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Volume I: Foundations

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Vector based similarity measure for intuitionistic fuzzy sets

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Abstract

In this paper, in order to represent their connotative meaning, we explore vector based interpretations of the elements of an intuitionistic fuzzy set (IFS). Using one of these interpretations, we propose the *spot-difference* concept as a measure of differences between preferences about the membership and non-membership of each element. Considering the similarity as a concept related to the differences (the more differences, the less similarity), we make use of all the spot-differences to obtain a similarity measure. Additionally, we introduce the *spot-differences footprint* as a supplement to analyze the similarity between two IFSs.

Keywords: Connotation of preferences, Differences footprint, Semantic richer similarity

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	Wine 1	*Wine 2*	*Wine 3*	*Wine 4*
Alice	TASTED	TASTED	TASTED	
Bob	TASTED	TASTED		TASTED

Table 1: All wines tasted by Alice or Bob *(Wine preferences example).*

1 Introduction

Picture the following: two friends, Alice and Bob, want to compare their preferences about wine considering or not one of their viewpoints, e.g., when Alice makes a comparison she could or not take her point of view as referent. Therefore, they have compiled a list of all wines tasted by either one of them or both as is shown in Table 1. Using such a list, each friend must record the belongingness of a particular wine to her or his individual preferred list. *How can these friends record and compare their preferences even though they have not tasted all the wines and, moreover, considering or not one of their points of view in the comparison?*

The first part of this question, i.e., the recording or representation, could be answered using the intuitionistic fuzzy sets (IFSs) concept presented by Atanassov in [1]. Thus, with the degree of membership, the degree of non-membership and any reference regarding to a given wine, Alice and Bob are able to record their preferences although they had already not tasted it; for instance, using any clue that she might know about $wine\ 4$, Alice may assign 0.3 to the degree of membership, as well as 0.5 to the degree of non-membership, in order to reflect the level to which this wine could belong, or not, to her preferred list. Moreover, these preferences could be represented by using one of the existing geometrical interpretations of an IFS, e.g., the *unit segment* or the *IFS-interpretational triangle* both given by Atanassov in [2], or the *three-dimensional geometrical interpretation* given by Szmidt and Kacprzyk in [3] and [4].

To answer the second part of the question, i.e., the comparison, which is the main motivation of this work, we found in [5] some options using geometrical interpretations of IFSs to perform comparisons between two of them, which could be used to compare Alice and Bob's preferred lists. However, because the symmetrical approach assumed in those options is not able to handle different points of view in comparisons, this part of the question would be just partially answered if one of them were used —i.e., those options could be used to assess the degree to which Alice and Bob's lists are similar to each other, but not to assess the degree to which Alice's list is similar to Bob's, or vice versa.

To manage different viewpoints in comparisons, our aim is to use a vector based interpretation to represent the levels of membership and non-membership of an IFS not just with magnitudes, but also with directions that denote their connotative meaning. Thus, with this interpretation, we try to take account of the psychological view of similarity presented by Tversky in [6], which considers directionality and asymmetry in comparison judgments, i.e., it could be used e.g., to assess the degree to which Alice's preferred list is like Bob's, and to explain the reason why if "Alice's preferred list is like Bob's" and "Bob's preferred list is like Harry's," it is possible that "Alice's preferred list is not like Harry's." In other words, what we are proposing is *a vector based interpretation that represents the connotative meaning (or sense) of the elements of an IFS, and try to use it to assess holistically an observed similarity relation between objects that are not well known or not well defined.*

The interesting thing in our approach is that, considering what the preferences connote, we could obtain more reliable results in similarity-related process where the connotative meaning is important. For instance, imagine the following situations: a) despite of having almost no difference in a large number of wines, using an unit interval scale, Bob's list is 0.8 like Alice's because there are remarkable differences in a few number of wines; and b) using the same scale, Charlie's list is 0.8 like Alice's because there are tiny differences in almost all the wines. Now, image that someone asks Alice to evaluate a new group of wines according to her preferences in a new list, but she is not available to do that. *Which friend, Bob or Charlie, should Alice choose to do the evaluation in her behalf?* If only the magnitude of similarity had been considered, maybe there is no difference in choosing any friend, but due to the given reasons, Alice should choose the friend who represents in a better way the connotations of her preferences. This illustrates how the suggested meaning in comparisons could be considered in the rating of pairs, which is a similarity-related process. This also gives us an idea of a potential application. Suppose that you are an expert in classification of plants according to their presumed natural relationships. Also suppose that you are rating a group of volunteers according to the analysis that they have performed on images showing plants in such a way that who provides the more similar analysis to yours will obtain a better evaluation. If you can find anyone who performs an analysis connoting what you connote in yours, you could trust his or her future jobs more than others. Put this in a crowdsourcing context and you have a way to assess the quality of collaborators, as well as, the quality of the data they provided.

For the sake of clearness, we use the *Wine preferences* example throughout the paper, which is structured as follows. Section 2 introduces preliminary concepts. Section 3 presents our proposed vector based interpretation and defines the spot-differences concept, the spot-differences footprint, and the similarity measure.

Section 4 concludes the paper, and gives some directions for future work.

2 Preliminaries

For the purpose of answering the question *"how can Alice and Bob record their preferences?,"* this section introduces the IFS concept and two of its geometrical interpretations and, moreover, it introduces the approach of measuring similarity to be used.

2.1 IFS concept

As an extension of a fuzzy set ([7]), an IFS A^* in E ([1], [2, pp. 1,2]) is defined as an object such that

$$A^* = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in E\} \tag{1}$$

where sets E and A are considered to be fixed, $A \subset E$, functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the *degree of membership* and the *degree of non-membership* of $x_i \in E$ to the set A respectively, and for each element $x_i \in E$

$$0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1. \tag{2}$$

The lack of knowledge about the membership (or non-membership) of element $x_i \in E$ to set A is expressed by

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) \tag{3}$$

and it is defined as the *degree of non-determinacy* (in [5] it is called *hesitation margin*).

It is important to note that, although the definition shows the difference between the IFS A^* and the set A, for simplicity ([1], [2, p. 2]) the expression (1) will be denoted by

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in E\}, \tag{4}$$

in the reminder of the paper.

At this point, we are able to apply the IFS concept to model the *Wine preferences* example and, thus, we could answer the question *"how can Alice and Bob record their preferences?"* Applying the analogies shown in Table 2, Alice and Bob could record their preferences about each wine like is shown in Table 3. For a better understanding, let us take a look into Alice's thoughts about `wine 4` to

Example's component	***Model as ...***
`wine i`	x_i
All wines tasted by Alice or Bob	$E = \{x_1, x_2, x_3, x_4\}$
Alice's preferred list	$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle \| x_i \in E\}$
Degree of membership of `wine i` in Alice's list	$\mu_A(x_i)$
Degree of non-membership of `wine i` in Alice's list	$\nu_A(x_i)$
Hesitation margin of `wine i` in Alice's list	$\pi_A(x_i)$
Bob's preferred list	$B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle \| x_i \in E\}$
Degree of membership of `wine i` in Bob's list	$\mu_B(x_i)$
Degree of non-membership of `wine i` in Bob's list	$\nu_B(x_i)$
Hesitation margin of `wine i` in Bob's list	$\pi_B(x_i)$

Table 2: An IFS model of *Wine preferences* example.

find out how the IFS model is applied. She thinks: *"Even though* I haven't tasted `wine 4` *yet, I know it is produced by* `Winery 4 Inc.` *so* it would be good. *I also know that it's 60% Merlot so* it could be a little bit tasteless for me.*"* The fact that `wine 4` has been produced by `Winery 4 Inc.` suggests a 0.3 degree of membership to Alice, but the fact that the wine is 60% Merlot puts a 0.5 degree of non-membership into her mind.

