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Volume I: Foundations

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π-ordering and index of indeterminacy for intuitionistic fuzzy sets

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Abstract

A new ordering on the class of intuitionistic fuzzy sets, the so called π-ordering, is introduced in this paper. As a start point we employ the modal operators on IFSs and investigate their properties in respect of the π-ordering. In the last section the new ordering is used for the definition of the index of indeterminacy over IFSs, which is supposed to satisfy three corresponding axioms. A few versions of the index of indeterminacy are proposed according the the structure of the underlying universe X, over which the IFSs are considered. This index measures how far (close) is an IFS from (to) the family of the usual FSs on the same universe X.

Keywords: π-ordering, Index of indeterminacy, Modal quasi-orderings.

1 Introduction to intuitionistic fuzzy sets and modal operators

A fuzzy set in X (cf. Zadeh [7]) is given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{1}$$

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where $\mu_{A'}(x) \in [0,1]$ is the *membership function* of the fuzzy set A'. As opposed to the Zadeh's fuzzy set (abbreviated FS), Atanassov (cf. [1], [2]) extended its definition to an intuitionistic fuzzy set (abbreviated IFS) A, given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a *degree of membership* and a *degree of non-membership* of $x \in A$, respectively. An additional concept for each IFS in X, that is an obvious result of (2) and (3), is called

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

a *degree of uncertainty* of $x \in A$. It expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [1]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$. Uncertainty degree turn out to be relevant for both - applications and the development of theory of IFSs. For instance, distances between IFSs are calculated in the literature in two ways, using two parameters only (cf. Atanassov [1]) or all three parameters (cf. Szmidt and Kacprzyk [6]).

Talking about partial ordering in IFSs, we will by default mean $(IFS(X), \leq)$ where $\leq$ stands for the standard partial ordering in IFS(X). That is, for any two A and $B \in IFS(X)$: $A \leq B$ is satisfied if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$. On Fig. 1 one may see the triangular representation of the two chosen A and B in a particular point $x \in X$, where $f_A(x)$ stands for the point on the plane with coordinates $(\mu_A(x), \nu_A(x))$.

Let us recall the definitions and some properties of the modal operators on intuitionistic fuzzy sets as introduced by Atanassov. For more detailed descriptions and properties the reader may refer to [2], Ch. 4.1., although we introduce now some new statements and put light on them from various points of view. "Necessity" and "possibility" operators (denoted $\Box$ and $\Diamond$ respectively) applied on an intuitionistic fuzzy set $A \in IFS(X)$ have been defined as:

$$\begin{array}{ll} \Box A & = \{\langle \; x, \; \mu_A(x), \quad 1 - \mu_A(x) \; \rangle | x \in X\}, \\ \Diamond A & = \{\langle \; x, \; 1 - \nu(x), \; \nu_A(x) \qquad \rangle | x \in X\} \end{array}$$

From the above definition it is evident that

$$\star \colon IFS(X) \longrightarrow FS(X) \tag{5}$$

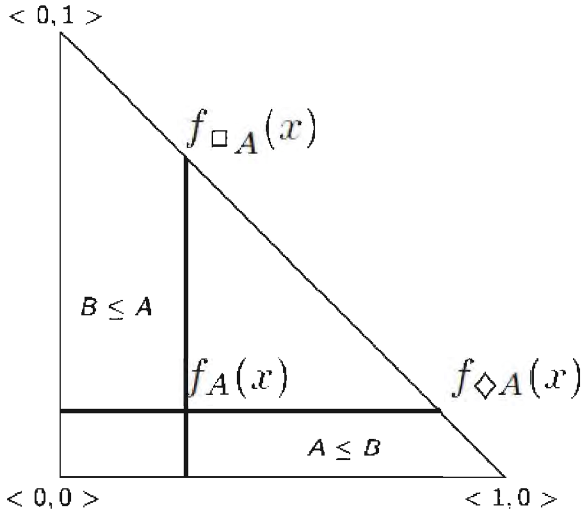

Figure 1: Triangular representation of the the intuitionistic fuzzy sets A and $B \in IFS(X)$ in a particular point $x \in X$, where $f_A(x)$ stands for the point on the plane with coordinates $(\mu_A(x), \nu_A(x))$. $\Box A$ and $\Diamond A$ stand for the two modal operators "necessity" and "possibility" acting on A.

