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Volume I: Foundations

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Maximal and minimal intuitionistic fuzzy negations

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Abstract

In this paper, is presented the ordering of the numbers of currently existing intuitionistic fuzzy (non-parametric) negations. The ordering has been obtained on the basis of intuitionistic fuzzy ordering relations. The forms of its minimal and maximal elements are determined and some of their properties are studied.

Keywords: Intuitionistic fuzzy logic, Negation

1 Introduction

In a series of papers, different forms of Intuitionistic Fuzzy Negations (IFNs) have been defined and their properties have been discussed. There, the complete list of

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these negations is given and for the set of all intuitionistic fuzzy (non-parametric) negations the forms of its minimal and maximal elements are determined.

Initially, we give some remarks on Intuitionistic Fuzzy Logic (IFL, see, e.g., [1, 2]).

In some of the definitions below, we use the functions sg and $\overline{\text{sg}}$ that are defined by:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}, \quad \overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \le 0 \end{cases}$$

In IFL, if x is a variable, then its truth-value is represented by the ordered pair

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a+b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x.

Everywhere below, we assume that for the tho variables x and y there hold the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle$, where $(a, b, c, d, a+b, c+d \in [0, 1])$.

For the needs of the discussion below, we define the notion of Intuitionistic Fuzzy Tautology (IFT, see [1, 2]) by:

$$x \text{ is an IFT if and only if } a \ge b,$$

while x will be a tautology iff $a = 1$ and $b = 0$.

For two variables x and y operations "conjunction" (&) and "disjunction" ($\vee$) are defined by

$$V(x \& y) = V(x) \& V(y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = V(x) \vee V(y) = \langle \max(a, c), \min(b, d) \rangle$$

and the relation $\le$ is defined by

$$V(x) \le V(y) \text{ if and only if } a \le c \text{ and } b \ge d.$$

In Table 1, the existing currently 45 negations (by the end of 2013) are given.

Table 1: List of known negations

$\neg_1$	$\langle x, b, a \rangle$
$\neg_2$	$\langle x, \overline{\text{sg}}(a), \text{sg}(a) \rangle$
$\neg_3$	$\langle x, b, a.b + a^2 \rangle$

Continued on next page

Table 1: List of known negations

$\neg_4$	$\langle x, b, 1-b\rangle$
$\neg_5$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(1-b)\rangle$
$\neg_6$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(a)\rangle$
$\neg_7$	$\langle x, \overline{\text{sg}}(1-b), a\rangle$
$\neg_8$	$\langle x, 1-a, a\rangle$
$\neg_9$	$\langle x, \overline{\text{sg}}(a), a\rangle$
$\neg_{10}$	$\langle x, \overline{\text{sg}}(1-b), 1-b\rangle$
$\neg_{11}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(b)\rangle$
$\neg_{12}$	$\langle x, b.(b+a), \min(1, a.(b^2+a+b.a))\rangle$
$\neg_{13}$	$\langle x, \text{sg}(1-a), \overline{\text{sg}}(1-a)\rangle$
$\neg_{14}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(1-a)\rangle$
$\neg_{15}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(1-a)\rangle$
$\neg_{16}$	$\langle x, \overline{\text{sg}}(a), \overline{\text{sg}}(1-a)\rangle$
$\neg_{17}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(b)\rangle$
$\neg_{18}$	$\langle x, b.\text{sg}(a), a.\text{sg}(b)\rangle$
$\neg_{19}$	$\langle x, b.\text{sg}(a), 0\rangle$
$\neg_{20}$	$\langle x, b, 0\rangle$
$\neg_{21}$	$\langle x, \min(1-a, \text{sg}(a)), \min(a, \text{sg}(1-a))\rangle$
$\neg_{22}$	$\langle x, \min(1-a, \text{sg}(a)), 0\rangle$
$\neg_{23}$	$\langle x, 1-a, 0\rangle$
$\neg_{24}$	$\langle x, \min(b, \text{sg}(1-b)), \min(1-b, \text{sg}(b))\rangle$
$\neg_{25}$	$\langle x, \min(b, \text{sg}(1-b)), 0\rangle$
$\neg_{26}$	$\langle x, b, a.b+\overline{\text{sg}}(1-a)\rangle$
$\neg_{27}$	$\langle x, 1-a, a.(1-a)+\overline{\text{sg}}(1-a)\rangle$
$\neg_{28}$	$\langle x, b, (1-b).b+\overline{\text{sg}}(b)\rangle$
$\neg_{29}$	$\langle x, \max(0, b.a+\overline{\text{sg}}(1-b)), \min(1, a.(b.a+\overline{\text{sg}}(1-b))+\overline{\text{sg}}(1-a))\rangle$
$\neg_{30}$	$\langle x, a.b, a.(a.b+\overline{\text{sg}}(1-b))+\overline{\text{sg}}(1-a)\rangle$
$\neg_{31}$	$\langle x, \max(0, (1-a).a+\overline{\text{sg}}(a)), \min(1, a.((1-a).a+\overline{\text{sg}}(a))$ $+\overline{\text{sg}}(1-a))\rangle$
$\neg_{32}$	$\langle x, (1-a).a, a.((1-a).a+\overline{\text{sg}}(a))+\overline{\text{sg}}(1-a)\rangle$
$\neg_{33}$	$\langle x, b.(1-b)+\overline{\text{sg}}(1-b), (1-b).(b.(1-b)+\overline{\text{sg}}(1-b))+\overline{\text{sg}}(b))\rangle$
$\neg_{34}$	$\langle x, b.(1-b), (1-b).(b.(1-b)+\overline{\text{sg}}(1-b))+\overline{\text{sg}}(b)\rangle$
$\neg_{35}$	$\langle \frac{b}{2}, \frac{1+a}{2}\rangle$
$\neg_{36}$	$\langle \frac{b}{3}, \frac{2+a}{3}\rangle$
$\neg_{37}$	$\langle \frac{2b}{3}, \frac{2a+1}{3}\rangle$