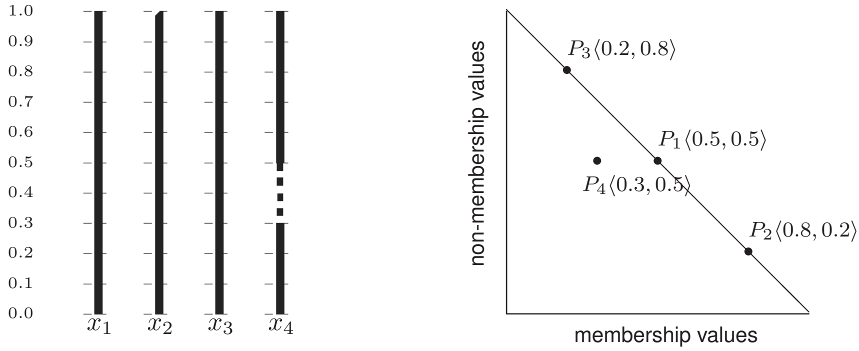
x_i	x_1	x_2	x_3	x_4
$\mu_A(x_i)$	0.5	0.8	0.2	0.3
$\nu_A(x_i)$	0.5	0.2	0.8	0.5
$\pi_A(x_i)$	0.0	0.0	0.0	0.2

(a) Alice's preferred list: IFS A.

x_i	x_1	x_2	x_3	x_4
$\mu_B(x_i)$	0.3	0.9	0.4	0.7
$\nu_B(x_i)$	0.7	0.1	0.0	0.1
$\pi_B(x_i)$	0.0	0.0	0.6	0.2

(b) Bob's preferred list: IFS B.

Table 3: Recorded preferences using IFSs (*Wine preferences* example).



(a) Unit segments.

(b) IFS-interpretational triangle.

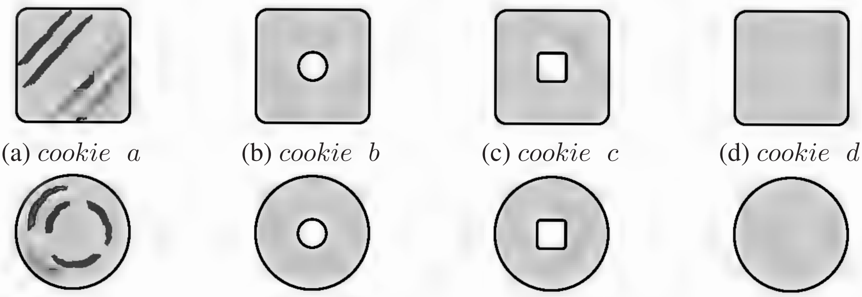
Figure 1: Geometrical interpretations of Alice's preferences.

2.2 Geometrical interpretations of an IFS

An IFS has several geometrical interpretations ([2, pp. 37,38]). One of them is the mapping of the degrees of membership, non-membership and the hesitation margin of each element to an *unit segment*. Figure 1a depicts Alice's preferences using this interpretation. Here, the black-solid part of each unit segment denotes the degree of membership, the gray-solid part denotes the degree of non-membership, and the black-dotted one denotes the hesitation margin, respectively. Another geometrical interpretation given in [2, pp. 38,39] is the so called *IFS-interpretational triangle*, in which the degree of membership and non-membership of each x_i are coordinates of a point P_i. Within this interpretation, Alice's preferences about `wine i` could be represented as $P_i\langle \mu_A(x_i), \nu_A(x_i)\rangle$ as is shown in Figure 1b.

2.3 Similarity concept

Using a set-theoretical approach, in [6] the similarity between two objects o_1 and o_2 —which are represented as a collection of features O_1 and O_2 respectively— is described as a *feature-matching* process. The features may correspond to components (e.g., eyes, mouth), represent properties (e.g., size, color), or reflect abstract attributes (e.g., quality, complexity).



(a) *cookie* a (b) *cookie* b (c) *cookie* c (d) *cookie* d

(e) *cookie* a' (f) *cookie* b' (g) *cookie* c' (h) *cookie* d'

Figure 2: Cookies matching (assumptions in a feature-matching process).

2.3.1 Feature-matching process

Let $s(o_1, o_2)$ be a measure of the similarity of o_1 to o_2 defined for all distinct o_1, o_2 over a universe of discourse U. The scale s is treated as an ordinal measure of similarity, i.e., $s(o_1, o_2) > s(o_3, o_4)$ means that o_1 is more similar to o_2 than o_3 is to o_4. In this way, the feature-matching process is based on the following assumptions (to illustrate them, we will use the *cookies matching* example shown in Figure 2 as an adaptation of Tversky's approach):

- Matching: The observed similarity between objects o_1 and o_2 is expressed as a function F that depends on the common features (i.e., $O_1 \cap O_2$), the features that belong exclusively to o_1 (i.e., $O_1 - O_2$), and the features that belong exclusively to o_2 (i.e., $O_2 - O_1$). This could be denoted as

$$s(o_1, o_2) = F(O_1 \cap O_2, O_1 - O_2, O_2 - O_1). \tag{5}$$

  For example, a common feature between $cookie\ a$ (Figure 2a) and $cookie\ b$ (Figure 2b) is the *square shape*, a feature that belongs to $cookie\ a$ and not to $cookie\ b$ is the *straight icing*, and a feature that belongs to $cookie\ b$ and not to $cookie\ a$ is the *round hole*.

- Monotonicity: The similarity increases by adding common features and/or by decreasing distinctive features. For instance, making a round hole in the $cookie\ a$ (Figure 2a) removes a distinctive feature between it and the $cookie\ b$ (Figure 2b), which increases the similarity between them.

