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Volume I: Foundations

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Generalized nets with pairwise capacities of the places

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Abstract

Generalized nets for which the transfer of tokens from input to output place of a given transition depends on the total number of tokens in the two places are discussed. When the number of tokens in the pair of places reaches a predefined limit no tokens can be transferred through the arc connecting the places. The basic cases of transitions with such restriction are studied. This restriction may cause problems related to the flow of the tokens into the net. Solutions to these problems are proposed.
Keywords: generalized nets, pairwise capacity, transfer of tokens.

1 Introduction

The focus of this paper is on Generalized Nets (GNs) for which if the number of tokens in a pair of places — one input and one output for a given transition — exceeds a fixed number then no tokens can be transferred through the arc connecting them. In a standard GN a token can be transferred from input place to output place of a given active transition if the respective predicate of the Index Matrix (IM) of the transition's conditions has truth value $true$ and the capacities of the arc and the output place allow the transfer. In the modelling of real process we can encounter a situation where an upper limit for the sum of the tokens in a pair of input-output places must be set. This limit is determined by the modelled process

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and reflects its characteristics. Here we study the case where this limit restricts only the transfer of the tokens from the input to the output place in the pair in a sense that the number of tokens in the pair can exceed the limit but if the limit is reached then transfer cannot occur. In such GN depending on the structure of the transitions problems related to the flow of the tokens may appear. Before we proceed with the discussion of these problems let us remind the basic notation.

A transition Z is the seven-tuple

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle .$$

A GN E is the ordered four-tuple

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \rangle .$$

For detailed definition of transition and GN as well as the algorithms for their functioning the reader can refer to [1, 2]. Let l'_i be an input and l''_j be an output place for arbitrary transition Z (see Fig. 1).

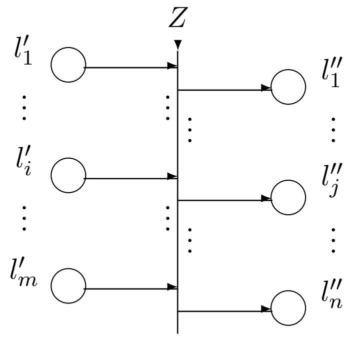

Fig. 1

Let the capacities of the places l'_i and l''_j be $c(l'_i) = n_i$ and $c(l''_j) = n_j$. Let $n_{i,j}$ be the maximum number of the sum of the tokens in places l'_i and l''_j beyond which the transfer of tokens from l'_i to l''_j is not allowed. This means that if all other conditions for successful transfer are present but there are too many tokens in the pair of places the tokens will not be transferred. Here and below n_i, n_j and $n_{i,j}$ are natural numbers. In order for the restriction over the sum of the capacities to make sense, we should further impose the condition $n_{i,j} < n_i + n_j$. If $n_{i,j} \geq n_i + n_j$, then the additional restriction for the sum of the capacities is useless because the tokens in the two places can not be more than $n_i + n_j$ and the transfer will be possible when all other conditions allow it. What is important is that even when the capacity of the output place l''_j is not reached but the sum of the tokens in the pair is greater or equal to $n_{i,j}$ no tokens can be transferred. It is useful to give the following definition:

Definition 1. *Pairwise capacity of a pair of one input and one output place for a given transition is an integer number n, $n > 1$ such that when the sum of the tokens in the two places is greater or equal to it no tokens can be transferred through the arc connecting the places.*

We do not impose the condition that the pairwise capacity be less than the sum of the capacities of the places. The pairwise capacity can be given by some function and this definition allows us to extend the function, if needed, over all pairs of input-output places by defining its value to be the sum of the capacities of the places for the pairs which do not have pairwise capacity.

2 Possible problems related to the flow of the tokens

The pairwise capacity of the places can cause some problems related to the flow of the tokens in the net. In the present paper we shall consider that the splitting of tokens is not allowed. Let again l'_i and l''_j be two places with capacities n_i and n_j and pairwise capacity $n_{i,j}$. When the pairwise capacity of the places is reached, depending on the GN model, the tokens in place l'_i may or may not be transferred to other output places for the transition. If the transfer of the tokens from l'_i to other output places is not possible or the number of tokens leaving l'_i is too small compared to the number of the incoming tokens, then the place l'_i may reach its capacity very fast. As a result the flow of the tokens through this place will be interrupted, i.e. the transfer of tokens from other places to l'_i will be impossible. For brevity we shall say that the set of places $\{l_i, l_j, ..., l_k\}$ forms a cycle if a token can pass consecutively through each of them, starting from l_i and ending again there. If the cycle consists of only one place, we will call it a 1-cycle and generally a cycle with n places will be referred to as n-cycle. With k_i we denote the number of tokens in place l_i at the beginning of the current time moment $TIME$. With $k_{i,j}$ we denote the number of tokens that has been transferred from l_i to l_j at the current time step. First we study some basic cases of transitions with pairwise capacity.