Continued on next page

Table 1: List of known negations

$\neg_{38}$	$\langle \frac{1-a}{3}, \frac{2+a}{3} \rangle$
$\neg_{39}$	$\langle \frac{b}{3}, \frac{3-b}{3} \rangle$
$\neg_{40}$	$\langle \frac{2-2a}{3}, \frac{1+2a}{3} \rangle$
$\neg_{41}$	$\langle \frac{2b}{3}, \frac{3-2b}{3} \rangle$
$\neg_{\lambda}$	$\langle \frac{b+\lambda-1}{2\lambda}, \frac{a+\lambda}{2\lambda} \rangle$, where $\lambda \geq 1$
$\neg_{\gamma}$	$\langle \frac{b+\gamma}{2\gamma+1}, \frac{a+\gamma}{2\gamma+1} \rangle$, where $\gamma \geq 1$
$\neg_{\alpha,\beta}$	$\langle \frac{b+\alpha-1}{\alpha+\beta}, \frac{a+\beta}{\alpha+\beta} \rangle$, where $\alpha \geq 1, \beta \in [0, \alpha]$
$\neg_{\varepsilon,\eta}$	$\langle \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta) \rangle$

2 Main results

Using D. Dimitrov's program "IFS Tool" (see, [3]) we check for the existence or non-existence of an ordering relation between every two IFNs and after this, we construct the graph on Fig. 1.

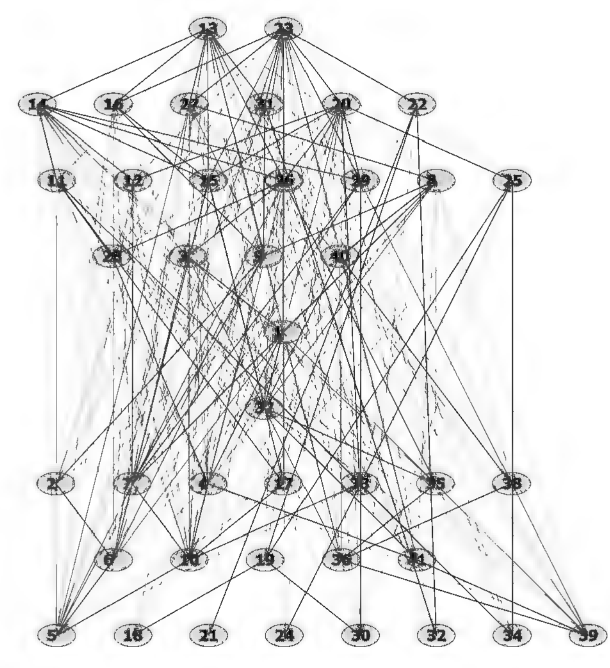

Figure 1: Ordering of relations between the IFNs

We see that the graph consists of 9 levels. If vertex x stays in the first (highest) level, then there is no vertex y, such that $V(x) \leq V(y)$. If vertex x stays in the ninth (lowest) level, then there is no vertex y, such that $V(y) \leq V(x)$. If vertex x stays in the i-th level ($2 \leq i \leq 8$), then there is a vertex y, such that $V(x) \leq V(y)$.