Any function $F : U^2 \to \mathbb{R}$ that satisfies these assumptions is called a *matching function* and it could be used to measure the degree to which two objects match. In order to determine the functional form of a matching function, additional assumptions are introduced:

- Independence: Let X, Y and Z be components respectively denoting features present in objects o_1 and o_2, features present in o_1 but not in o_2, and features present in o_2 but not in o_1 —i.e., $X = O_1 \cap O_2$, $Y = O_1 - O_2$ and $Z = O_2 - O_1$. Let $s(o_1, o_2) > s(o'_1, o'_2)$ be a particular ordering that denotes that objects o_1 and o_2 are more similar than objects o'_1 and o'_2. Then, the effect upon a given ordering when two components join each other (e.g., X and Y, X' and Y') is *independent* of the fixed level of the third component (e.g., Z or Z') ([6]). For example, let A, B, C, D, A', B', C' and D' be feature sets present in $cookies\ a, b, c, d, a', b', c'$ and d' (Figure 2) respectively. Thus, the common features between $cookie\ a$ and $cookie\ b$, i.e., the singleton $\{$*square shape*$\}$, could be denoted by a component $X = A \cap B$; the feature set present in $cookie\ a$ but not in $cookie\ b$, i.e., $\{$*linear icing*$\}$, could be denoted by a component $Y = A - B$; and the feature set present in $cookie\ b$ but not in $cookie\ a$, i.e., $\{$*round hole*$\}$, could be denoted by a component $Z = B - A$. In a similar way, it could state that
$$\begin{array}{lllllll} A \cap B & = & A \cap C & = & \{\textit{square shape}\} & = & X, \\ A' \cap B' & = & A' \cap C' & = & \{\textit{round shape}\} & = & X', \\ A - B & = & A - C & = & \{\textit{linear icing}\} & = & Y, \\ A' - B' & = & A' - C' & = & \{\textit{curved icing}\} & = & Y', \\ B - A & = & B' - A' & = & \{\textit{round hole}\} & = & Z,\ and \\ C - A & = & C' - A' & = & \{\textit{square hole}\} & = & Z'. \end{array}$$
Now, let us imagine that, initially, $cookies\ a$, b and c, as well as $cookies\ a'$, b' and c', do not have distinctive features, i.e., $cookies\ a$, b and c look like $cookie\ d$, and $cookies\ a'$, b' and c' look like $cookie\ d'$; therefore, we might say that the similarity between $cookies\ a$ and b is equal to the similarity between $cookies\ a'$ and b', i.e., $s(a, b) = s(a', b')$. Putting a *linear icing* on $cookie\ a$, as well as a *curved icing* on $cookie\ a'$, may or may not change the order of similarities $s(a, b)$ and $s(a', b')$ regardless of making or not a *round hole* in the $cookies\ b$ and b', or making or not a *square hole* in the $cookies\ c$ and c' —i.e., when X and Y, as well as X' and Y', have joined each other respectively, the order of $s(a, b)$ and $s(a', b')$ may or may not change independently of Z or Z'.

- Solvability: The feature space under study must be sufficiently rich, so that a given number of (similarity) equations can be solved ([6]). This assumption does not impose constraints on an observed similarity, but just asserts that the corresponding matching function F can be solved. For instance, the fact that there is a common feature between $cookie\ b$ and $cookie\ b'$, i.e., $B \cap B' = \{$*round hole*$\}$, let equation $s(b, b') \geq s(d, d')$ be solved —recalling the assumption of monotonicity, making a *round hole* in both

$cookies$ d and d' should increase the similarity between them.

- Invariance: Let $I = [n_L, n_R] = \{n : n_L \leq n \leq n_R, n \in \mathbb{R}\}$ be an interval with limits n_L and n_R. Let f_1, f_2 and f_3 be nonnegative elements in I such that f_1, f_2 and f_3 measure the contribution of factors $O_1 \cap O_2$, $O_1 - O_2$ and $O_2 - O_1$, respectively, in F. Given $V = O_1 \cap O_2$, $V' = O_1^{'} \cap O_2^{'}$, $W = O_1 - O_2$, $W' = O_1^{'} - O_2^{'}$, $X = O_2 - O_1$ and $X' = O_2^{'} - O_1^{'}$, by invariance is meant that $f_i(V) - f_i(V') = f_i(W) - f_i(W') = f_i(X) - f_i(X')$ if and only if $f_j(V) - f_j(V') = f_j(W) - f_j(W') = f_j(X) - f_j(X')$, where $i, j = 1, 2, 3$ ([6]), that is, the equivalence of intervals is preserved across factors. This assumption also does not impose constraints on an observed similarity, but just states that it is possible to represent it by using any interval scale that measures the contribution of factors in F.

Under these assumptions and considering S as an interval similarity scale (or measure) such that $S(o_1, o_2) \geq S(o_3, o_4)$ if and only if $s(o_1, o_2) \geq s(o_3, o_4)$ (i.e., S preserves the observed similarity order), it is possible to represent matching-functions with models such as the *contrast model* that expresses the similarity between two objects as a linear combination of the measures of the common and the distinctive features, and the *ratio model* that expresses the similarity as a proportion between the common and the distinctive features in a normalized form. Using the *contrast model*, matching-functions have the form

$$S(o_1, o_2) = \lambda_1 \cdot f(O_1 \cap O_2) - \lambda_2 \cdot f(O_1 - O_2) - \lambda_3 \cdot f(O_2 - O_1) \quad (6)$$

where $\lambda_1, \lambda_2, \lambda_3 \geq 0$; and $f \in I$ is a non-negative measure of the contribution of common features, or the contribution of features that belong exclusively to o_1, or the contribution of features that belong exclusively to o_2. On the other hand, with the *ratio model* matching-functions have the form

$$S(o_1, o_2) = \frac{f(O_1 \cap O_2)}{f(O_1 \cap O_2) + \lambda_2 \cdot f(O_1 - O_2) + \lambda_3 \cdot f(O_2 - O_1)} \quad (7)$$

where λ_2, λ_3 and f have the same meaning as written above.

It is important to note that the contrast and the ratio models define a family of scales (or measures) characterized by different values of the parameters λ_1, λ_2 and λ_3 ([6]). For example, using the contrast model, if $\lambda_1 = 1$, $\lambda_2 = 0$ and $\lambda_3 = 0$ then $S(o_1, o_2) = f(O_1 \cap O_2)$; that is, the similarity between objects o_1 and o_2 is just given by the measure of the common features. On the other hand, if $\lambda_1 = 0$, $\lambda_2 = 1$ and $\lambda_3 = 1$ then $-S(o_1, o_2) = f(O_1 - O_2) + f(O_2 - O_1)$; that is, the dissimilarity between objects o_1 and o_2 is given by the measure of the distinctive features.

According to both models, similarity is not necessarily a *symmetric relation* ([6]), i.e., $s(o_1, o_2) = s(o_2, o_1)$. From either the contrast or the ratio model, it follows that $s(o_1, o_2) = s(o_2, o_1)$ if and only if $(\lambda_2 - \lambda_3) \cdot f(O_1 - O_2) = (\lambda_2 - \lambda_3) \cdot f(O_2 - O_1)$. Therefore, $s(o_1, o_2)$ is a symmetric relation whenever the objects o_1 and o_2 are equal in measure, i.e., $f(O_1) = f(O_2)$, or the task is *nondirectional*, i.e., $\lambda_2 = \lambda_3$. To interpret the intended meaning of a *nondirectional task*, it is suggested ([6]) to compare the following two forms: (a)assess the degree to which o_1 and o_2 are similar to each other, and (b)assess the degree to which o_1 is similar to o_2. In (a), neither o_1 nor o_2 is taken as referent, i.e., the task is nondirectional. In contrast, in (b), o_2 is taken as referent and o_1 is the subject of the comparison, i.e., the task is directional with respect to o_2.