2.1 Pairwise capacity with no cycle

In the simplest case we consider the three transitions presented in Fig. 2.

$$Z_1 = \langle \{l_1\}, \{l_2\}, t_1^1, t_2^1, r_1, M_1, \Box_1 \rangle ,$$

$$Z_2 = \langle \{l_2\}, \{l_3\}, t_1^2, t_2^2, r_2, M_2, \Box_2 \rangle \,,$$
$$Z_3 = \langle \{l_3\}, \{l_4\}, t_1^3, t_2^3, r_3, M_3, \Box_3 \rangle \,.$$

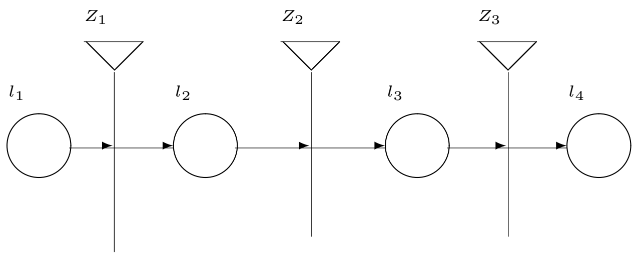

Fig. 2

Let $n_{2,3}$ be the pairwise capacity of the pair of places $\langle l_2, l_3\rangle$. In this simplest case the transition Z_2 has only two places and they have pairwise capacity. At the end of the current time step the number of tokens in the pair is

$$k_2 + k_{1,2} - k_{2,3} + k_3 + k_{2,3} - k_{3,4} = k_2 + k_{1,2} + k_3 - k_{3,4}.$$

If the pairwise capacity has been reached, then $k_{2,3} = 0$ and the number of tokens in place l_2 becomes $k_2 + k_{1,2}$. In such case l_2 may reach its capacity very fast. When this capacity is reached no tokens can be transferred from l_1 to l_2. The following two cases should be considered:

C1. The pairwise capacity is less than the capacity of the first place of the pair.

C2. The pairwise capacity is greater than the capacity of the first place of the pair.

In the first case the flow of the tokens through l_2 can not be restored within the net. This is an example of a conflict situation in GNs that should be avoided when constructing the net. To resolve the problem modification of the transitions' components is required. In the second case the flow of the tokens through l_2 can possibly be restored within the net. If sufficiently enough tokens leave place l_3 so that the number of tokens in the pair drops below the pairwise capacity, then the tokens from l_2 can be transferred to l_3 and as a result the number of tokens in l_2 becomes less than its capacity which may eventually allow the transfer of tokens from l_1 to l_2. However, a change in the transitions' components may still be needed.

A more general case of pairwise capacity with no cycle is presented in Fig. 3. Again we consider three transitions with pairwise capacity $n_{2,3}$ for the pair $\langle l_2, l_3\rangle$ but now Z_2 has one more output place. This new place l_4 can be input for

Z_3, for another transition or output for the whole net. We shall not distinguish between these possibilities because they are not related to the problem discussed here. What is important is that the presence of place l_4 may have impact on the number of tokens in l_2 (if the predicate allows it).

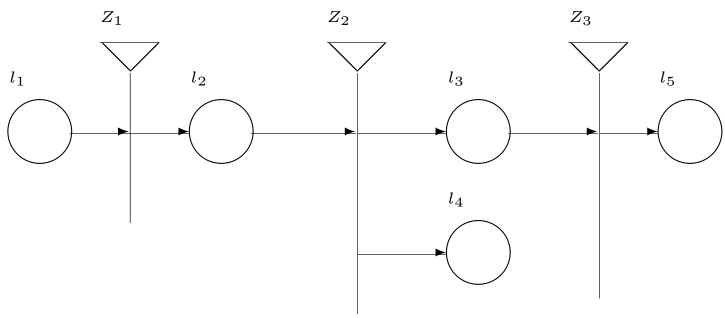

Fig. 3

At the end of the current time step the number of tokens in the pair $\langle l_2, l_3\rangle$ is

$$k_2 + k_{1,2} - k_{2,3} - k_{2,4} + k_3 + k_{2,3} - k_{3,5} = k_2 + k_{1,2} - k_{2,4} + k_3 - k_{3,5}.$$

If the pairwise capacity has been reached, at the end of the current time step the number of tokens in place l_2 becomes

$$k_2 + k_{1,2} - k_{2,4}.$$

In this case, depending on the modelled proccess, some of the tokens may leave l_2 even though the pairwise capacity is reached. The number of tokens leaving l_2 and going to l_4 may be enough to compensate for the reached pairwise capacity. This should be taken into account when we look for ways to restore the flow of the tokens through l_2.

2.2 Pairwise capacity with 2-cycle

Let us consider two transitions Z_1 and Z_2 (see Fig. 4).

$$Z_1 = \langle \{l_1, l_4\}, \{l_2\}, t_1^1, t_2^1, r_1, M_1, \Box_1\rangle\,,$$

$$Z_2 = \langle \{l_2\}, \{l_3, l_4\}, t_1^2, t_2^2, r_2, M_2, \Box_2\rangle\,.$$

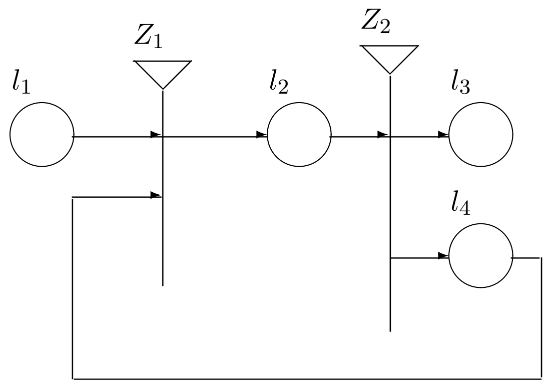

Fig. 4

The set $\{l_2, l_4\}$ forms a 2-cycle. As before, let the capacities of l_2 and l_4 be n_2 and n_4 respectively and their pairwise capacity be $1 < n_{2,4} < n_2 + n_4$. During the active state of the first transition, at a single time step and when the conditions for the transfer allow it, some tokens from l_1 and l_4 will enter place l_2. Let us denote their numbers by $k_{1,2}$ and $k_{4,2}$. During the active state of Z_2 some of the tokens in place l_2 enter places l_3 and l_4. Let us denote their numbers by $k_{2,3}$ and $k_{2,4}$ respectively. At the end of the current time step the sum of the tokens in l_2 and l_4 is

$$k_2 + k_4 - (k_{2,3} + k_{2,4} + k_{4,2}) + (k_{1,2} + k_{4,2} + k_{2,4}) = k_2 + k_4 - k_{2,3} + k_{1,2}.$$