Now, we can construct two new negations: $\neg_{\max}$ and $\neg_{\min}$. They have the forms:

$$\begin{aligned}\neg_{\max}\langle a, b\rangle &= \neg_{13}\langle a, b\rangle \vee \neg_{23}\langle a, b\rangle \\ &= \langle \max(\mathrm{sg}(1-a), 1-a), \min(\overline{\mathrm{sg}}(1-a), 0)\rangle \\ &= \langle \mathrm{sg}(1-a), 0\rangle,\end{aligned}$$

because for each $a \in [0,1]$: $\mathrm{sg}(1-a) \geq 1-a$;

$$\begin{aligned}&\neg_{\min}\langle a, b\rangle \\ &= \neg_5\langle a, b\rangle \& \neg_{18}\langle a, b\rangle \& \neg_{21}\langle a, b\rangle \& \neg_{24}\langle a, b\rangle \& \neg_{30}\langle a, b\rangle \& \neg_{32}\langle a, b\rangle \\ &\quad \& \neg_{34}\langle a, b\rangle \& \neg_{39}\langle a, b\rangle \\ &= \langle \min(\overline{\mathrm{sg}}(1-b), b.\mathrm{sg}(a), \min(1-a, \mathrm{sg}(a)), \\ &\quad \min(b, \mathrm{sg}(1-b)), a.b, (1-a).a, b.(1-b), \frac{b}{3}), \\ &\quad \max(\mathrm{sg}(1-b), a.\mathrm{sg}(b), \min(a, \mathrm{sg}(1-a)), \\ &\quad \min(1-b, \mathrm{sg}(b)), a.(a.b + \overline{\mathrm{sg}}(1-b)) + \overline{\mathrm{sg}}(1-a), \\ &\quad a.((1-a).a + \overline{\mathrm{sg}}(a)) + \overline{\mathrm{sg}}(1-a), (1-b).(b.(1-b) + \overline{\mathrm{sg}}(1-b)) + \overline{\mathrm{sg}}(b), \\ &\quad \frac{3-b}{3}))\rangle \\ &= \langle \min(\overline{\mathrm{sg}}(1-b), b.\mathrm{sg}(a), 1-a, \mathrm{sg}(a), b, \mathrm{sg}(1-b), a.b, (1-a).a, b.(1-b), \\ &\quad \frac{b}{3}), \max(\mathrm{sg}(1-b), a.\mathrm{sg}(b), \min(a, \mathrm{sg}(1-a)), \min(1-b, \mathrm{sg}(b)), \\ &\quad a.(a.b + \overline{\mathrm{sg}}(1-b)) + \overline{\mathrm{sg}}(1-a), a.((1-a).a + \overline{\mathrm{sg}}(a)) + \overline{\mathrm{sg}}(1-a), \\ &\quad (1-b).(b.(1-b) + \overline{\mathrm{sg}}(1-b)) + \overline{\mathrm{sg}}(b), \frac{3-b}{3}))\rangle.\end{aligned}$$

It is directly checked that

$$a.b \leq b.\mathrm{sg}(a),$$

$$a.b \leq 1-a,$$

$$a.b \leq \mathrm{sg}(a),$$

$$a.b \leq b,$$

$$a.b \leq \text{sg}(1-b),$$

$$a.b \leq (1-a).a,$$

$$a.b \leq b.(1-b)$$

$$\text{sg}(1-b) \geq a \geq \min(a, \text{sg}(1-a)),$$

$$\text{sg}(1-b) \geq 1-b \geq \min(1-b, \text{sg}(b)).$$

The validity of the inequalities

$$\text{sg}(1-b) \geq a.(a.b + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(1-a),$$

$$\text{sg}(1-b) \geq a.((1-a).a + \overline{\text{sg}}(a)) + \overline{\text{sg}}(1-a),$$

$$\text{sg}(1-b) \geq (1-b).(b.(1-b) + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(b)$$

is checked, e.g., in the following way. Let

$$X \equiv \text{sg}(1-b) - a.(a.b + \overline{\text{sg}}(1-b)) - \overline{\text{sg}}(1-a).$$

If $b = 1$, then $a = 0$ and

$$X = 0 - 0.(0+1) - 0 = 0.$$

If $b < 1$ and $a < 1$, then

$$X = 1 - a.a.b - 0 \geq 0.$$

If $a = 1$, then $b = 0$ and

$$X = 1 - 1.(0+0) - 1 = 0.$$

Therefore, in all cases $X \geq 0$. The two other inequalities are checked analogous.