At this point we could determine the degree of similarity between some cookies in Figure 2, in order to illustrate how the contrast model could be applied. Let us start by defining as a measure of common features, as well as distinctive ones, the counting of them; thus, $f(A \cap B) = 1$, $f(A - B) = 1$ and $f(B - A) = 1$ are the measures for the common features between $cookies\ a$ and b, features present in $cookie\ a$ and not in $cookie\ b$, and features present in $cookie\ b$ and not in $cookie\ a$, respectively. In a similar way, it follows that $f(A \cap B') = 0$, $f(A - B') = 2$ and $f(B' - A) = 2$. As a first case, let us assess the degree to which $cookies\ a$ and b, as well as $cookies\ a$ and b', are similar to each other. Since, in this case, no cookie has been taken as a referent, we could assign an equal value to λ_2 and λ_3. Thus, with $\lambda_1 = 1$, $\lambda_2 = 1$ and $\lambda_3 = 1$, $S(a, b) = f(A \cap B) - f(A - B) - f(B - A)$ and $S(a, b') = f(A \cap B') - f(A - B') - f(B' - A)$ hold, and, therefore, $S(a, b) = -1$ and $S(a, b') = -4$, i.e., $cookies\ a$ and b are more similar to each other than $cookies\ a$ and b'. Also, it follows readily that $S(b, a) = S(a, b)$ and $S(b', a) = S(a, b')$, which means that, in this case, both similarities are symmetric relations. As a second case, let us assess the degree to which $cookie\ b$, as well as $cookie\ b'$ are similar to $cookie\ a$. In this case, $cookie\ a$ has been taken a referent, thus, it is possible to do more evident the features present in $cookies\ b$ or b' and not in $cookie\ a$ by giving λ_2 a value greater than λ_3. Thus, with $\lambda_1 = 1$, $\lambda_2 = 1.5$ and $\lambda_3 = 0.75$, $S(b, a) = f(B \cap A) - 1.5 \cdot f(B - A) - 0.75 \cdot f(A - B)$ and $S(b', a) = f(B' \cap A) - 1.5 \cdot f(B' - A) - 0.75 \cdot f(A - B')$ hold, and, therefore, $S(b, a) = -1.25$ and $S(b', a) = -4.5$, i.e., in comparison to $cookie\ a$, $cookie\ b$ is more similar than $cookie\ b'$. Although having a directional task, in the latter case $S(b, a) = S(a, b)$ and $S(b', a) = S(a, b')$, i.e., they are symmetric relations, which is due to $cookies\ b$ and a, as well as $cookies\ b'$ and a, are equal in measure, that is, $f(B - A) = f(A - B)$ and $f(B' - A) = f(A - B')$.

2.3.2 Similarity vs. Difference

In [6] it is also considered that the similarity and difference are complementary, that is, perceived difference is a linear function of perceived similarity with a slope of -1. Thus, an increase in the measure of common features increases similarity and decreases difference. However, it is pointed out that the relative weight assigned to the common and the distinctive features may differ depending on whether the assessment is about similarity or about difference. In assessment of similarity between two objects, the assessor may attend more to the common features, whereas in assessment of difference, the assessor may pay more attention to the distinctive features ([6]). For instance, evaluating the difference between $cookies$ a and d, we might pay more attention to the *linear icing*, which is a feature present only in $cookie$ a, rather than the *square shape*, which is a common feature between them.

3 A Vector Based Similarity Measure

This section presents our proposed vector based interpretations of an IFS and, using one of these interpretations, defines the spot-difference concept, the spot-differences footprint, and a spot-differences based similarity measure. We will adopt these definitions to answer the second part of *Wine preferences* example's question: *how can Alice and Bob compare their preferences considering or not one of their viewpoints?*

3.1 Vector based interpretations of an IFS

Let us begin with a straightforward approach. Rather than considering the degrees of membership and non-membership as coordinates of a point into the IFS-interpretational triangle, we consider them as scalar components of a vector $\mathbf{a_i^*}$ in $[0,1]^2$. From a semantic point of view, these components represent the magnitude of the membership and non-membership meanings (directions) respectively. In this way, Alice's preferences about `wine i` could be interpreted as $\mathbf{a_i^*} = \begin{pmatrix} \mu_A(x_i) \\ \nu_A(x_i) \end{pmatrix}$ as is shown in Figure 3.

The question now is *"how could the hesitation margin be expressed in this interpretation?"* Recalling from Section 2.1 that the hesitation margin represents the lack of knowledge about the membership and non-membership of element $x_i \in E$ to IFS A, we propose to split it into two parts: one corresponding to membership, and the other, corresponding to the non-membership of element $x_i \in E$ to IFS A, respectively. Let us consider $\alpha_A \in [0,1]$ as the *hesitation splitter*,

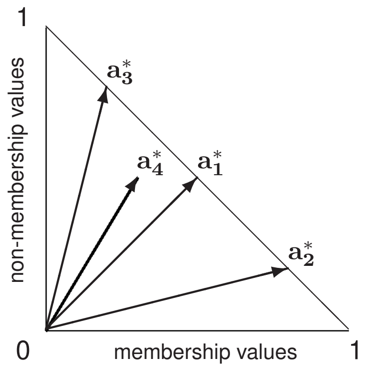


Figure 3: Straightforward vector based interpretation of Alice's preferences.

thus we could express the hesitation margin as

$$\pi_A(x_i) = \alpha_A \cdot \pi_A(x_i) + (1 - \alpha_A) \cdot \pi_A(x_i) \tag{8}$$

and introduce each part into a holistic vector based interpretation such that

$$\mathbf{a_i} = \begin{pmatrix} \mu_A(x_i) & + & \alpha_A \cdot \pi_A(x_i) \\ \nu_A(x_i) & + & (1 - \alpha_A) \cdot \pi_A(x_i) \end{pmatrix} \tag{9}$$

represents Alice's preferences about `wine i`. This approach is depicted in Figure 4.