where $\star$ is the prefix operator $\star \in \{\Box, \Diamond\}$, operating on the class of intuitionistic fuzzy sets. Let us take any $A, B \in IFS(X)$ and define $A \leq_{\Box} B$ iff $\mu_A \leq \mu_B$ on X, respectively $A \leq_{\Diamond} B$ iff $\nu_A \geq \nu_B$ on X. Obviously both $\leq_{\Box}$ and $\leq_{\Diamond}$ are reflexive and transitive. That is, they are both quasi-orderings in IFS(X) which will be called *quasi* $\Box$*-ordering* and *quasi* $\Diamond$*-ordering* respectively. For more information and examples of quasi-orderings the reader may consult the book of Birkhoff [3], Ch. II.1.

2 π-ordering for intuitionistic fuzzy sets

Let us define in this section a new ordering in IFS(X) and examine some if its properties and applications.

Definition 1 *For any two intuitionistic fuzzy sets A and $B \in IFS(X)$, we define the following binary relation :*

$$A \preceq_{\pi} B \textit{ iff } (\forall x \in X)(\mu_A(x) \leq \mu_B(x) \;\&\; \nu_A(x) \leq \nu_B(x))$$

On Fig. 2 one may see the triangular representation of the A and B in a particular point x from X if $A \preceq_{\pi} B$ or $B \preceq_{\pi} A$.

Definition 2 *For any two intuitionistic fuzzy sets A and $B \in IFS(X)$ and a particular $x_0 \in X$, we define the following binary relation :*

$$A \preceq_\pi B \text{ in } x_0 \text{ iff } (\mu_A(x_0) \leq \mu_B(x_0) \;\&\; \nu_A(x_0) \leq \nu_B(x_0))$$

As an equivalent way to denote that $A \preceq_\pi B$ in x_0 let us write $f_A(x_0) \preceq_\pi f_B(x_0)$ or $(\mu_A(x_0), \nu_A(x_0)) \preceq_\pi (\mu_A(x_0), \nu_A(x_0))$ as well. By default, $A \preceq_\pi B$ means that the above proposition is satisfied for each point from the universe X.

As an easy exercise the reader may show that $\preceq_\pi$ satisfies the three axioms for a partial ordering, i.e. the reflexive, transitive and anti-symmetric properties which would prove the following proposition.

Proposition 1 *The above defined relation $\preceq_\pi$ is a partial ordering on $IFS(X)$.*

Let us state, in terms of the already defined quasi-orderings, some equivalent conditions for an $A \in IFS(X)$ to be π-included in another $B \in IFS(X)$.

Proposition 2 *Let A and $B \in IFS(X)$, then the following statements are equivalent:*

1. $A \preceq_\pi B$,
2. $A \leq_\Box B$ *and* $A \geq_\Diamond B$,
3. $A \sqsupseteq B$ *and* $A \leq_\Box B$,
4. $A \sqsupseteq B$ *and* $A \geq_\Diamond B$.

The proof of the last proposition is an almost direct rewriting of the definitions.