While the number of tokens in place l_2 is

$$k_2 + k_{1,2} + k_{4,2} - k_{2,3} - k_{2,4}.$$

If the pairwise capacity $n_{2,4}$ has been reached, then $k_{2,4} = 0$. At the current time step it does not have an effect on the first sum. The second sum however becomes

$$k_2 + k_{1,2} + k_{4,2} - k_{2,3}.$$

Now if the transfer of tokens from l_2 to l_3 is also not possible or $k_2 + k_{1,2} + k_{4,2}$ is sufficiently greater than $k_{2,3}$, then place l_2 will reach its capacity and the flow of the tokens from l_1 and possibly other input places of the transition to l_2 will be interrupted, i.e.

$$k_2 + k_{1,2} + k_{4,2} - k_{2,3} = n_2\,,$$

where n_2 is the capacity of the place l_2. The reason for this is that once the pairwise capacity has been reached, all tokens from l_4 can only be transferred to l_2 but no tokens from l_2 can be transferred to l_4.

2.3 Pairwise capacity with 1-cycle

We consider two transitions (see Fig. 5)

$$Z_1 = \langle \{l_1\}, \{l_2\}, t_1^1, t_2^1, r_1, M_1, \Box_1 \rangle \,,$$

$$Z_2 = \langle \{l_2, l_4\}, \{l_3, l_4\}, t_1^2, t_2^2, r_2, M_2, \Box_2 \rangle \,.$$

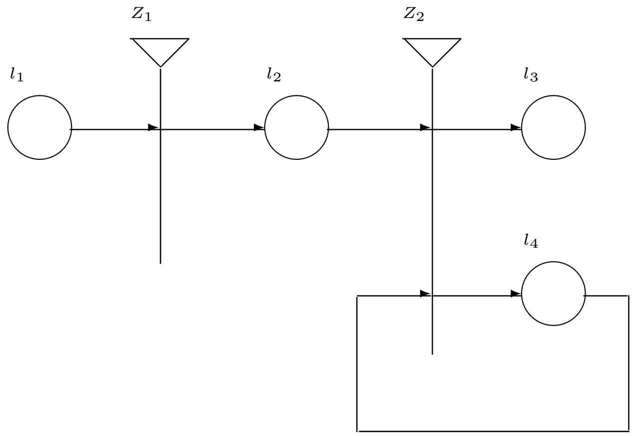

Fig. 5

For the transition above $\{l_4\}$ is a 1-cycle. Let again the pairwise capacity of l_2 and l_4 be $n_{2,4}$. At the end of the current time step the number of tokens in the pair $\langle l_2, l_4 \rangle$ is

$$k_2 + k_4 - (k_{2,3} + k_{2,4} + k_{4,3}) + (k_{1,2} + k_{2,4}) = k_2 + k_4 + k_{1,2} - k_{4,3} - k_{2,3}.$$

If $k_{4,3}$ and $k_{2,3}$ are small compared to $k_{1,2}$, i.e. the number of ingoing tokens for the pair $\langle l_2, l_4 \rangle$ is greater than the number of outgoing tokens, then the pair $\langle l_2, l_4 \rangle$ may reach its pairwise capacity very fast. At the end of the the current time step the number of tokens in place l_2 is

$$k_2 + k_{1,2} - k_{2,3} - k_{2,4}.$$

If the pairwise capacity has been reached, then $k_{2,4} = 0$. And if $k_{1,2}$ is sufficiently greater than $k_{2,3}$, i.e. the number of ingoing tokens for place l_2 is greater than the number of outgoing tokens, place l_2 will reach its capacity:

$$k_2 + k_{1,2} - k_{2,3} = c(l_2) \,.$$

The transfer of tokens from l_1 to l_2 will not be possible. In this case the reason for the break of the flow of the tokens from l_1 to l_2 is that tokens are not leaving place l_2 fast enough due to the pairwise capacity of the pair $\langle l_2, l_4 \rangle$.

3 Managing the flow of tokens in GN with pairwise capacities

We now proceed with the analysis of possible solutions to the problems described in the previous section. Only the basic cases described there will be discussed. The more general cases in which the transitions have more input and output places are treated analogously. For instance, the case when transition Z_1 in Fig. 3 has more input places is not different with regard to the pairwise capacity. The more input places may lead to more tokens entering place l_2 which can be viewed as greater value of $k_{1,2}$ in our basic case. The other more general cases can be treated similarly.

3.1 The case of pairwise capacity with 2-cycle

In the case discussed in 2.2 (see Fig. 4) place l_2 reaches its capacity because the tokens in this place cannot be transferred to l_4 (since the pairwise capacity is reached) and the tokens going to l_3 are not enough to compensate for the tokens entering l_2. There are different ways to maintain the flow of tokens through l_2. Let us discuss some of them.

3.1.1 Change in the duration of the active state of the transitions

When the pairwise capacity is reached during the active state of Z_2 tokens can only be transferred from l_2 to l_3. This reduces the number of tokens in l_2. Therefore, if we change the duration of active state of Z_2 so that it is still active when Z_1 is not, the number of tokens in l_2 will be reduced enough to make the transfer of tokens from l_1 to l_2 possible when Z_1 becomes active. Formally,

$$Z_2' = \langle \{l_2\}, \{l_3, l_4\}, t_1^2, t_2^{2,*}, r_2, M_2, \Box_2 \rangle \, ,$$

where $t_2^{2,*} > t_2^2$. Another way to restore the flow of tokens through l_2 is to make the duration of the active state of Z_1 shorter: Formally,

$$Z_1' = \langle \{l_1, l_4\}, \{l_2\}, t_1^1, t_2^{1,*}, r_1, M_1, \Box_1 \rangle \, ,$$

where $t_2^{1,*} < t_2^1$. In this way less tokens may enter l_2.