Although they have different intentions, the *hesitation splitter* is somehow similar to the *extended modal operator* D_α, which is defined in [2, p. 77] for an IFS A as

$$D_\alpha(A) = \{\langle x_i, \mu_A(x_i) + \alpha.\pi_A(x_i), \nu_A(x_i) + (1 - \alpha).\pi_A(x_i)\rangle | x_i \in E\}. \tag{10}$$

Consequently, taking as reference the extended modal operator $F_{\alpha,\beta}$, which is defined in [2, p. 77] for an IFS A as

$$F_{\alpha,\beta}(A) = \{\langle x_i, \mu_A(x_i) + \alpha.\pi_A(x_i), \nu_A(x_i) + \beta.\pi_A(x_i)\rangle | x_i \in E\}, \tag{11}$$

it is possible to consider α_A and β_A as splitters such that $\alpha_A + \beta_A \leq 1$ where $\alpha_A, \beta_A \in [0, 1]$. We will call α_A a *membership hesitation splitter* in A, and β_A a *non-membership hesitation splitter* in A. Thus, the vector $\mathbf{a_i}$ could also be expressed as

$$\mathbf{a_i} = \begin{pmatrix} \mu_A(x_i) & + & \alpha_A \cdot \pi_A(x_i) \\ \nu_A(x_i) & + & \beta_A \cdot \pi_A(x_i) \end{pmatrix}. \tag{12}$$

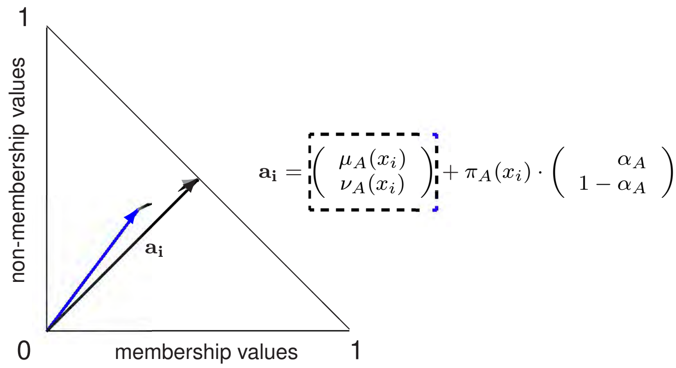


Figure 4: (Holistic) vector based interpretation.

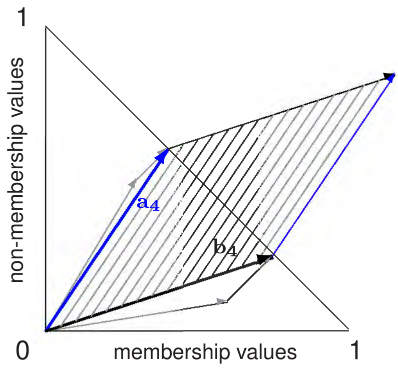
3.2 Comparing differences: Spot Difference

In order to compare Alice and Bob's preferred lists to each other, first we have to deal with the difference between their preferences about a particular wine. Let us choose `wine 4` as instance. Using (9), Alice and Bob's preferences about `wine 4` are depicted respectively as vectors $\mathbf{a_4}$ and $\mathbf{b_4}$ in Figure 5. We will use the area of the parallelogram formed by $\mathbf{a_4}$ and $\mathbf{b_4}$ as a reference to measure the difference between the modeled preferences about the wine. Thus, the larger this area, the larger the difference between $\mathbf{a_4}$ and $\mathbf{b_4}$. Within this approach, the largest area is given by the vectors $\mathbf{m_f} = \left(\begin{array}{c}1\\0\end{array}\right)$ and $\mathbf{n_f} = \left(\begin{array}{c}0\\1\end{array}\right)$; we will call them *full membership vector* and *full non-membership vector* respectively. Using the given idea, in this point we are able to define the spot-difference concept.

Definition 1 (Spot-difference) *Let $\mathbf{a_i}$ and $\mathbf{b_i}$ be two vectors each representing the degree of membership and non-membership of $x_i \in E$ to the IFS A and IFS B respectively, a measure of their differences is known as* spot-difference *and is given by*

$$dif(\mathbf{a_i}, \mathbf{b_i}) = \frac{\mathbf{a_i} \times \mathbf{b_i}}{\mathbf{m_f} \times \mathbf{n_f}}, \tag{13}$$

where $\times$ denotes the vector product.



$$\mathbf{a_4} = \begin{pmatrix} 0.3 & + & \alpha_A \cdot 0.2 \\ 0.5 & + & (1-\alpha_A)\cdot 0.2 \end{pmatrix}$$

$$\mathbf{b_4} = \begin{pmatrix} 0.6 & + & \alpha_B \cdot 0.3 \\ 0.1 & + & (1-\alpha_B)\cdot 0.3 \end{pmatrix}$$

Figure 5: Idea behind the spot-difference concept.

Using (9), the above definition could be expressed in detail as

$$dif(\mathbf{a_i},\mathbf{b_i}) = \frac{\begin{pmatrix} \mu_A(x_i)+ & \alpha_A\cdot\pi_A(x_i) \\ \nu_A(x_i)+ & (1-\alpha_A)\cdot\pi_A(x_i) \end{pmatrix} \times \begin{pmatrix} \mu_B(x_i)+ & \alpha_B\cdot\pi_B(x_i) \\ \nu_B(x_i)+ & (1-\alpha_B)\cdot\pi_B(x_i) \end{pmatrix}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix}}, \tag{14}$$

and, doing some calculations, we can obtain the following expressions:

$$dif(\mathbf{a_i},\mathbf{b_i}) = (\mu_A(x_i) - \mu_B(x_i)) + (\alpha_A\cdot\pi_A(x_i) - \alpha_B\cdot\pi_B(x_i)), \tag{15}$$

based on the degrees of membership and hesitation margins;

$$dif(\mathbf{a_i},\mathbf{b_i}) = -[(\nu_A(x_i) - \nu_B(x_i)) + ((1-\alpha_A)\cdot\pi_A(x_i) - (1-\alpha_B)\cdot\pi_B(x_i))], \tag{16}$$

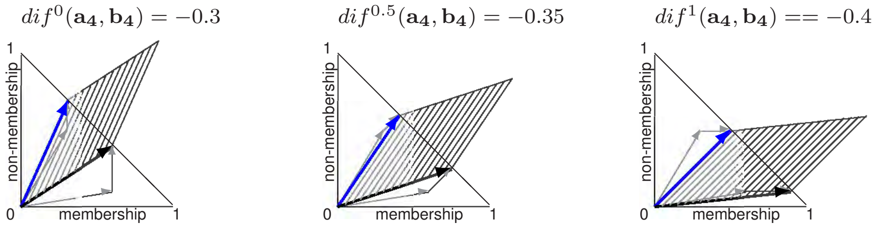
based on the degrees of non-membership and hesitation margins; and

$$\begin{aligned} dif(\mathbf{a_i},\mathbf{b_i}) = {} & [(1-\alpha_A)\cdot\mu_A(x_i) - (1-\alpha_B)\cdot\mu_B(x_i)] \\ & - [\alpha_A\cdot(1-\nu_A(x_i)) - \alpha_B\cdot(1-\nu_B(x_i))], \end{aligned} \tag{17}$$

based on the degrees of membership and non-membership. In a similar way, using (12), it is possible to obtain the expression

$$\begin{aligned} dif(\mathbf{a_i},\mathbf{b_i}) = {} & (\mu_A(x_i) + \alpha_A\cdot\pi_A(x_i))\cdot(\nu_B(x_i) + \beta_B\cdot\pi_B(x_i)) \\ & - (\mu_B(x_i) + \alpha_B\cdot\pi_B(x_i))\cdot(\nu_A(x_i) + \beta_A\cdot\pi_A(x_i)), \end{aligned} \tag{18}$$

which is based on the membership hesitation splitters α_A and α_B, and the non-membership hesitation splitters β_A and β_B.



