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Intuitionistic Fuzzy Preference Structures

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Abstract

Here we examine the preference predicate and its decomposition into preference structures from the perspective of Atanassov's Intuitionistic Fuzzy Sets (A-IFS). We focus on the way the meaning/semantics of the preference predicate is measured according to the properties of being desired and of being non-desired, such that some type of uncertainty may exist regarding the way in which both properties are understood. Hence, we explore the existence of not just one but different types of uncertainties, which rise from the particular semantics assigned to the preference predicate.

Keywords: Preference predicate; preference structures; Atanassov's Intuitionistic Fuzzy Sets; incompleteness; hesitation; decision neutrality.

1 Introduction

Under standard preference modeling, a preference structure is defined on a set of alternatives A, decomposing the meaning of the preference predicate into basic and one-dimensional concepts (see e.g., [32]), such as strict preference, indifference or incomparability.

It is well known that using common fuzzy operators [34], these structures can be extended to a fuzzy setting (see e.g., [26]), where the verification of the properties of preference can be verified up to a degree of intensity. A key aspect

of this extension (see e.g., [13]) is the use of logical operators that guarantee the fulfilment of the classical laws of thought [7].
Here we review, first, fuzzy preference structures and the different solutions that can be obtained for the standard model (see e.g., [13], [26], [39]). Then, we explore the standard approach making use of Atanassov's Intuitionistic Fuzzy Sets (A-IFS) [3], where a negative degree is added to the meaning of the preference predicate decomposition, i.e., to the preference structure, in order to measure both its positive and negative properties. Under this intuitionistic setting, some type of *uncertainty* rises due to the semantic relation existing between both *reference poles*, i.e., *meaningful opposites* in the sense of [23], [28].
This component of *uncertainty* (see e.g., [11]) needs careful attention. For example, it may refer to the different epistemic states that appear along the subjective process of decision making. Hence, it is a definite type of *semantic uncertainty* which can be classified under the general concepts of *hesitation* or *decision neutrality*.
Here, *hesitation* (as it has already been used by [33]) refers to the impossibility of identifying one and only one basic situation from the preference structure to properly describe the decision problem; while *decision neutrality* refers to any situation from the preference structure where there is no alternative that is more *desired* or *non desired* than all the others.
Besides, the component of *uncertainty/ignorance* depends explicitly on the structure used for understanding the preference predicate (see e.g., [8], [25]). Then, depending on its particular decomposition, the model will be able to represent different kinds of hesitation and of decision neutrality.
In this paper we review in Section 2, the general definition of *classical* preference structures. Then, in Section 3, we examine fuzzy preference structures, in order to explore, in Section 4, the intuitionistic fuzzy approach. Finally, in Section 5, we propose a general definition for *intuitionistic fuzzy preference structures*, and end up making some concluding remarks.

2 Preference Structures

A preference structure is generally defined on a set of alternatives *A*, by the decomposition of the preference predicate into three basic binary relations (*P, I, J*). In this way, the preferences of the individual can be described for any pair $(a,b) \in A \times A$ by (see e.g., [32]),

- Strict preference (*P*): The pair of alternatives (*a,b*) belongs to the strict preference relation *P* if and only if the individual desires *a* more than *b*.

We denote the inverse strict preference, "*b* is more desired than *a*", as P^{-1}.

- Indifference (*I*): The pair of alternatives (*a,b*) belongs to the indifference relation *I* if and only if the individual desires *a* as much as *b*.
- Incomparability (*J*): The pair of alternatives (*a,b*) belongs to the incomparability relation *J* if and only if the individual cannot compare the alternatives *a* and *b*.