Alternatively, we can shorten the duration of the active state of Z_1 and at the same time prolong the duration of the active state of Z_2.

3.1.2 Change in the priorities of the transitions

If the priority of Z_1 is greater than that of Z_2, then it makes sense to change these priorities. We would like to transfer the tokens from l_2 to l_3 first to reduce the number of tokens in l_2. This can be achieved by increasing the priority of Z_2 or decreasing the priority of Z_1 so that $\pi_A(Z_2) > \pi_A(Z_1)$.

3.1.3 Change in the capacities of the arcs

Another way to restore the flow of tokens through l_2 is to control the number of ingoing and outgoing tokens to place l_2 by means of the capacities of the arcs. If the sum of the capacities of the arcs (l_4, l_2) and (l_1, l_2) is greater than the capacity of the arc (l_2, l_3), we can decrease this sum by decreasing the capacity of the arc (l_4, l_2) or that of (l_1, l_2). Alternatively, we can decrease the capacities of both arcs. The same result can be achieved if we increase the capacity of the arc (l_2, l_3). Depending on the model, one of these solutions can be better than the other.

3.1.4 Change in the priorities of the places

The reason for the pair $\langle l_2, l_4\rangle$ to reach its pairwise capacity can be the priorities of the output places of Z_2. If the priority of place l_4 is greater than that of l_3, more tokens will remain in the pair $\langle l_2, l_4\rangle$ and this may be the reason for the pair to reach its pairwise capacity. By decreasing the priority of l_4 or increasing that of l_3 so that $\pi_L(l_3) > \pi_L(l_4)$ we can keep the number of tokens in the pair $\langle l_2, l_4\rangle$ lower. Once however the pairwise capacity has been reached, this change in the priorities of the places will not have effect on the number of tokens in the pair because no tokens can be transferred from l_2 to l_4 regardless of the priorities of the output places. Therefore the change of the priorities of the output places may only be used to prevent the pair from reaching its pairwise capacity and in this way indirectly maintain the flow the tokens through l_2.

3.1.5 Use of additional place

Again we consider the two transitions

$$Z_1 = \langle \{l_1, l_4\}, \{l_2\}, t_1^1, t_2^1, r_1, M_1, \Box_1\rangle ,$$

$$Z_2 = \langle \{l_2\}, \{l_3, l_4\}, t_1^2, t_2^2, r_2, M_2, \Box_2\rangle ,$$

with pairwise capacity for the pair $\langle l_2, l_4\rangle$ (see Fig. 4). We add an extra place to Z_1 so that it becomes (see Fig. 6)

$$Z'_1 = \langle \{l_1, l_4, l_5\}, \{l_2, l_5\}, t^1_1, t^1_2, r'_1, M'_1, \square'_1\rangle\,.$$

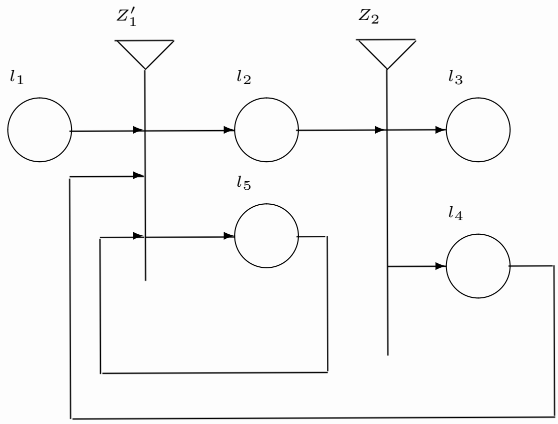

Fig. 6

The new place l_5 plays the role of a buffer for l_2. The temporal components t^1_1 and t^1_2 remain the same. The other components are obtained as follows.

The IM of the transition's conditions is

$$r'_1 = \begin{array}{c|cc} & l_2 & l_5 \\ \hline l_1 & r'_{1,2} & r'_{1,5} \\ l_4 & r'_{4,2} & r'_{4,5} \\ l_5 & r'_{5,2} & r'_{5,5} \end{array}\,,$$

where

$$r'_{1,5} = false,$$

$$r'_{1,2} = r_{1,2},$$

$$r'_{4,2} = r_{4,2}\,\&\,\text{“}c(l_2) > k_2 + m'_{4,2}\text{”},$$

$$r'_{4,5} = r_{4,2}\,\&\,\text{“}c(l_2) \leq k_2 + m'_{4,2}\text{”},$$

$$r'_{5,2} = \text{”}c(l_2) > k_2 + m'_{5,2}\text{”},$$

$$r'_{5,5} = \neg r'_{5,2}.$$

Here $r_{i,j}$ is the predicate corresponding to the i-th input and j-th output place of the original transition and $m'_{i,j}$ is the capacity of the arc between the i-th input and j-th output place of the modified transition. The IM of the capacities of the arcs is

$$M_1' = \begin{array}{c|cc} & l_2 & l_5 \\ \hline l_1 & m'_{1,2} & m'_{1,5} \\ l_4 & m'_{4,2} & m'_{4,5} \\ l_5 & m'_{5,2} & m'_{5,5} \end{array},$$

where

$$m'_{1,2} = m_{1,2},$$
$$m'_{4,2} = m'_{4,5} = m'_{5,2} = m_{4,2},$$
$$m'_{5,5} = \infty\,.$$

Here $m_{i,j}$ is the capacity of the arc from the i-th input place to the j-th output place of the original transition.

$$\square'_1 = \vee(\square_1, l_5)\,.$$

The priority of the new place l_5 should satisfy the condition $\pi_L(l_5) > \pi_L(l_4)$ so that the tokens that entered l_5 on previous steps be transferred before the tokens in l_4.