(a) $\alpha = 0$ (b) $\alpha = 0.5$ (c) $\alpha = 1.0$

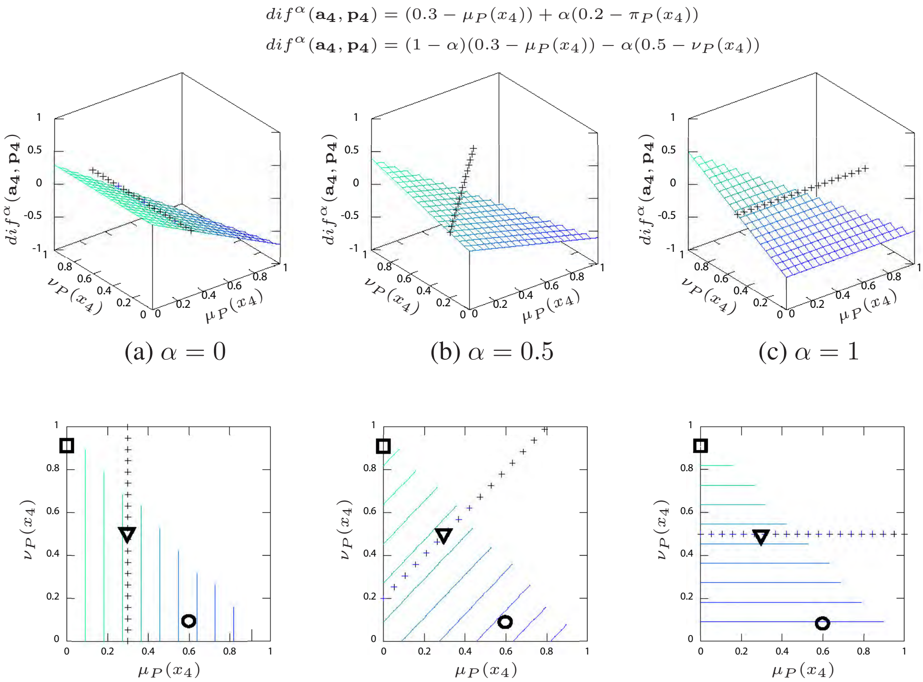
Figure 6: Managing the hesitation splitter.

Now, let us explore a little further about how (15) could be interpreted semantically. The first part of the expression, $(\mu_A(x_i) - \mu_B(x_i))$, denotes that the difference between Alice and Bob's preferences about `wine i` is determined by the degrees of membership to their individual lists. The second part, $(\alpha_A \cdot \pi_A(x_i) - \alpha_B \cdot \pi_B(x_i))$, denotes that the difference is also influenced by any doubts about the belongingness of `wine i` to both lists, and furthermore, this part could be affected by managing both Alice (α_A) and Bob (α_B)'s hesitation splitters. Considering as managing strategy to apply the same rule for Alice and Bob, we could assume that $\alpha_A = \alpha_B = \alpha$ and express (15) as

$$dif^{\alpha}(\mathbf{a_i}, \mathbf{b_i}) = (\mu_A(x_i) - \mu_B(x_i)) + \alpha(\pi_A(x_i) - \pi_B(x_i)). \quad (19)$$

Studying in Figure 6 how changing α-values affects the spot-difference results about `wine 4`, we may say that $\alpha \to 0$ is a kind of *pro non-membership* strategy, while $\alpha \to 1$ is a *pro membership* one. With respect to the semantic meaning of the (+/-) sign in spot-difference results, it denotes the *relative difference* between Alice and Bob's preferences. For example, $dif^{0.5}(\mathbf{a_4}, \mathbf{b_4}) = -0.35$ means that, from Alice's view, her preference about `wine 4` is 0.35 *less than* Bob's preferences. On the other hand, $dif^{0.5}(\mathbf{b_4}, \mathbf{a_4}) = +0.35$ means that, from Bob's view, his preference is 0.35 *more than* Alice's.

Another way of visualization is depicted in Figure 7, which, from Alice's view, compares her preferences with the preferences of anybody else, say P. Using (19), these comparisons can be expressed as $dif^{\alpha}(\mathbf{a_4}, \mathbf{p_4}) = (0.3 - \mu_P(x_4)) + \alpha(0.2 - \pi_P(x_4))$ where $\mathbf{p_4}$ is the vector representing the preferences for `wine 4` in the preferred list of the person P with who Alice's preferences are compared —using (17) with $\alpha_A = \alpha_P = \alpha$, those comparisons can also be expressed as $dif^{\alpha}(\mathbf{a_4}, \mathbf{p_4}) = (1-\alpha)(0.3 - \mu_P(x_4)) - \alpha(0.5 - \nu_P(x_4))$, which is used to obtain the plots. The plus-dotted line represents values where $dif^{\alpha}(\mathbf{a_4}, \mathbf{p_4}) = 0$, that is to say preferences of P are equal to Alice's. We can note once more how changing α-values affects the spot-difference results. For $\alpha = 0$ the plus-dotted



(a) $\alpha = 0$ (b) $\alpha = 0.5$ (c) $\alpha = 1$

(d) $\alpha = 0$ (e) $\alpha = 0.5$ (f) $\alpha = 1$

Figure 7: Spot-differences about $wine$ 4 between Alice and anybody's preferences.

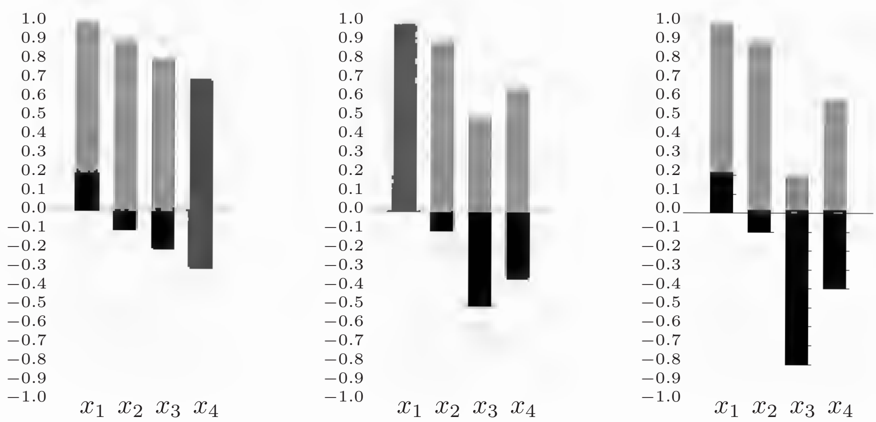
line is parallel to the non-membership line, labeled "$\nu_P(x_4)$"(see Figure 7a and its contour plot Figure 7d), which means that any doubt about the membership or non-membership will be favorable to the latter. On the other hand, for $\alpha = 1$ the plus-dotted line is parallel to the membership line, labeled "$\mu_P(x_4)$"(see Figure 7c and its contour plot Figure 7f), therefore any doubt will promote the membership. Because of these last reasons, we consider the α-tuning as strategical aspect of comparisons.