Then, standard preference modeling (see again [32]) assumes that the relations (*P, I, J*) satisfy additional conditions, such as, *I* and *J* are symmetrical, *I* is reflexive, *J* irreflexive, *P* asymmetrical, and

$$P \cap I = \varnothing, \quad \textbf{(P1)}$$

$$P \cap J = \varnothing, \quad \textbf{(P2)}$$

$$J \cap I = \varnothing, \quad \textbf{(P3)}$$

$$P \cup P^{-1} \cup I \cup J = A \times A. \quad \textbf{(P4)}$$

Under this approach, the binary relation $R = P \cup I$ is commonly referred to as the *large/weak preference relation* of the structure $\langle P, I, J \rangle$. Hence, *R* represents the preference predicate whose characteristic property is the one of "being at least as desired as". In this way, the following properties are satisfied,

$$P \cup I = R, \quad \textbf{(R1)}$$

$$P \cup I \cup P^{-1} = R \cup R^{-1}, \quad \textbf{(R2)}$$

$$P \cup J = R^{d}, \quad \textbf{(R3)}$$

where R^{-1} is the inverse of *R* (i.e., $R^{-1}(a,b)=R(b,a)$) and R^{d} is the dual of *R*, i.e., $R^{d} = not\ R^{-1}$.

From P1-P3, it can be seen that for any couple of alternatives, they have to belong to one and only one of the binary relations from the preference structure $\langle P, I, J \rangle$. Besides, there is the completeness condition P4, where it is assumed that every decision problem can be properly described by any of the four basic binary relations of the preference structure.

Hence, under such *crisp* setting, there is no room for *hesitation* [33], i.e., every decision problem has to be described by one unique category from the preference structure. In this way, the individual is assumed to understand his decision under the given preference decomposition, by just one situation of (*P, I, J*).

Regarding *decision neutrality*, we have two types of states. The first one is indifference *I*, where no alternative is better than the other one. Here, the alternatives have equivalently desired attributes, but no *best* decision is available. And the second one is incomparability *J*, where the alternatives cannot be compared either because some kind of *conflict*, *ignorance* or *lack of desired attributes*.

As we have seen, *classical* preference structures are restricted to binary relations that hold or don't hold in an absolute manner. On the contrary, if we allow degrees of intensity in the evaluation of *R*, then strict preference, indifference or incomparability may simultaneously hold, representing hesitation in the individual's decision process. For doing so, we review in the next Section *fuzzy set theory* [18], [40] applied to preference structures.

3 Fuzzy Preference Structures

Standard fuzzy preference models (see e.g. [13], [26], [39]) examine the subjective decision-making process using fuzzy binary relations over a finite set of alternatives *A*. Such models explore preference relations as gradual predicates (in the sense of [18], [40], [36]), where some basic properties can be verified up to a certain degree.

From this standing point, the characterization of a binary fuzzy preference relation is given by

$$R(a,b) = \left\{ \left\langle a,b,\mu_R(a,b) \right\rangle \middle| a,b \in A \right\},$$

where $\mu_R : A \times A \to [0,1]$ is the membership function for the fuzzy relation *R*. Hence, $\mu_R \in [0,1]$ is the membership intensity for every $(a,b) \in A \times A$, according to the verification of the property of "being at least *as desired* as".

Then, the preference structure $\langle P,I,J \rangle$ can be extended to a fuzzy preference structure assuming the existence of the three functions (see e.g., [13], [26])

$$p,i,j : [0,1]^2 \to [0,1],$$

defined as,

$$P = p\left(\mu_R, \mu_{R^{-1}}\right) = T\left(\mu_R, n\left(\mu_{R^{-1}}\right)\right),$$

$$I = i\left(\mu_R, \mu_{R^{-1}}\right) = T\left(\mu_R, \mu_{R^{-1}}\right),$$

$$J = j\left(\mu_R, \mu_{R^{-1}}\right) = T\left(n\left(\mu_R\right), n\left(\mu_{R^{-1}}\right)\right),$$

where T is a continuous t-norm [34], n is a strict negation and i and j are symmetrical functions.
Therefore, the three classical properties (R1)-(R3) can be translated by means of some continuous t-conorm S [34] as,