Considering the partial ordered set $(IFS(X), \preceq_\pi)$, for any two A and $B \in IFS(X)$, by $A \wedge_\pi B$ and $A \vee_\pi B$ we will denote (if they exist) the greatest lower bound (infimum) and respectively, the least upper bound (supremum) in respect of the π-ordering. Or more generally, for a subfamily of IFSs $G \subset IFS(X)$, let us write $\wedge_\pi G$ and $\vee_\pi G$ for the supremum and respectively the infimum of G. Let us also remind that if some of the last mentioned bounds exists and belongs to G, it is called the *minimal* and respectively, the *maximal* element of G. A partially ordered set $(E, \preceq)$ is called *left semi-lattice* (*right semi-lattice*) iff for all $e_1, e_2 \in E$ their infimum (supremum) exists. From the definition, by induction it follows that every finite subset of a left (right) semi-lattice has an infimum (supremum). A partially ordered set, which is a left and right semi-lattice, is called lattice. If moreover, the partially ordered set $(E, \preceq)$ has the property that for every subset $E' \subset E$, E' has an infimum (supremum), then E is called complete left (right) semi-lattice with respect to $\preceq$. If E is a complete left and right semi-lattice it is called then a complete lattice.

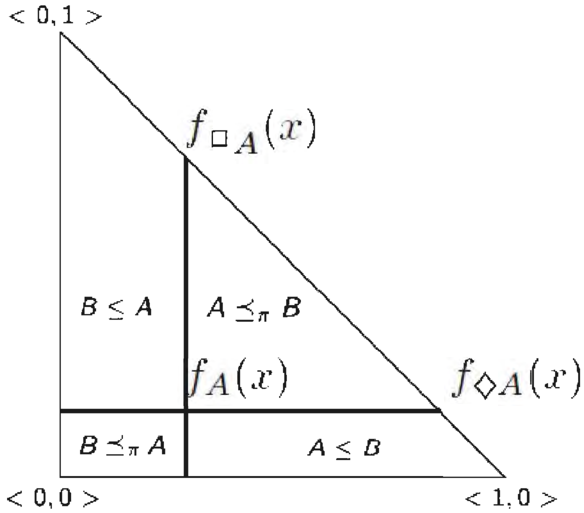

Figure 2: Triangular representation of the the intuitionistic fuzzy sets A and $B \in IFS(X)$ in a particular point $x \in X$. In contrast of Fig. 1, we see the position of $f_B(x)$ if $A \preceq_\pi B$ or $B \preceq_\pi A$.

Proposition 3 *The π-ordering is a left semi-lattice but not a right semi-lattice. The minimal element is $0_\pi :=< 0, 0 >\in IFS(X)$.*

Proof From the definition of least upper bound it is an easy exercise to show that for any $A, B \in IFS(X)$ there exists $A \wedge_\pi B$. Moreover, $\mu_{A\wedge_\pi B} = \mu_{A\wedge B}$ and $\nu_{A\wedge_\pi B} = \nu_{A\vee B}$, that is, $\mu_{A\wedge_\pi B} = \min(\mu_A, \mu_B)$ and $\nu_{A\wedge_\pi B} = \min(\nu_A, \nu_B)$. The last formulas imply that $< \mu, \nu \mid \mu = \nu = 0 >$ is really the minimal element.
To show the second statement of the proposition it suffices to take A and B such that $\exists x_0 \in X$ with $\mu_A(x_0) > 1 - \nu_B(x_0)$. Let us check that there is no $C \in IFS(X)$ such that $A \preceq_\pi C$ and $B \preceq_\pi C$ which would imply that that in particular $A \vee_\pi B$ does not exist. Supposing the existence of such C, let us observe what happens in the point x_0. From the definition of $\preceq_\pi$ it follows that in x_0: $\mu_A(x_0) \leq \mu_C(x_0)$ and $\nu_B(x_0) \leq \nu_C(x_0)$. But since $\mu_A(x_0) > 1 - \nu_B(x_0)$ then $1 - \nu_B(x_0) < \mu_C(x_0)$. On the other side $1 - \nu_B(x_0) \geq 1 - \nu_C(x_0)$ implies that $1 - \nu_C(x_0) < \mu_C(x_0)$, which contradicts with the definition of an intuitionistic fuzzy set. This leads to a contradiction with our assumption that there exists $C \in IFS(X)$ with $A \preceq_\pi C$ and $B \preceq_\pi C$. $\Box$