Hence,

$$\neg_{\min}\langle a, b\rangle = \langle \min(\overline{\text{sg}}(1-b), a.b, \frac{b}{3}), \max(\text{sg}(1-b), a.\text{sg}(b), \frac{3-b}{3})\rangle.$$

But, we see that if $b = 1$, then $a = 0$ and

$$\min(\overline{\text{sg}}(1-b), a.b, \frac{b}{3}) = 0,$$

$$\max(\text{sg}(1-b), a.\text{sg}(b), \frac{3-b}{3}) = \frac{2}{3};$$

if $b < 1$, then $\overline{\text{sg}}(1-b) = 0, \text{sg}(1-b) = 1$ and

$$\min(\overline{\text{sg}}(1-b), a.b, \frac{b}{3}) = 0,$$

$$\max(\text{sg}(1-b), a.\text{sg}(b), \frac{3-b}{3}) = 1.$$

Therefore, finally, we obtain that

$$\neg_{\min}\langle a, b\rangle = \langle 0, \frac{2+\text{sg}(1-b)}{3}\rangle.$$

Now, we calculate

$$\neg_{\min}\neg_{\min}\langle a, b\rangle = \neg_{\min}\langle 0, \frac{2+\text{sg}(1-b)}{3}\rangle$$

$$= \langle 0, \frac{2+\text{sg}(1-\frac{2+\text{sg}(1-b)}{3})}{3}\rangle = \langle 0, \frac{2+\overline{\text{sg}}(1-b)}{3}\rangle.$$

The following three properties of the negations are important are they are studied for all IFNs (see, e.g., [2]):

Property P1: $x \to \neg\neg x$,

Property P2: $\neg\neg x \to x$,

Property P3: $\neg\neg\neg x = \neg x$.

Now, we must check these properties for the new IFNs ($\neg_{\max}$ and $\neg_{\min}$), too. For each of these properties, we must use some implication. Here, we introduce four implications, generated by the two new negations, using the schemes:

$$X \to_1 Y = \neg X \vee Y,$$

$$X \to_2 Y = \neg X \vee \neg\neg Y.$$

Therefore, we construct the implications

$$\langle a, b\rangle \to_{\max,1} \langle c, d\rangle = \langle \max(\text{sg}(1-a), c), 0\rangle,$$

$$\langle a, b\rangle \to_{\max,2} \langle c, d\rangle = \langle \max(\text{sg}(1-a), \overline{\text{sg}}(1-c)), 0\rangle,$$

$$\langle a, b\rangle \to_{\min,1} \langle c, d\rangle = \langle c, \min(\frac{2+\text{sg}(1-b)}{3}, d)\rangle,$$

$$\langle a, b\rangle \to_{\min,2} \langle c, d\rangle = \langle 0, \frac{2}{3} + \frac{1}{3}\min(\text{sg}(1-b), \overline{\text{sg}}(1-d))\rangle.$$

Theorem 1. Negation $\neg_{\max}$ satisfies Properties 1, 2 and 3 as tautologies (and, therefore, satisfies them as IFTs, too) with each of the two implications $\to_{\max,1}$