$$S\left(p\left(\mu_R, \mu_{R^{-1}}\right), i\left(\mu_R, \mu_{R^{-1}}\right)\right) = \mu_R, \tag{1}$$

$$S\left(p\left(\mu_R, \mu_{R^{-1}}\right), i\left(\mu_R, \mu_{R^{-1}}\right), p\left(\mu_{R^{-1}}, \mu_R\right)\right) = S\left(\mu_R, \mu_{R^{-1}}\right), \tag{2}$$

$$S\left(p\left(\mu_{R^{-1}}, \mu_R\right), j\left(\mu_{R^{-1}}, \mu_R\right)\right) = n\left(\mu_R\right). \tag{3}$$

As a consequence, the only solution for $\langle p, i, j\rangle$ that completely fulfills equations (1)-(3) such that i and j are mutually exclusive, is given by (see [13], [26]),

$$p\left(\mu_R, \mu_{R^{-1}}\right) = T^M\left(\mu_R, n\left(\mu_{R^{-1}}\right)\right), \tag{4}$$

$$i\left(\mu_R, \mu_{R^{-1}}\right) = T^L\left(\mu_R, \mu_{R^{-1}}\right), \tag{5}$$

$$j\left(\mu_R, \mu_{R^{-1}}\right) = T^L\left(n\left(\mu_R\right), n\left(\mu_{R^{-1}}\right)\right), \tag{6}$$

where for every $x, y \in [0,1]$, $T^L(x, y) = \max(x + y - 1, 0)$, $T^M(x, y) = \min(x, y)$, $S = S^L = \min(x + y, 1)$, and n is a strong negation. This is, using the Lukasiewicz De Morgan triple $\langle T^L, S^L, n\rangle$ (see again [13]), along with its respective residual given by T^M, the model finds a solution for (1)-(3) such that the completeness condition holds, i.e.,

$$S^L\left(p, p^{-1}, i, j\right) = 1. \tag{7}$$

Hence, the preference structure $\langle P, I, J\rangle$ can be verified up to a degree of intensity, allowing in this way, different types of hesitation, i.e., it allows hesitation between p, p^{-1} and i or j, such that i and j can never hold simultaneously. This mutual exclusion between i and j may be understood as the impossibility of finding two alternatives similarly desired but at the same time incomparable on their desired attributes.

In order to study other types of hesitation, there exist different *limit* solutions for fuzzy preference structures (see e.g., [13], [39]). For example, if instead of defining the function *p* by means of the min *t*-norm T^M, it is defined by means of the Lukasiewicz *t*-norm T^L, and then *i* and *j* are defined using T^M, then *p* is defined as an asymmetrical relation (if $p > 0$ then $p^{-1} = 0$), fulfilling equations (1) and (3), but not (2). In this way, any type of hesitation between *i*, *j* and *p* or p^{-1} can be represented, maintaining the classical asymmetric property of *p*.
Another example of a particular limit solution can be given (see e.g., [39]), which allows verifying any of the different kinds of hesitations between *p*, p^{-1}, *i* and *j*, although the equations (1)-(3) are not satisfied. Such solution uses the probabilistic/multiplicative De Morgan triple $\langle T^p, S^p, n\rangle$, where for any $x, y \in [0,1]$, $T^p = x \cdot y$ and $S^p = x + y - x \cdot y$. This solution is given by,

$$p\left(\mu_R, \mu_{R^{-1}}\right) = T^p\left(\mu_R, n\left(\mu_{R^{-1}}\right)\right), \quad (8)$$

$$i\left(\mu_R, \mu_{R^{-1}}\right) = T^p\left(\mu_R, \mu_{R^{-1}}\right), \quad (9)$$

$$j\left(\mu_R, \mu_{R^{-1}}\right) = T^p\left(n\left(\mu_R\right), n\left(\mu_{R^{-1}}\right)\right). \quad (10)$$

such that

$$S^p\left(p, p^{-1}, i, j\right) = 1. \quad (11)$$

From these different solutions for the fuzzy preference structure $\langle p, i, j\rangle$, we can see that the completeness condition always holds. This is, it is always the case that the decision problem can be certainly described by any of the four basic relations of the preference structure (see e.g., [1]). In the following section we examine a different kind of preference structure, where this completeness condition may not hold.