Remark 1 *Employing the properties of the greatest lower bound, the reader can easily show that $(IFS(X), \preceq_\pi)$ is a complete left semi-lattice. On the other hand,* *as already proved that $(IFS(X), \preceq_\pi)$ is not a right semi-lattice it can not be a complete right semi-lattice.*

Definition 3 *For any usual fuzzy set $F \in FS(X)$, i.e. $F \in IFS(X)$ such that $\pi_F = 1 - \mu_F - \nu_F = 0$ on X, let us define*

$$\Theta(F) := \{A \mid A \in IFS(X) \,\&\, A \preceq_\pi F\}.$$

We are ready now to summerize some important properties of the π-ordering in terms of the above defined $\Theta(F)$. The lower index $\leq$ to inf and sup, i.e. $\inf_\leq$ and $\sup_\leq$, means that the greatest lower (least upper) bound is taken with respect to the corresponding partial ordering.

Proposition 4 *Let X be any universe for IFSs and FSs, and $F \in FS(X)$, $C \in IFS(X)$. Then the following propositions hold:*

1. *There is a bijective correspondence between the maximal elements of $(IFS(X), \preceq_\pi)$ and the family of fuzzy sets $FS(X)$. Moreover, the family of maximal elements is exactly FS(X),*
2. *Any two different usual fuzzy sets, considered as IFSs, are incomparable with respect to the partial ordering $\preceq_\pi$,*
3. *$(\Theta(F), \preceq_\pi)$ considered as a subfamily of $(IFS(X), \preceq_\pi)$ is a lattice,*
4. *$\inf_\leq\{G \mid G \in FS(X) \,\&\, C \in \Theta(G)\} = \Box C$,*
5. *$\sup_\leq\{G \mid G \in FS(X) \,\&\, C \in \Theta(G)\} = \Diamond C$.*

Proof Let us start with the proof of the first item and check that every usual FS is a maximal element. Suppose that for $F \in FS(X)$ there is $B \in IFS(X)$ such that $F \preceq_\pi B$. This implies that $\mu_F \leq \mu_B$ and $1 - \mu_F \leq \nu_B$. But since $1-\mu_B \leq 1-\mu_F$ then $1-\mu_B \leq \nu_B$. The last expression implies that $1-\mu_B \equiv \nu_B$ on X and therefore $B \in FS(X)$ and since $1 - \mu_F \leq 1 - \mu_B$, then $\mu_F \geq \mu_B$. This together with $\mu_F \leq \mu_B$ implies that $F = B$. Therefore, for any usual FS there are no π-successors, which means that F is maximal. Let us now prove that every maximal element is a usual fuzzy set. Supposing its contradiction, i.e. that there is an IFS which is not usual FS, we will show that it can not be a maximal element. If $A \in IFS(X) \backslash FS(X)$, i.e. there is at least one $x_0 \in X$ such that $\mu_A(x_0) < 1 - \nu_A(x_0)$. Then $\Box A \neq A$ and obviously since $\mu_A \leq \mu_{\Box A} = \mu_A$ and $\nu_A \lneq \nu_{\Box A} = 1 - \mu_A$, therefore $A \preceq_\pi \Box A$ and $A \neq \Box A$. The last expression implies that A can not be a maximal element with respect to $\preceq_\pi$. The first item is

proved.
To show the validity of the second item, let us take $F, G \in FS(X)$ with $F \preceq_\pi G$. Therefore, $\mu_F \leq \mu_G$ and $1 - \mu_F \leq 1 - \mu_G$. This means that $\mu_F \geq \mu_G$ and then $\mu_F = \mu_G$ which finishes the proof of the second item.
To prove the third item, let us take $A, B \in \Theta(F)$. But $A \wedge_\pi B \preceq_\pi B$ and $B \preceq_\pi F$ implies that $A \wedge_\pi B \in \Theta(F)$. And hence $(IFS(X), \preceq_\pi)$ is a left semi-lattice. One can easily check that $\mu_{A \vee_\pi B} = \max(\mu_A, \mu_B)$ and $\nu_{A \vee_\pi B} = \max(\nu_A, \nu_B)$. $A \vee_\pi B$ is now well defined and one can not come to a contradiction described in the previous proposition. The third item is proved.
The last two items are left to be checked by the reader. □

Our observations in the proof of the last proposition let us state some remarks. The infimum and supremum as defined over $(IFS(X), \preceq_\pi)$ can be considered as mappings with domains which are IFSs.

