (16), (17).

A_0:

$$\begin{aligned} a_0 &= \underline{a_0} + \alpha_{a0}\left(\overline{a_0} - \underline{a_0}\right), \alpha_{a_0} \in [0,1] \\ a_0 &= 0.5 + 0.4\alpha_{a0} \end{aligned} \tag{15}$$

B_0:

$$\begin{aligned} b_0 &= \underline{b_0} + \alpha_{b_0}\left(\overline{b_0} - \underline{b_0}\right), \alpha_{b_0} \in [0,1] \\ b_0 &= 0.3 + 0.4\alpha_{b_0} \end{aligned} \tag{16}$$

C_0:

$$\begin{aligned} c_0 &= \underline{c_0} + \alpha_{c_0}\left(\overline{c_0} - \underline{c_0}\right), \alpha_{c_0} \in [0,1] \\ c_0 &= 1 + \alpha_{c0} \end{aligned} \tag{17}$$

With use of RDM-variable the support-interval-equation (14) can be transformed in (18).

$$\begin{aligned} &A_0 + B_0 X_0 = C_0 \\ &(0.5 + 0.4\alpha_{a_0}) + (0.3 + 0.4\alpha_{b_0})x_0 = 1 + \alpha_{c0} \\ &\alpha_{a0} \in [0,1], \alpha_{b_0} \in [0,1], \alpha_{c_0} \in [0,1] \end{aligned} \tag{18}$$

On the basis of (18) solution X_0 can be calculated.

$$\begin{aligned} &X_0 = \tfrac{C_0 - A_0}{B_0} = \tfrac{0.5 - 0.4\alpha_{a_0} + \alpha_{c_0}}{0.3 + 0.4\alpha_{b_0}} \\ &\alpha_{a0} \in [0,1], \alpha_{b_0} \in [0,1], \alpha_{c_0} \in [0,1] \end{aligned} \tag{19}$$

The solution X_0 is not interval! As formula (19) shows solution of equation (18) is not a function of only one variable but of 3 variables α_{a_0}, α_{b_0}, α_{c_0}, or, which is tantamount, of 3 constrained variables a_0, b_0, c_0. Table 1 shows $x_0$values

Table 1: Values of variable x_0 for border values of RDM-variables α_{a_0}, α_{b_0} for $\alpha_{c_0} = 0$.

α_{a_0} a_0	0 0.5	0 0.5	1 0.9	1 0.9
α_{b_0} b_0	0 0.3	1 0.7	0 0.3	1 0.7
x_0	5/3 ≈ 1.67	5/7 ≈ 0.71	1/3 ≈ 0.33	1/7 ≈ 0.14

Table 2: Values of variable x_0 for border values of RDM-variables α_{a_0}, α_{b_0} for $\alpha_{c_0} = 1$.

α_{a_0} a_0	0 0.5	0 0.5	1 0.9	1 0.9
α_{b_0} b_0	0 0.3	1 0.7	0 0.3	1 0.7
x_0	5	15/7 ≈ 2.14	11/3 ≈ 3.67	11/7 ≈ 1.57

for border values of the RDM-variables α_{a_0}, α_{b_0} for $\alpha_{c_0} = 0$ and Table 2 for $\alpha_{c_0} = 1$.

Solution of the interval-equation (14) $A_0 + B_0X_0 = C_0$ for supports of fuzzy-equations (13) $A + BX = C$ is visualized in Fig. 3.

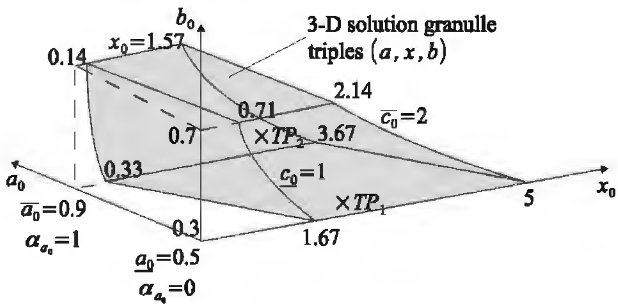

Figure 3: Non-regular, non-rectangular solution granule (also the solution-domain) of the support equation $A_0 + B_0X_0 = C_0$ (equation (14) and (15)) achieved wit use of multidimensional RDM interval-arithmetic, test-points $TP_1(a_0, b_0, c_0, x_0) = (0.51, 0.31, 1.11, 1.935)$, $TP_2(0.70, 0.69, 1.9, 1.739)$.

For comparison, Fig. 4 shows the 1D-solution of the problem achieved with the MI-method.

As can be seen in Fig. 3 solution domain of equation (14) is a set of an infinitive number of point-solutions contained in the non-regular, non-rectangular

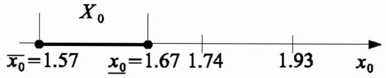

Figure 4: One-dimensional solution of support-equation $A_0 + B_0X_0 = C_0$ achieved with the modal-interval method (the solution domain)

solution-granule. The minimal value of variable x_0 contained in the solution-domain equals 0.14 and the maximal value equals 5. Thus, the domain $X_0 = [1.67, 1.57]$ suggested by the MI-method is not correct: it creates only a small fragment of possible values of variable x_0. However, the main error of the MI-method is suggesting that the solution-domain is of 1-dimensional character, that shows Fig. 4. Really, the solution domain of equation (14) is not an interval but a multidimensional information-granule. This granule can be described with formulas (20). All particular solution-points contained in this granule satisfy conditions (20).