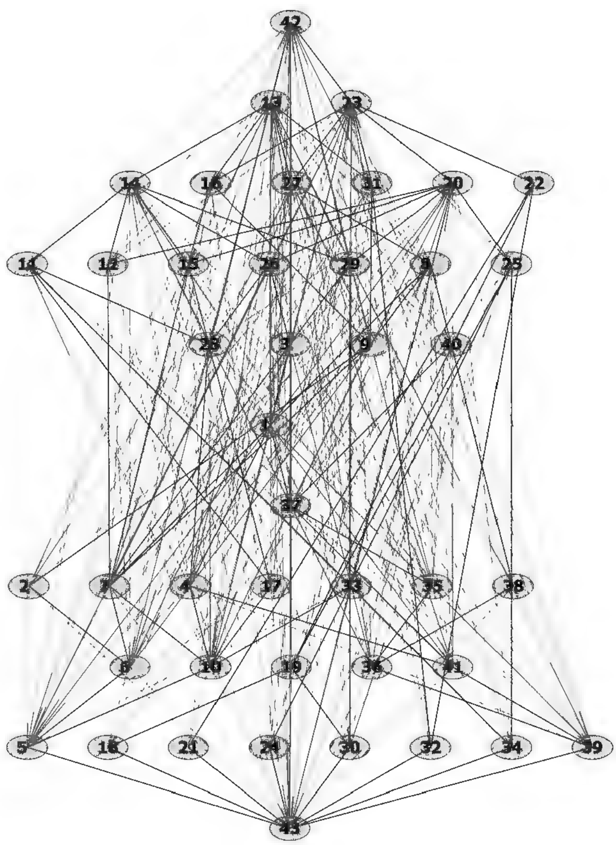

Figure 2: Extended ordering of relations including $\neg_{\max}$ and $\neg_{\min}$

and $\rightarrow_{\max,2}$ generated by it.
Proof. Sequentially, we check that

$$\langle a,b\rangle \rightarrow_{\max,1} \neg_{\max}\neg_{\max}\langle a,b\rangle$$

$$= \langle a,b\rangle \rightarrow_{\max,1} \neg_{\max}\langle \mathrm{sg}(1-a),0\rangle$$

$$= \langle a,b\rangle \rightarrow_{\max,1} \langle \overline{\mathrm{sg}}(1-a),0\rangle$$

$$= \langle \max(\mathrm{sg}(1-a),\overline{\mathrm{sg}}(1-a)),0\rangle = \langle 1,0\rangle;$$

$$\langle a,b\rangle \rightarrow_{\max,2} \neg_{\max}\neg_{\max}\langle a,b\rangle$$

$$= \langle a, b\rangle \to_{\max,2} \langle \overline{\text{sg}}(1-a), 0\rangle$$
$$= \langle \max(\text{sg}(1-a), \overline{\text{sg}}(1-\overline{\text{sg}}(1-a))), 0\rangle = \langle 1, 0\rangle,$$

because, if $a = 1$, then $\max(\text{sg}(1-a), \overline{\text{sg}}(1-\overline{\text{sg}}(1-a))) = \max(0,1) = 1$ and if $a < 1$, then $\max(\text{sg}(1-a), \overline{\text{sg}}(1-\overline{\text{sg}}(1-a))) = \max(1,0) = 1$;

$$\neg_{\max}\neg_{\max}\langle a, b\rangle \to_{\max,1} \langle a, b\rangle$$
$$= \langle \overline{\text{sg}}(1-a), 0\rangle \to_{\max,1} \langle a, b\rangle$$
$$= \langle \max(\text{sg}(1-\overline{\text{sg}}(1-a)), a), 0\rangle = \langle 1, 0\rangle,$$

because, if $a = 1$, then $\max(\text{sg}(1-\overline{\text{sg}}(1-a)), a) = \max(0,1) = 1$ and if $a < 1$, then $\max(\text{sg}(1-\overline{\text{sg}}(1-a)), a) = \max(1, a) = 1$;

$$\neg_{\max}\neg_{\max}\langle a, b\rangle \to_{\max,2} \langle a, b\rangle$$
$$= \langle \overline{\text{sg}}(1-a), 0\rangle \to_{\max,2} \langle a, b\rangle$$
$$= \langle \max(\text{sg}(1-\overline{\text{sg}}(1-a)), \overline{\text{sg}}(1-a)), 0\rangle = \langle 1, 0\rangle,$$

because, if $a = 1$, then $\max(\text{sg}(1-\overline{\text{sg}}(1-a)), \overline{\text{sg}}(1-a)) = \max(0,1) = 1$ and if $a < 1$, then $\max(\text{sg}(1-\overline{\text{sg}}(1-a)), \overline{\text{sg}}(1-a)) = \max(1,0) = 1$;

$$\neg_{\max}\neg_{\max}\neg_{\max}\langle a, b\rangle = \neg_{\max}\neg_{\max}\langle \text{sg}(1-a), 0\rangle$$
$$= \neg_{\max}\langle \overline{\text{sg}}(1-a), 0\rangle = \langle \text{sg}(1-a), 0\rangle = \neg_{\max}\langle a, b\rangle.$$

Therefore, all properties are valid as tautologies and, therefore, are valid as IFTs.