Remark 2 $(IFS(X), \preceq_\pi)$ *is a left semi-lattice which exactly means that the infimum* ($\wedge_\pi$) *is defined over the whole direct product of* $IFS(X)$, *i.e.* $IFS(X) \times IFS(X)$. *Moreover,* $\wedge_\pi$ *is surjective (onto) on* $IFS(X)$, *e.g. for any* $A \in IFS(X)$ *we have that* $A \wedge_\pi A = A$.

Remark 3 *The supremum* ($\vee_\pi$) *in IFS(X) is not defined over the whole direct product of* $IFS(X)$ *because* $(IFS(X), \preceq_\pi)$ *is not a right semi-lattice. The last proposition implies exactly that* $Dom(\vee_\pi)$ *coincides with the pairs of IFSs the two members of which belong to some* $\Theta(F_0)$, *where* $F_0 \in FS(X)$. *That is,*

$$\begin{array}{rrcl} \vee_\pi : & \bigcup\limits_{F \in FS(X)} \Theta(F) \times \Theta(F) & \longrightarrow & IFS(X) \\ & (A, B) & \longmapsto & A \vee_\pi B \in \Theta(F) \end{array}$$

3 Index of indeterminacy

We are going to define an index of indeterminacy, i.e. degree of indeterminacy of an intuitionistic fuzzy set, which measures how far this IFS from a usual FS is. In Dubois and Prade [4], Chap. 1.5. has been discussed the index of fuzziness (to be denoted by ind_f in this paper) which measures the “degree of fuzziness” of a $F \in FS(X)$ where X is finite. It measures the degree of how far a usual fuzzy set from a crisp set is. Three axioms are proposed for the definition of such index. Taking any $F, F' \in FS(X)$ let us remind these axioms, slightly modifying them by the assumption that the range of ind_f should be the interval $[0, 1]$:

1. $ind_f(F) = 0$ iff F is a crisp set,

2. $ind_f(F) = 1$ iff $\mu_F = 0.5$ on X,

3. $ind_f(F) \leq ind_f(F')$ if $(\forall x \in X)(|\mu_F(x) - \mu_{\overline{F}}(x)| \geq |\mu_{F'}(x) - \mu_{\overline{F'}}(x)|)$.

Let us remark that $|\mu_F(x) - \mu_{\overline{F}}(x)| = 2|\mu_F(x) - 0.5|$ and a consequence of the third axiom is that $ind_f(F) = ind_f(\overline{F})$. Higashi and Klir [5] proposed and proved that a solution of the above three axioms is the formula $ind_{f,D}(F) = 1 - D(F, \overline{F})$, where D is some normalized distance between F and $\overline{F}$. If D is the Hamming distance, i.e. $D(F, \overline{F}) = \frac{1}{|X|}\sum_{x \in X} |\mu_F(x) - \mu_{\overline{F}}(x)|$, we get the index of fuzziness of Kaufmann.

Let us introduce now the term *index of indeterminacy* for an intuitionistic fuzzy set trough the following three axioms.

Definition 4 (Index of indeterminacy) *Take A and B to be any two elements from $IFS(X)$ and 0_π - the minimal element of $(IFS(X), \preceq_\pi)$. An index of indeterminacy over $IFS(X)$, to be denoted by ind_π and ranging over the interval $[0, 1]$, has to satisfy the following axioms:*

1. $ind_\pi(A) = 0$ iff $A \in FS(X)$,

2. $ind_\pi(A) = 1$ iff $A = 0_\pi$,

3. $ind_\pi(A) \leq ind_\pi(B)$ if $B \preceq_\pi A$.