Another way to control the number of tokens in place l_2 is by adding an extra place to Z_2 (see Fig. 7).

$$Z'_2 = \langle\{l_2, l_5\}, \{l_3, l_4, l_5\}, t_1^2, t_2^2, r'_2, M'_2, \square'_2\rangle$$

$$r'_2 = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_2 & r'_{2,3} & r'_{2,4} & r'_{2,5} \\ l_5 & r'_{5,3} & r'_{5,4} & r'_{5,5} \end{array},$$

where

$$r'_{2,3} = r_{2,3}\,,$$
$$r'_{2,4} = r_{2,4}\,,$$
$$r'_{2,5} = \text{“}k_2 + k_4 \geq n_{2,4}\text{”}\,\&\,\text{“}k_2 = c(l_2)\text{”}\,,$$
$$r'_{5,3} = r_{2,3}\,,$$
$$r'_{5,4} = r_{2,4}\,,$$
$$r'_{5,5} = \neg r'_{5,3} \& \neg r'_{5,4}\,.$$

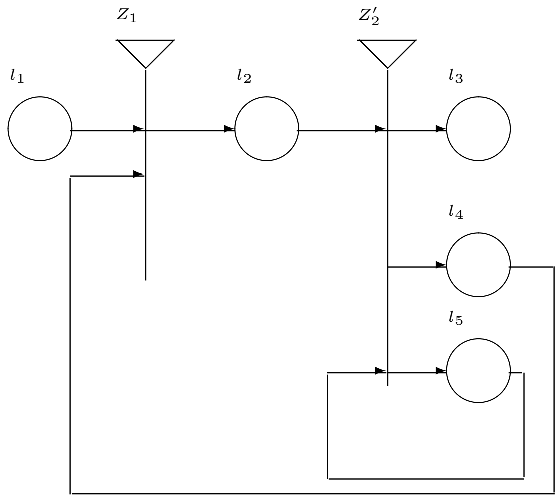

Fig. 7

The additional place l_5 place the role of a buffer for l_4. The IM of the capacities of the arcs is:

$$M_1' = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_2 & m'_{2,3} & m'_{2,4} & m'_{2,5} \\ l_5 & m'_{5,3} & m'_{5,4} & m'_{5,5} \end{array} ,$$

where

$$m'_{2,3} = m'_{5,3} = m_{2,3} \,,$$

$$m'_{2,4} = m'_{2,5} = m'_{5,4} = m'_{5,5} = m_{2,4} \,.$$

$$\Box'_2 = \vee(\Box_2, l_5) \,.$$

The priority of the new place l_5 must satisfy the condition $\pi_L(l_2) < \pi_L(l_5)$ so that the tokens that has entered l_5 on the previous steps be processed before the tokens that can be transferred from l_2 to l_4 and l_3. In place l_5 the tokens do not obtain new characteristics.

3.2 The case of pairwise capacity with 1-cycle

In the case discussed in 2.3 (see Fig. 5) the reason for place l_2 to reach its capacity is that when the pairwise capacity is reached the number of tokens transferred from l_2 to l_3 is not enough to compensate for the tokens entering place l_2. We proceed with discussion of different ways to restore the flow of the tokens through place l_2.

3.2.1 Change in the duration of the active state of the transitions

When the pairwise capacity is reached at the end of the current time step the number of tokens in place l_2 depends only on the tokens entering l_2 through l_1 and leaving l_2 to l_3. If place l_2 reaches its capacity because the number of ingoing tokens is greater than that of the outgoing as a result of the limitation imposed by the pairwise capacity, we can change the duration of the active states of the transitions to reduce the number of tokens in l_2. As in the case with pairwise capacity and 2-cycle we can reduce the duration of the active state of Z_1:

$$Z_1' = \langle \{l_1\}, \{l_2\}, t_1^1, t_2^{1,*}, r_1, M_1, \square_1 \rangle \,,$$

where $t_2^{1,*} < t_2^1$ or prolong the duration of the active state of Z_2:

$$Z_2' = \langle \{l_2, l_4\}, \{l_3, l_4\}, t_1^2, t_2^{2,*}, r_2, M_2, \square_2 \rangle \,,$$

where $t_2^{2,*} > t_2^2$.

In the first case less tokens can enter l_2 while in the second more tokens can leave l_2. Alternatively, the shortening of the duration of active state of Z_1 can be combined with prolonging of the duration of the active state of Z_2.

3.2.2 Change in the priorities of the transitions

If the priorities of the transitions are such that $\pi_A(Z_1) > \pi_A(Z_2)$, it makes sense to change the priorities of Z_1 and Z_2 so that $\pi_A(Z_1) < \pi_A(Z_2)$. Now, if the other conditions allow it, the tokens from l_2 to l_3 will be transferred before the tokens from l_1 (and possibly other input places for Z_1) to l_2. In this way the number of tokens in l_2 may drop below the capacity of l_2 which would allow transfer of tokens from l_1 to l_2 at the current time step.