Theorem 2. Negation $\neg_{\min}$ does not satisfy Properties 1 and 2 as IFT (and, therefore, does not satisfy them as regular tautologies, too) with each of the two implications $\to_{\min,1}$ and $\to_{\min,2}$ generated by it. It satisfies only Property 3.

Proof. Sequentially, we check that

$$\langle a, b\rangle \to_{\min,1} \neg_{\min}\neg_{\min}\langle a, b\rangle$$
$$= \langle a, b\rangle \to_{\min,1} \neg_{\min}\langle 0, \frac{2+\text{sg}(1-b)}{3}\rangle$$
$$= \langle a, b\rangle \to_{\min,1} \langle 0, \frac{2+\overline{\text{sg}}(1-b)}{3}\rangle$$
$$= \langle 0, \min(\frac{2+\text{sg}(1-b)}{3}, \frac{2+\overline{\text{sg}}(1-b)}{3})\rangle$$
$$= \langle 0, \frac{2}{3} + \frac{1}{3}\min(\text{sg}(1-b), \overline{\text{sg}}(1-b))\rangle = \langle 0, \frac{2}{3}\rangle;$$

$$\langle a,b\rangle \to_{\min,2} \neg_{\min}\neg_{\min}\langle a,b\rangle$$

$$= \langle a,b\rangle \to_{\min,2} \langle 0, \frac{2+\overline{\mathrm{sg}}(1-b)}{3}\rangle$$

$$= \langle 0, \frac{2}{3} + \frac{1}{3}\min(\mathrm{sg}(1-b), \overline{\mathrm{sg}}(\frac{1-\overline{\mathrm{sg}}(1-b)}{3}))\rangle = \langle 0, \frac{2}{3}\rangle;$$

$$\neg_{\min}\neg_{\min}\langle a,b\rangle \to_{\min,1} \langle a,b\rangle$$

$$= \langle 0, \frac{2+\overline{\mathrm{sg}}(1-b)}{3}\rangle \to_{\min,1} \langle a,b\rangle$$

$$= \langle a, \min(\frac{2+\mathrm{sg}(1-\frac{2+\overline{\mathrm{sg}}(1-b)}{3})}{3}, b)\rangle$$

$$= \begin{cases} \langle 0, \frac{2}{3}\rangle, & \text{if } b = 1 \text{ and hence } a = 0 \\ \langle a,b\rangle, & \text{if } b < 1 \end{cases};$$

$$\neg_{\min}\neg_{\min}\langle a,b\rangle \to_{\min,2} \langle a,b\rangle$$

$$= \langle 0, \frac{2+\overline{\mathrm{sg}}(1-b)}{3}\rangle \to_{\min,2} 1\langle a,b\rangle$$

$$= \langle 0, \frac{2}{3} + \frac{1}{3}\min(\mathrm{sg}(\frac{1-\overline{\mathrm{sg}}(1-b)}{3}), \overline{\mathrm{sg}}(1-b)) = \langle 0, \frac{2}{3}\rangle.$$

Therefore, the negation $\neg_{\min}$ does not satisfy Properties 1 and 2 as IFTs and, therefore, as tautologies. On the other hand,

$$\neg_{\min}\neg_{\min}\neg_{\min}\langle a,b\rangle = \neg_{\min}\neg_{\min}\langle 0, \frac{2+\mathrm{sg}(1-b)}{3}\rangle$$

$$= \neg_{\min}\langle 0, \frac{2+\overline{\mathrm{sg}}(1-b)}{3}\rangle = \langle \frac{2+\mathrm{sg}(1-b)}{3}\rangle = \neg_{\min}\langle a,b\rangle,$$

i.e., the negation $\neg_{\min}$ satisfies Property 1.

3 Conclusion

We showed that the number of all (non-parametric) IFNs generates a (discrete) lattice with a maximal element $\neg_{\max}$ and a minimal element $\neg_{\min}$.

In a next research, we will study the lattice generated by the set of all (non-parametric) intuitionistic fuzzy implications.

References

[1] Atanassov K. (1999) Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag, Heidelberg.

[2] Atanassov K. (2012) On Intuitionistic Fuzzy Sets Theory, Springer, Berlin.

[3] Dimitrov D. (2011) IFS Tool – software for intuitionistic fuzzy sets. Issues in Intuitionistic Fuzzy Sets and Generalized Nets, 9, 61–69.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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