Apropos of the same Figure 7, looking into the contour plots, 7d, 7e and 7f, one can observe the relative differences between preferences. Let us suppose for a moment that Caroline, another friend of Alice, records her preferences about `wine 4` as $\mu_C(x_4) = 0$ and $\nu_C(x_4) = 0.9$; these preferences together with Alice's and Bob's are represented by $\square$, $\bigtriangledown$ and $\bigcirc$ respectively. With $\alpha = 0$ we obtain $dif^0(\mathbf{a_4}, \mathbf{c_4}) = 0.3$, which means that, from Alice's view, her preferences about `wine 4` are 0.3 *more than* Caroline's; in contrast to $dif^0(\mathbf{a_4}, \mathbf{b_4}) = -0.3$ where her preferences are 0.3 *less than* Bob's; therefore, it may be expected that, from Caroline's view, her preferences will be around 0.6 *more than* Bob's . If a distance based approach had been used and just the magnitude had been considered to compare the preferences, we might obtain the same absolute value as difference but no conclusions with respect to relative comparisons could be drawn; so we could, e.g., observe that the difference between Alice and Caroline's preferences is equal to the difference between Alice and Bob's preferences, but this tells us nothing about the difference between Caroline and Bob's preferences. This observation illustrates the added value of the proposed approach, in which the relative notion in the spot-difference concept permits the comparison of two preferences regarding to a particular (known or unknown) object —which are modeled as vectors using an interpretation of elements of an IFS— when the assessment is *directional*, i.e., when one (vector) of the two given preferences (vectors) is taken as referent (cf. the written in Section 2.3 about directional and nondirectional tasks in the assessment of similarity).

3.3 Spot-Differences Footprint

We already studied how to measure the difference between Alice and Bob's preferences about a *particular* wine using the spot-difference concept. Now, we have to put all the spot-differences together in order to assess the difference when comparing both, Alice and Bob's, preferred lists to each other, or when comparing one list with respect to the other. Let us start with a visual representation that denotes the relative notion of difference given in the spot-difference concept. Imagine a ruler marked off in "difference"-units with a length equal to the maximal value expected —recalling the full membership and full non-membership vectors, we know that the magnitude of the maximal difference value is 1. This ruler has a black region that denotes the *used* "difference"-units, i.e., the units representing the magnitude of a spot-difference, and a gray region that denotes the *unused* "difference"-units. Hence, such a ruler could be represented by a rectangle of height one. If we place this ruler perpendicularly on a line that represents no-difference, we could move it, also perpendicularly, to denote the relative difference. Thus, using a ruler for each wine and considering the Alice's view, we could obtain a representation of the difference between her preferred list and Bob's as is shown in Figure 8. We call this representation a *spot-differences footprint*.

According to the above, we could say that the spot-differences footprint reflects the internal composition of the difference between two IFSs somehow. The spot-differences footprint hence provides us with detailed information about the difference between to IFSs. For example, let us consider two spot-differences footprints representing the differences between Alice and Harry (Figure 9a), and Alice and Emma preferred lists (Figure 9b), respectively. Assuming that we add



(a) $\alpha = 0$ (b) $\alpha = 0.5$ (c) $\alpha = 1$

Figure 8: Spot-differences footprints using Alice's view.

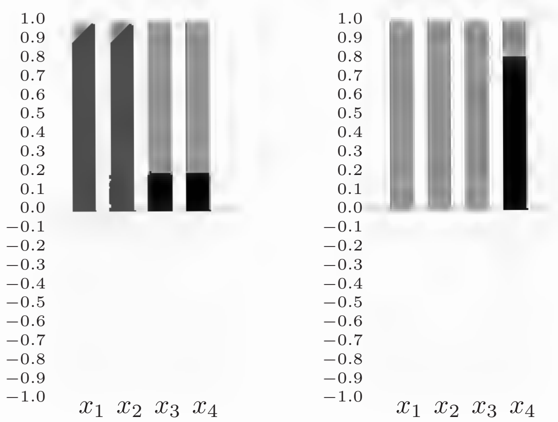
all the "difference"-units as a measure that represents the difference between two preferred lists, we obtain the same value for both Alice vs. Harry and Alice vs. Emma comparisons. However, looking at the spot-differences footprints, we can note the uniformity among the all wines' preferences in the first comparison, and the abrupt difference in preferences about a particular wine (x_4) in the second one. Distinguishing such situations is the motivation for the similarity measure for IFSs that is proposed in the next Subsection.

3.4 Spot-Differences and Similarity

For the sake of illustration, let us represent each "difference"-unit, presented in Section 3.3, by a unit square. Thus, by subtracting the different squares from all available ones, and dividing this result between all squares, we could get a ratio that represents the degree of similarity between Alice and Bob's preferred lists. Such a ratio could be expressed by

$$S = \frac{n \cdot q_{ruler} - \sum_{i=1}^{n} q_i}{n \cdot q_{ruler}}, \tag{20}$$

where q_{ruler} denotes the total number of squares in a ruler, q_i denotes the number of different squares in each ruler (i.e., those depicted in black), and n denotes the number of rulers to be used. Reckoning that q_{ruler} is the ruler's maximal length



(a) Alice vs. Harry (b) Alice vs. Emma

Figure 9: Comparing spot-differences footprints.

(i.e., 1) and q_i is $|dif^{\alpha}(\mathbf{a_i}, \mathbf{b_i})|$, the ratio could be expressed by

$$S^{\alpha}(A, B) = 1 - \frac{1}{n}\sum_{i=1}^{n} |dif^{\alpha}(\mathbf{a_i}, \mathbf{b_i})| \tag{21}$$

that denotes the degree of similarity between IFSs A and B. In order to show that this expression holds the ratio model for matching-functions given in (7), we could express (20) by

$$S = \frac{n \cdot q_{ruler} - \sum_{i=1}^{n^+} q_i^+ - \sum_{i=1}^{n^-} q_i^-}{\left(n \cdot q_{ruler} - \sum_{i=1}^{n^+} q_i^+ - \sum_{i=1}^{n^-} q_i^-\right) + \lambda_2 \left(\sum_{i=1}^{n^+} q_i^+\right) + \lambda_3 \left(\sum_{i=1}^{n^-} q_i^-\right)}, \tag{22}$$

where q_i^+ denotes the number of different squares placed in the upper side of the no-difference line (i.e., above the line that represents no difference), q_i^- denotes the number of squares on the lower side (i.e., below the line), n^+ denotes the number of rulers with squares placed in the q_i^+-side, n^- denotes the number of rulers with squares placed in the q_i^--side, n^0 denotes the number of rulers with no different squares, and $n = n^+ + n^- + n^0$. In this way, it is possible visualize that

$$\begin{aligned}
&f_{O_1 \cap O_2} = n \cdot q_{ruler} - \textstyle\sum_{i=1}^{n^+} q_i^+ - \sum_{i=1}^{n^-} q_i^-;\\
&f_{O_1 - O_2} = \textstyle\sum_{i=1}^{n^+} q_i^+;\\
&f_{O_2 - O_1} = \textstyle\sum_{i=1}^{n^-} q_i^-; and\\
&\lambda_2 = \lambda_3 = 1.
\end{aligned}$$