3.2.3 Change in the capacities of the arcs

As in the case with pairwise capacity and 2-cycle we can increase the capacity of the arc (l_2, l_3) to allow more tokens to leave l_2. More formally,

$$Z_2' = \langle \{l_2, l_4\}, \{l_3, l_4\}, t_1^2, t_2^2, r_2, M_2', \square_2 \rangle$$

$$M_2' = \begin{array}{c|cc} & l_3 & l_4 \\ \hline l_2 & m_{2,3}' & m_{2,4}' \\ l_4 & m_{4,3}' & m_{4,4}' \end{array} \,,$$

where $m'_{2,4} = m_{2,4}$, $m'_{4,3} = m_{4,3}$, $m'_{4,4} = m_{4,4}$ and $m'_{2,3} > m_{2,3}$. The exact increase of the capacity depends on the modelled process. Alternatively, we can decrease the capacity of the arc (l_1, l_2) so that less tokens can enter l_2.

$$Z_1' = \langle \{l_1\}, \{l_2\}, t_1^1, t_2^1, r_1, M_1', \Box_1 \rangle$$

$$M_2' = \begin{array}{c|c} & l_2 \\ \hline l_1 & m'_{1,2} \end{array},$$

where $m'_{1,2} < m_{1,2}$. Or we can combine these two changes. These changes of the capacities of the arcs in some sense neutralize the effect of the pairwise capacity on the flow of the tokens through l_2.

3.2.4 Change in the priorities of the places

Again we consider the case when the pairwise capacity has been reached and as a result place l_2 has reached its capacity. It makes sense to change the priorities of the places in such way that $\pi_L(l_3) > \pi_L(l_4)$. In this way more tokens are allowed to leave the pair $\langle l_2, l_4 \rangle$.

3.2.5 Use of additional place

Again as in the case of pairwise capacity and 2-cycle, we study the possibility of maintaining the flow of tokens through l_2 with the help of additional place which is both input and output for the transition Z_2 in Fig. 5. We denote the new transition by Z_2' (see Fig. 8).

$$Z_2' = \langle \{l_2, l_4, l_5\}, \{l_3, l_4, l_5\}, t_1^2, t_2^2, r_2', M_2', \Box_2' \rangle$$

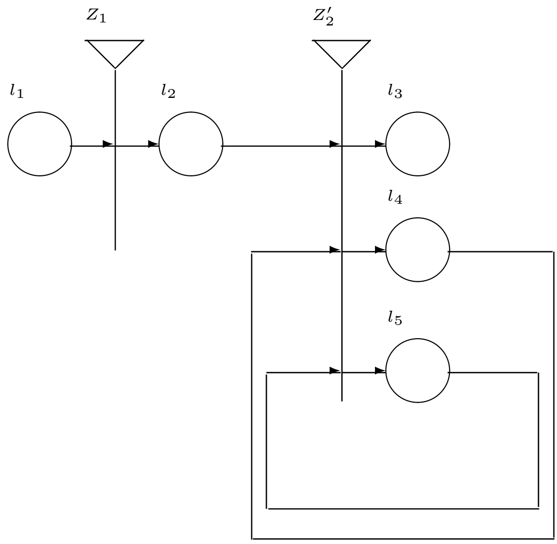

Fig. 8

The temporal components t_1^2 and t_2^2 remain the same.

$$r_2' = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_2 & r'_{2,3} & r'_{2,4} & r'_{2,5} \\ l_4 & r'_{4,3} & r'_{4,4} & r'_{4,5} \\ l_5 & r'_{5,3} & r'_{5,4} & r'_{5,5} \end{array},$$

where

$$r'_{2,3} = r_{2,3}\,,$$

$$r'_{2,4} = r_{2,4}\,,$$

$$r'_{4,3} = r_{4,3}\,,$$

$$r'_{4,4} = r_{4,4}\,,$$

$$r'_{2,5} = \text{“}k_2 = c(l_2)\text{”} \ \& \ \text{“}k_2 + k_4 \geq n_{2,4}\text{”}\,,$$

$$r'_{4,5} = false\,,$$

$$r'_{5,3} = r_{2,3}\,,$$

$$r'_{5,4} = r_{2,4}\,,$$

$$r'_{5,5} = \neg r_{5,4} \,\&\, \neg r_{5,3}\,.$$

The IM of the capacities of the arcs is:

$$M_2' = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_2 & m'_{2,3} & m'_{2,4} & m'_{2,5} \\ l_4 & m'_{4,3} & m'_{4,4} & m'_{4,5} \\ l_5 & m'_{5,3} & m'_{5,4} & m'_{5,5} \end{array},$$

where

$$m'_{2,3} = m'_{5,3} = m_{2,3}\,,$$

$$m'_{4,3} = m_{4,3}, m'_{4,4} = m_{4,4}, m'_{4,5} = 0\,,$$

$$m'_{2,5} = m'_{2,4} = m'_{5,4} = m_{2,4}\,,$$

$$m'_{5,5} = \infty\,.$$

$$\Box'_2 = \vee(\Box_2, l_5)\,.$$

The priority of the places should satisfy the conditions

$$\pi_L(l_3) > \pi_L(l_5) > \pi_L(l_2)\,,$$

and

$$\pi_L(l_4) > \pi_L(l_5)$$

so that the tokens that entered l_5 on previous steps be transferred before the tokens in l_2. In l_5 the tokens do not obtain new characteristics.

3.3 The case of pairwise capacity with no cycle

In the case discussed in Section 2.1(see Fig. 3), the two places on which pairwise capacity is set, take part in three transitions. The pair $\langle l_2, l_3\rangle$ has reached its pairwise capacity $n_{2,3}$ and as a result of this place l_2 has reached its capacity.