Remark 4 *Let us remark that the index of indeterminacy takes its maximal value, i.e. 1, over the minimal element of $(IFS(X), \preceq_\pi)$. On the other side, ind_π takes its minimal value, i.e. 0, over the maximal elements of $(IFS(X), \preceq_\pi)$, i.e. FS(X). The third axiom says exactly that ind_π is non-increasing.*

We are going to define few examples of such measure of indeterminacy, depending on the structure and cardinality of the underlying set X and the type of membership and non-membership degrees.
Suppose first that X is finite and let us define the index of indeterminacy in the following way:

$$\begin{array}{rcl} ind_{\pi,1} \colon IFS(X) & \longrightarrow & [0, 1] \\ A & \longmapsto & \frac{1}{|X|} \sum\limits_{x \in X} \pi_A(x) \end{array} \tag{6}$$

With the same assumption of finiteness of the universe we can introduce another version of the indeterminacy measure, corresponding to the max or sup-metric:

$$\begin{array}{rcl} ind_{\pi,\infty} \colon IFS(X) & \longrightarrow & [0, 1] \\ A & \longmapsto & \sup\limits_{x \in X} \pi_A(x) \end{array} \tag{7}$$

It is easy to check that both of the two above defined indexes satisfy the three axioms for indeterminacy. Moreover, $ind_{\pi,1}$ is $\preceq_\pi$-decreasing whereas $ind_{\pi,\infty}$ is non-increasing.

Suppose now that X is a measurable space with a Lebesgue measure η and let us consider IFSs with η-integrable degrees of membership and non-membership. Therefore, the uncertainty degree $\pi = 1 - \mu - \nu$ would be integrable as well. Let us then, assuming that $\eta(X) < \infty$, define an integral form of the indeterminacy index as:

$$\begin{array}{rcl} ind_{\pi,\int}\colon IFS(X) & \longrightarrow & [0,1] \\ A & \longmapsto & \frac{1}{\eta(X)} \int\limits_X \pi_A d\eta \end{array} \tag{8}$$

Since $\forall A \in IFS(X)\colon \pi_A \geq 0$, therefore for the Lebesgue integral we have that $ind_{\pi,\int}(A) = \int\limits_X \pi_A d\eta \geq 0$ and $ind_\pi(A) = 0 \Leftrightarrow \pi_A = 0$ almost everywhere. In the last case we say that A coincides almost everywhere with a maximal element of $(IFS(X), \preceq_\pi)$. Slightly modifying the range of ind_π to be interval $[0,\infty)$ and ind_π and skipping the second axiom from the definition of the index of indeterminacy we can state the following integral form:

$$\begin{array}{rcl} ind_{\pi,\int,2}\colon IFS(X) & \longrightarrow & [0,1] \\ A & \longmapsto & \int\limits_X \pi_A d\eta \end{array} \tag{9}$$

That way, we see that if $\eta(X) = \infty$, then $ind_{\pi,\int,2}(0_\pi) = \int\limits_X 1 d\eta = \eta(X) = \infty$, which is the maximal value of the range $[0,\infty)$.

4 Conclusion

In this paper we have introduced two main notions - the π-ordering $\preceq_\pi$ and the index of indeterminacy. As a start point we employ the modal operators on IFSs and investigate their properties in respect of the π-ordering. $(IFS(X), \preceq_\pi)$ is a left semi-lattice but not a right semi-lattice, i.e. it is not a complete lattice. This ordering has a very good geometrical representation in terms of the triangular form of the IFSs as shown on Fig. 2. In the last section the new ordering is used for the definition of the index of indeterminacy over IFSs, which is supposed to satisfy three corresponding axioms. A few versions of the index of indeterminacy were proposed according the the structure of the underlying universe X, over which the IFSs are considered. This index measures how far (close) is an IFS from (to) the family of the usual FSs on the same universe X.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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