4 Intuitionistic Fuzzy Preference Structures

Atanassov's intuitionistic fuzzy sets (A-IFS) were introduced in 1983 (see [3]), following the idea that from the *membership* degree $\mu_A(x)$ of a given object *x* to a fuzzy set *A*, it does not necessarily hold that *a* does *not belong* to *A* with an intensity of $1 - \mu_A(x)$. In this way, the standard view of fuzzy sets, where it is implicitly accepted that $1-\mu_A(x)$ is the degree of *non membership* of an object x to A, is challenged by the intuitionistic view.

A-IFS assign to each element of the universe of discourse a degree of membership $\mu_A(x)$ and a degree of non membership $\nu_A(x)$, under the restriction that $\mu_A(x)+\nu_A(x)\leq 1$ should hold [3]. Hence, the implicitly forced duality of standard fuzzy sets, given by $\mu_A(x)+\nu_A(x)=1$, is relaxed by the IFS's inequality restriction, considering separately the positive and negative sides of *inexact* and *gradual* concepts (in the sense of [18], [36]).

Therefore, the characterization of an intuitionistic fuzzy preference relation (initially explored in [16]), is given by

$$R(a,b)=\left\{\left\langle a,b,\mu_R(a,b),\nu_R(a,b)\right\rangle \middle| a,b\in A\right\},$$

where $\mu_R,\nu_R : A\times A\rightarrow[0,1]$ are respectively the membership and non membership functions for the fuzzy relation *R*, where $\mu_R(a,b)+\nu_R(a,b)\leq 1$. Hence, $\mu_R,\nu_R\in[0,1]$ are respectively the membership and non membership intensities for every $(a,b)\in A\times A$, according to the verification of the property of "being at least as *desired* as" for the former, and of "being at least as *non desired* as" for the latter.

Now, the relevance of the negative verification of the preference predicate rests on the meaning/use [36] that the individual assigns to it. As pointed out in [25], the *non membership* degree of A-IFS, or in this case the property of being *non desired*, has some inherent ambiguity regarding the way in which it is directly understood in common language. In this way, making reference to everything that is not desire, the negative predicate can use any type of negation (including the strong one), which determines the particular semantic relation between both positive and negative poles of preference.

Hence, determining such relation between μ_R and ν_R by means of some intuitionistic negation n_i (see e.g., [4]), we have that $\nu_R=n_i(\mu_R)$. Under such intuitionistic environment, and considering the standard fuzzy preference system given by (1)-(3), we have that instead of equation (3), the following condition holds

$$S\left(p\left(\mu_{R^{-1}},\mu_R\right),j\left(\mu_{R^{-1}},\mu_R\right)\right)=\nu_R, \tag{12}$$

such that the completeness equalities of (7) and (11) are replaced by the following inequality (see Proposition 1),

$$S\left(p,p^{-1},i,j\right)\leq 1. \tag{13}$$

Notice that by taking a strict negation for representing the semantic relation between μ_R and ν_R, such that $n_i = n$, we obtain the same formulation of the standard fuzzy preference model given by (1)-(3), where the different limit solutions (4)-(6) and (8)-(10), given in Section 3, hold true if and only if the negation is a strong one [13], [39].

Proposition 1. For any intuitionistic fuzzy preference relation, where $\nu_R = n_i\left(\mu_R\right)$, both completeness conditions (7) and (11), respectively given for the limit solutions (4)-(6) and (8)-(10), do not hold for any *t*-conorm *S*, and they are replaced by the inequality given in (13).