3.3.1 Change in the duration of the active state of the transitions

One way to restore the flow of tokens through l_2 is to prolong the duration of active state of Z_2. In this way more tokens may be transferred from l_2 to l_4 and the number of tokens in l_2 will drop below the capacity of the place. Formally,

$$Z_2' = \langle\{l_2\}, \{l_3, l_4\}, t_1^2, t_2^{2,*}, r_2, M_2, \Box_2\rangle\,,$$

where $t_2^{2,*} > t_2^2$. Another solution may be to reduce the duration of active state of Z_1 so that less tokens enter place l_2:

$$Z_1' = \langle\{l_1\}, \{l_2\}, t_1^1, t_2^{1,*}, r_1, M_1, \Box_1\rangle\,,$$

where $t_2^{1,*} < t_2^1$. The number of tokens in l_2 can also be reduced indirectly by prolonging the duration of active state of Z_3 :

$$Z_3' = \langle \{l_3\}, \{l_5\}, t_1^3, t_2^{3,*}, r_3, M_3, \Box_3 \rangle \,,$$

where $t_2^{3,*} > t_2^3$. This may lead to more tokens leaving l_3 and the number of tokens in the pair $\langle l_2, l_3 \rangle$ may drop below its pairwise capacity. Once this happens the transfer of tokens from l_2 to l_3 will be possible and the number of tokens in l_2 will be reduced.

3.3.2 Change in the priorities of the transitions

If $\pi_A(Z_1) > \pi_A(Z_2)$, no tokens can be transferred to l_2 at the current time step. The number of tokens in l_2 can be reduced directly if we change the priorities of the transitions so that $\pi_A(Z_2) > \pi_A(Z_1)$. In this way tokens can be transferred from l_2 to l_4 freeing space for tokens to enter l_2. However, if the transfer of tokens from l_2 to l_4 is not possible, this change of the priorities of the transitions will not have effect on the flow of the tokens. This is so because no tokens can be transferred from l_2 to l_3 due to the pairwise capacity. In such case the priorities of the transitions should be changed so that $\pi_A(Z_3) > \pi_A(Z_2) > \pi_A(Z_1)$. Now enough tokens may leave place l_3 so that the number of tokens in $\langle l_2, l_3 \rangle$ drops below the pairwise capacity. As a result, tokens from l_2 can be transferred to l_3 and the number of tokens in l_2 will drop below its capacity.

3.3.3 Change in the capacities of the arcs

As in the cases of pairwise capacity with 1-cycle and 2-cycle, the flow of the tokens can be maintained by means of the capacities of the arcs. Increasing the capacity of the arc (l_2, l_4) can directly reduce the number of tokens in l_2. Once the pairwise capacity is reached, change in the capacity of the arc (l_2, l_3) does not have effect on the flow of the tokens. The number of tokens in l_2 can be decreased indirectly by increasing the capacity of (l_3, l_5) which may allow more tokens to leave place l_3 so that the number of tokens in (l_2, l_3) drops below the pairwise capacity. This would allow transfer of tokens from l_2 to l_3 and the number of tokens in l_2 would drop below the capacity of the place.

3.3.4 Change in the priorities of the places

Once the pairwise capacity is reached and $\pi_L(l_3) > \pi_L(l_4)$, changing these priorities so that $\pi_L(l_3) < \pi_L(l_4)$ will not have effect on the flow of the tokens. Such change can be used to prevent the pair $\langle l_2, l_3 \rangle$ from reaching its pairwise capacity and in this way indirectly maintain the flow through l_2.

3.3.5 Use of additional place

In the cases of pairwise capacity with 1-cycle and 2-cycle we showed how an additional place can be used to maintain the flow of the tokens through the first place in the pair with pairwise capacity. The same method can also be applied in the case of pairwise capacity with no cycle. We add a new place l_6 to Z_2 which is both input and output for the transition. We denote the new transition by Z_2' (see Fig. 9).

$$Z_2' = \langle \{l_2, l_6\}, \{l_3, l_4, l_6\}, t_1^2, t_2^2, r_2', M_2', \Box_2' \rangle \, .$$

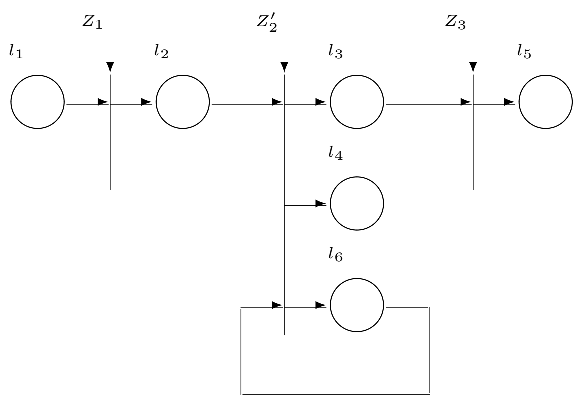

Fig. 9

The time components remain the same as in the original transition Z_2.

$$r_2' = \begin{array}{c|ccc} & l_3 & l_4 & l_6 \\ \hline l_2 & r'_{2,3} & r'_{2,4} & r'_{2,6} \\ l_6 & r'_{6,3} & r'_{6,4} & r'_{6,6} \end{array} \, ,$$

where

$$r'_{2,3} = r_{2,3} \, ,$$

$$r'_{2,4} = r_{2,4} \, ,$$

$$r'_{2,6} = \text{“}k_2 = c(l_2)\text{”} \;\&\; \text{“}k_2 + k_3 \geq n_{2,3} \, ,\text{”}$$

$$r'_{6,3} = r_{2,3}\,,$$
$$r'_{6,4} = r_{2,4}\,,$$
$$r'_{6,6} = \neg r_{6,3} \,\&\, \neg r_{6,4}\,.$$