Also, we could express (21) by

$$S^{\alpha}(A,B) = \frac{g^{\alpha}_{A\cap B}(A,B)}{g^{\alpha}_{A\cap B}(A,B) + \lambda_2 \cdot g^{\alpha}_{A-B}(A,B) + \lambda_3 \cdot g^{\alpha}_{B-A}(A,B)}, \tag{23}$$

where

$$g^{\alpha}_{A-B}(A,B) = \begin{cases} \sum_{i=1}^{n} |dif^{\alpha}(\mathbf{a_i},\mathbf{b_i})| & \text{if } dif^{\alpha}(\mathbf{a_i},\mathbf{b_i}) > 0 \\ 0 & \text{otherwise,} \end{cases}$$

$$g^{\alpha}_{B-A}(A,B) = \begin{cases} \sum_{i=1}^{n} |dif^{\alpha}(\mathbf{a_i},\mathbf{b_i})| & \text{if } dif^{\alpha}(\mathbf{a_i},\mathbf{b_i}) < 0 \\ 0 & \text{otherwise,} \end{cases}$$

and

$$g^{\alpha}_{A\cap B}(A,B) = n - g^{\alpha}_{A-B}(A,B) - g^{\alpha}_{B-A}(A,B).$$

Considering $O_1 = A$ and $O_2 = B$, also it follows that

$$\begin{aligned} &f_{A\cap B} = g^{\alpha}_{A\cap B}(A,B); \\ &f_{A-B} = g^{\alpha}_{A-B}(A,B); \\ &f_{B-A} = g^{\alpha}_{B-A}(A,B); and \\ &\lambda_2 = \lambda_3 = 1. \end{aligned}$$

Therefore, the assumptions made in the feature-matching process, introduced in Section 2.3, are carried out as follows.

- Matching: The similarity between IFSs A and B is expressed by a function $S^{\alpha}(A,B)$ that depends on the differences about membership and non-membership of each element x_i to the IFSs A and B respectively. Thus, we could say that, in (23), $g^{\alpha}_{A\cap B}(A,B)$, $g^{\alpha}_{A-B}(A,B)$ and $g^{\alpha}_{B-A}(A,B)$ denote common preferences, preferences that belong exclusively to A, and preferences that belong exclusively to B, respectively.

- Monotonicity: $S^{\alpha}(A,B) \geq S^{\alpha}(A,C)$ when $\sum_{i=1}^{n} |dif^{\alpha}(\mathbf{a_i},\mathbf{b_i})| \leq \sum_{i=1}^{n} |dif^{\alpha}(\mathbf{a_i},\mathbf{c_i})|$. It means that the similarity increases when the difference on preferences decreases. Recalling from (19), we know that the differences on preferences increase when the difference between the degrees of membership and/or the difference between the hesitation margins increase.

- Independence: $dif^{\alpha}(\mathbf{a_i}, \mathbf{b_i})$ is independent of $dif^{\alpha}(\mathbf{a_i}, \mathbf{c_i})$, that is, the difference between the measure of x_i belonging to IFS A and the measure of x_i belonging to IFS B, is independent of the difference between the measure of x_i belonging to IFS A and the measure of x_i belonging to IFS C. Thus, also it follows that $g^{\alpha}_{A-B}(A, B)$ is independent of $g^{\alpha}_{A-C}(A, C)$, as well as, $g^{\alpha}_{B-A}(A, B)$ is independent of $g^{\alpha}_{C-A}(A, C)$.

- Solvability: The equation $S^{\alpha}(A, B)$ can be solved despite of the number of spot-differences to be considered.

- Invariance: If A_1 matches A_2 and B_1 matches B_2 then $S^{\alpha}(A_1, B_1)$ matches $S^{\alpha}(A_2, B_2)$. It means that, e.g., $S^{\alpha}(A_1, B_1)$ represents the observed similarity between IFSs A_1 and B_1.

At this point, using (21), it is possible to calculate the degree of similarity between Alice and Bob's preferred lists; Table 4 shows the results.

α	$S^{\alpha}(A, B)$
0	0.80
0.5	0.71
1.0	0.63

Table 4: Similarity between Alice and Bob's preferred lists.

Although we have considered the spot-difference concept —which denotes magnitude and direction— in order to get (21), this expression by itself just denotes the magnitude of similarity. We have pointed out in Section 3.3 that, when representing differences in a whole, it is not possible to conclude about the correspondence in appearance, i.e., the similarity. As instance, let us bring back the spot-differences footprints in Figure 9. Using (21), the similarity between Alice and Harry preferred lists, A and H respectively, is given by

$$S^{\alpha}(A, H) = 1 - \frac{1}{4}(0.2 + 0.2 + 0.2 + 0.2) = 0.8,$$

as well as, the similarity between Alice and Emma preferred lists, A and M respectively, is given by

$$S^{\alpha}(A, M) = 1 - \frac{1}{4}(0 + 0 + 0 + 0.8) = 0.8,$$

i.e., although they differ in their spot-differences footprint, both Alice vs. Harry and Alice vs. Emma have the same value as a measure of similarity. Recalling from Section 3.1 our intention of capturing the semantic meaning of preferences by means of a vector interpretation of an IFS, we propose to make use of the corresponding spot-differences footprint as a suplement of (21) in order to know not just the magnitude, but also the semantic meaning (or sense) of similarity within a comparison. Our intention here is to distinguish between the *"0.8-uniform"* similarity in Alice vs. Harry and the *"0.8-with-a-peak"* similarity in Alice vs. Emma. We consider that making this distinction could help Alice to decide whether she follows a suggestion from Harry or a suggestion from Emma about a no-tasted-yet wine. How to use the spot-differences footprint to obtain a semantic richer similarity measure is a key motivation for our future work.

4 Conclusions

We have considered several vector based interpretations of IFS elements and, using one of them, we have defined the *spot-difference* as a measure of the differences between preferences about the membership and non-membership of each element. Figuring the spot-differences as differences between two objects' features, we made use of the set-theoretical approach presented in [6] to obtain a new similarity measure. Furthermore, we have introduced the *spot-differences footprint* as a supplement of the similarity measure in order to know not just the magnitude, but also the sense behind it and, thus, to achieve semantic richer results in the comparison. With such semantic richer results, it is possible to overcome difficulties as those presented by Szmidt and Kacprzyk in [5], resulting when similarity is understood as a dual concept of a distance. In our future work, we will further explore the applicability of the spot-differences footprint in order to represent in a better way what the comparison between two IFSs connotes.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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