$$\begin{array}{l} a_0 = 0.5 + 0.4\alpha_{a_0}, \alpha_{a_0} \in [0,1] \\ b_0 = 0.3 + 0.4\alpha_{b_0}, \alpha_{b_0} \in [0,1] \\ c_0 = 1 + \alpha_{c_0}, \alpha_{c_0} \in [0,1] \\ x_0 = \frac{0.5 - 0.4\alpha_{a_0} + \alpha_{c_0}}{0.3 + 0.4\alpha_{b_0}} \end{array} \tag{20}$$

An alternative way of defining the solution domain is using the lower $\underline{x_0}$ and the upper $\overline{x_0}$ limit of the multidimensional solution-domain (21).

$$\begin{array}{l} x_0 \in \left[\underline{x_0}(\alpha_{a_0}, \alpha_{b_0}, \alpha_{c_0} = 0), \overline{x_0}(\alpha_{a_0}, \alpha_{b_0}, \alpha_{c_0} = 1)\right] \\ \underline{x_0} = \frac{0.5 - 0.4\alpha_{a_0}}{0.3 + 0.4\alpha_{b_0}}, \overline{x_0} = \frac{1.5 - 0.4\alpha_{a_0}}{0.3 + 0.4\alpha_{b_0}} \\ a_0 = 0.5 + 0.4\alpha_{a_0}, \alpha_{a_0} \in [0,1] \\ b_0 = 0.3 + 0.4\alpha_{b_0}, \alpha_{b_0} \in [0,1] \\ c_0 = 1 + \alpha_{c_0}, \alpha_{c_0} \in [0,1] \end{array} \tag{21}$$

In Fig. 3 the lower limit $\underline{x_0}(\alpha_{a_0}, \alpha_{b_0}, \alpha_{c_0} = 0)$ is the left wall of the solution granule and the upper limit $\overline{x_0}$ the right wall (the walls are darkened). It means that in the multidimensional approach to interval solutions similarly as in the 1-dimensional approach lower and upper limits also occur: the difference consists only in dimensionality of the limits. In solving uncertain equations a very important thing is determining the notion of "problem solution". What is the solution? Which requirements has the solution to satisfy? In case of a problem without uncertainty solution of equation $A_0 + B_0X_0 = C_0$ is a point. E.g. equation $2 + 3x = 5$ has the one-dimensional solution $x = 1$. This solution seems to be independent. However, it can be presented in following way:

$$\begin{aligned}&\mathrm{IF}(a_0 = 2)\mathrm{AND}(b_0 = 3)\mathrm{AND}(c_0 = 5)\mathrm{AND}(a_0 + b_0 x_0 = c_0)\\&\mathrm{THEN}(x_0 = 1)\\&\mathrm{or}(x_0 = 1 \mid a_0 = 2, b_0 = 3, c_0 = 5, a_0 + b_0 x_0 = c_0)\end{aligned}$$

From the above one can see that the equation ($a_0 + b_0x_0 = c_0$)-solutions are dependent on parameter values and on type of dependence connecting them. Thus, they are not independent but dependent. Similarly as solutions of crisp equations also solutions of interval-equations are dependent on imposed conditions and they all should be taken into account in uncertain-problem solving. If we have to do with the uncertain interval-equation $[0.5, 0.9] + [0.3, 0.7]X_0 = [1.0, 2.0]$ then solution is not a one-dimensional interval but a multidimensional interval being set of points satisfying all conditions (20) imposed on the solution. These conditions (requirements or constraints) create our knowledge about the problem and the solution has to be consistent with these conditions. Let us notice that the test-point $TP_1(a_0, b_0, c_0, x_0) = TP_1(0.51, 0.31, 1.11, 1.935)$ satisfies all conditions (20) imposed on the problem ($0.51 \in [0.5, 0.9]$, $0.31 \in [0.3, 0.7]$, $1.11 \in [1.0, 2.0]$, $x_0 = (c_0 - a_0)/b_0 = 1.935$), similarly as the test-point $TP_2(0.70, 0.89, 1.90, 1.348)$, Fig. 3. Thus, both points belong to the solution domain defined by formulas (20) and shown in Fig. 3. This multidimensional solution-domain cannot be in any way precisely represented by 1-dimensional interval. Each such trial results either in overestimation or underestimation of the correct granular-solution S consisting of an infinitive number of quadruples (a_0, b_0, c_0, x_0) expressed by (22).

$$\begin{aligned}S = \{(a_0, b_0, c_0, x_0) : a_0 \in \left[\underline{a_0}, \overline{a_0}\right], b_0 \in \left[\underline{b_0}, \overline{b_0}\right], c_0 \in \left[\underline{c_0}, \overline{c_0}\right],\\ x_0 = (c_0 - a_0)/b_0\}\end{aligned} \tag{22}$$

5 Conclusions

The paper presented the method of test-points that can be used for correctness checking of solutions of uncertain fuzzy and interval equations delivered by various methods proposed for solving of such equations. The testing-point method was presented on example of examining results delivered by the interval-modal method published by R.J. Bhivani and B.M. Patre and published in [1]. It was shown that the method delivers incorrect solution domain. This feature results from the 1-dimensional character of solutions delivered by this method. The paper also presented a multidimensional RDM method of solving interval-equations, which delivers correct solutions satisfying all conditions and requirements imposed on solutions.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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