The IM of the capacities of the arcs is:

$$M'_2 = \begin{array}{c|ccc} & l_3 & l_4 & l_6 \\ \hline l_2 & m'_{2,3} & m'_{2,4} & m'_{2,6} \\ l_6 & m'_{6,3} & m'_{6,4} & m'_{6,6} \end{array}\,,$$

where

$$m'_{2,3} = m_{2,3}\,,$$
$$m'_{2,4} = m'_{2,6} = m'_{6,4} = m_{2,4}\,,$$
$$m'_{6,6} = \infty\,.$$
$$\Box'_2 = \vee(\Box_2, l_6)\,.$$

The priority of the new place l_6 must be such that $\pi_L(l_4) > \pi_L(l_6) > \pi_L(l_2)$ so that the tokens that entered l_6 on previous steps be transferred before the tokens in l_2 and also $\pi_L(l_3) > \pi_L(l_6)$. The capacity of the new place can be chosen to be equal to that of place l_2, i.e. $c(l_6) = c(l_2)$. In place l_6 the tokens do not obtain new characteristics.

4 A way to include the pairwise capacity in GNs

Up to now we have been studying the effect of the pairwise capacity on the flow of the tokens into the net on transition level. However, it is necessary to see how the pairwise capacitities of the places can be included in the GN's components. Suppose we have a transition (see Fig. 1)

$$Z = \langle L', L'', t_1, t_2, r, M, \Box\rangle$$

of some ordinary GN E. One way to impose a restriction in the sense of pairwise capacity for the pair $\langle l'_p, l''_q\rangle$ is by juxtaposing to Z the transition Z^* (see Fig. 10).

$$Z^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \Box^*\rangle\,.$$

The modified transition Z^* is obtained from Z with the addition of a place l_Z which is both input and output. In the initial time moment a token α_Z stays in this place and it has initial characteristic

$$\text{“}\langle l'_p, l''_q\rangle, n_{p,q}\text{”}\,,$$

where $n_{p,q}$ is the pairwise capacity imposed on the pair $\langle l'_p, l''_q \rangle$. Place l_Z has the lowest priority among the places of the transition. The temporal components t_1 and t_2 remain the same.

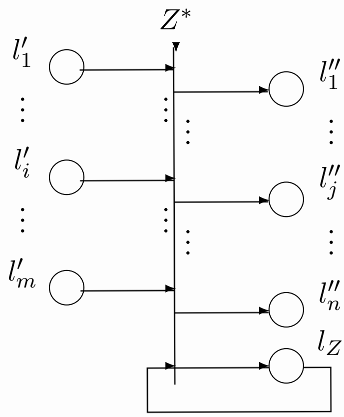

Fig. 10

$$L'^* = L' \cup \{l_Z\},$$
$$L''^* = L'' \cup \{l_Z\},$$
$$\Box^* = \wedge(\Box, l_Z).$$

If

$$r = pr_5 Z = [L', L'', \{r_{l_i,l_j}\}]$$

has the form of an IM, then

$$r^* = pr_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L' \setminus \{l'_p\})(\forall l_j \in L'' \setminus \{l''_q\})(r^*_{l_i,l_j} = r_{l_i,l_j});$$
$$(\forall l_i \in L')(r^*_{l_i,l_q} = r_{l_i,l_q});$$
$$(\forall l_j \in L'')(r^*_{l_p,l_j} = r_{l_p,l_j});$$
$$(\forall l_i \in L')(\forall l_j \in L'')(r^*_{l_i,l_Z} = r^*_{l_Z,l_j} = \text{“}false\text{”});$$
$$r^*_{l_Z,l_Z} = \text{“}true\text{”};$$

$r^*_{l_p,l_q} = r_{l_p,l_q}$ & “the number of tokens in the pair $\langle l_p, l_q \rangle$ is less than $n_{p,q}$”.

If

$$M = pr_6 Z = [L', L'', \{m_{l_i,l_j}\}]$$

has the form of an IM, then

$$M^* = pr_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_j} = m_{l_i,l_j});$$

$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_Z} = m^*_{l_Z,l_j} = 0);$$

$$m^*_{l_Z,l_Z} = 1\,.$$

The α_Z token in place l_Z does not obtain new characteristics during the functioning of the net. The new place l_Z has the lowest priority among the input places of the modified transition. All other components of the net remain the same. If pairwise capacity should be imposed over more than one pair of the transition, the pairwise capacities of all pairs can be given by the initial characteristic of the α_Z token. Suppose that the pairs are $\langle l'_{i,1}, l''_{j,1}\rangle, \langle l'_{i,2}, l''_{j,2}\rangle, ..., \langle l'_{i,k}, l''_{j,k}\rangle$ then the initial characteristic of the α_Z token is a list of all pairs with their corresponding pairwise capacities:

$$\text{“}\langle\langle l'_{i,1}, l''_{j,1}\rangle, n_{i,1,j,1}\rangle, \langle\langle l'_{i,2}, l''_{j,2}\rangle, n_{i,2,j,2}\rangle, ..., \langle\langle l'_{i,k}, l''_{j,k}\rangle, n_{i,k,j,k}\rangle\text{”}\,.$$

The other components of the transition in this case are obtained in a similar way as in the case of one pair.

5 Conclusions

In future we intend to study the problem discussed in this paper in the case when splitting of tokens is allowed. Other types of restrictions for the transfer of tokens from input to output place which are related to the number of tokens in the pair or, more generally, to the total number of tokens in the transition should also be studied. Results in this direction can be applied to the verification of object-oriented programs using GNs (see [3, 4, 5]).

6 Acknowledgments

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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