Proof: From [13] we know that the only *t*-conorm *S* that guarantees the completeness condition in (7) is the Lukasiewicz *t*-conorm S^L, and from [39] we know that the only *t*-conorm *S* that guarantees the completeness condition in (11) is the probabilistic *t*-conorm S^P. In both cases, they make use of the fact (initially pointed out in [26]) that $S\left(p,p^{-1},i,j\right)=S\left(\mu_R,n\left(\mu_R\right)\right)=1$, being *n* a strong negation. It is evident that this result does not hold if *n* lacks of the involutive property, like in the case where $n = n_i$, such that $S\left(p,p^{-1},i,j\right)=S\left(\mu_R,n_i\left(\mu_R\right)\right)\leq 1$.

The result of Proposition 1 suggests the importance of exploring the behavior of the standard preference structure $\langle P,I,J\rangle$ under different types of negations (see again [4] for some examples on intuitionistic fuzzy negations). Then, depending on the type of negation used, the opposite poles of preference, represented by μ_R and ν_R, will have a distinctive semantic relation. Only then, the meaning for the property of being *non desired* can be properly understood. This issue has the greatest relevance for us, because it depends on how we understand the negative properties of the predicate that we are modelling, that we choose the appropriate structure for representing its meaning.

In this sense, we understand that a truly and unambiguous fuzzy intuitionistic preference interpretation/semantics of the original membership and non-membership degrees, should allow the separate representation of the opposite properties of being “at least as *desired* as” and “at least as *rejected* as”, respectively (see [15] for a formal approach on this affirmation). Hence, the fuzzy intuitionistic model for preference structures should assign to ν_R the role/meaning of the antonym [36] or antagonist [31] intensity of μ_R, building the meaning of *R* from a dialectical process (in the sense of [21]) between its

opposite reference poles, like e.g., the ones of preference/desire and preference/aversion.
This approach has been explored in the Preference-Aversion (P-A) model [14], [15], where its semantics allow the separate evaluation of the preference/desire and preference/aversion intensities without the A-IFS characterisitc inequality $\mu_R + \nu_R \leq 1$. Therefore, the resulting structure rising from the particular P-A semantics under the A-IFS environment (where $\mu_R + \nu_R \leq 1$ holds) remains to be explored, and stands as an ongoing line of research.

5 The Preference-Aversion Model under Semantic Uncertainty and Ignorance

As we have seen, the intuitionistic fuzzy preference structure decomposes the preference predicate *R* into the opposite properties of being "at least as *desired* as", measured by the *membership* function μ_R, and that of being "at least as *non desired* as", measured by the *non membership* function ν_R, such that $\mu_R + \nu_R \leq 1$. But, as mentioned at the end of the last Section, the property of being "non desired" can be understood in different ways, such that it is not necessarily *dependent* of the property of being "desired".
Hence, one particularly general semantics is the one given by the P-A model, which understands the positive (μ_R) and negative (ν_R) components of preference without assuming any particular relation between them. In this way,

$$R(a,b) = \left\{ \left\langle a,b,\mu_R(a,b),\nu_R(a,b) \right\rangle \middle| a,b \in A \right\}$$

represents the preference predicate where $\mu_R, \nu_R : A \times A \to [0,1]$ are respectively the preference/desire and preference/aversion functions for the fuzzy relation *R*, being $\mu_R, \nu_R \in [0,1]$ the respective intensities for every $(a,b) \in A \times A$.
This approach just stresses the *independent* or *separate* nature of both positive and negative components (representing human perceptions [22], [38], attitudes [9], [23], [28] or stimuli [23], [27]), which can be understood and measured as opposite reference poles of one and the same preference predicate *R*. This independence is crucial for obtaining a general semantic space for *R* (see e.g., [14], [15]), where the cognitive learning of the individual recognizes preference regarding the two opposite attitudes of *desiring* and *rejecting*.
Therefore, the P-A methodology decomposes the preference predicate into independent preference and aversion dimensions, i.e., one partial order for preference/desire and the other for preference/aversion. As a result, we have that

$$R=\langle\mu_R,\nu_R\rangle=\langle(P,I,J),(Z,G,H)\rangle,$$

such that the preference predicate *R* is decomposed into the desire (μ_R) and the aversion (ν_R) intensities, being the former structured as in the standard model (1)-(3), while the latter is analogously structured into *strict aversion Z*, *indifference on aversion G*, and *incomparability on aversion H*. These aversion basic relations can then be defined analogously to the preference/desire ones, such that the functions

$$z,g,h:A\times A\to[0,1]$$

are defined as,

$$Z=z\left(\nu_R,\nu_{R^{-1}}\right)=T\left(\nu_R,n\left(\nu_{R^{-1}}\right)\right),$$

$$G=g\left(\nu_R,\nu_{R^{-1}}\right)=T\left(\nu_R,\nu_{R^{-1}}\right),$$

$$H=h\left(\nu_R,\nu_{R^{-1}}\right)=T\left(n\left(\nu_R\right),n\left(\nu_{R^{-1}}\right)\right),$$

being *g* and *h* symmetrical functions.
Then, the three preference equations from the standard fuzzy system (1)-(3) can be extended to the aversion case, such that

$$S\left(z\left(\nu_R,\nu_{R^{-1}}\right),g\left(\nu_R,\nu_{R^{-1}}\right)\right)=\nu_R, \tag{14}$$

$$S\left(z\left(\nu_R,\nu_{R^{-1}}\right),g\left(\nu_R,\nu_{R^{-1}}\right),z\left(\nu_{R^{-1}},\nu_R\right)\right)=S\left(\nu_R,\nu_{R^{-1}}\right), \tag{15}$$

$$S\left(z\left(\nu_{R^{-1}},\nu_R\right),h\left(\nu_{R^{-1}},\nu_R\right)\right)=n\left(\nu_R\right). \tag{16}$$

In consequence, under equations (1)-(3) and (14)-(16), both preference and aversion dimensions can be conjunctively examined, by means of a continuous aggregation operator such as a *t*-norm *T* (different type of operators remain to be examined where associativity or commutativity are not required [2], [19]), in order to obtain the respective *neutralities* that rise from the *tension* between P-A's opposite reference poles of preference. In this way, the fuzzy P-A structure

$$R_{P-A}=\langle\mu_R,\nu_R\rangle_{P-A}=T\left(\langle p,i,j\rangle,\langle z,g,h\rangle\right)=\langle pa,pz,pg,ph,iz,ig,ih,jz,jg,jh\rangle,$$

is obtained, representing the epistemic states of the individual towards decision, describing his preferences for any pair $(a,b)\in A\times A$ by,

- Strong preference: The pair of alternatives (*a,b*) belongs to the *strong preference* relation *PA* with an intensity of $pa = T\left(p, z^{-1}\right)$, if and only if the individual desires *a* more than *b* and at the same time rejects *b* more than *a*.
- Ambivalence: The pair of alternatives (*a,b*) belongs to the *ambivalence* relation *PZ* with an intensity of $pz = T\left(p, z\right)$, if and only if the individual desires *a* more than *b* and at the same time rejects *a* more than *b*.
- Pseudo-preference: The pair of alternatives (*a,b*) belongs to the *pseudo-preference* relation *PG* with an intensity of $pg = T\left(p, g\right)$, if and only if the individual desires *a* more than *b* and at the same time rejects *a* just as *b*.
- Semi-strong preference: The pair of alternatives (*a,b*) belongs to the *semi-strong preference* relation *PH* with an intensity of $ph = T\left(p, h\right)$, if and only if the individual desires *a* more than *b* and at the same time he cannot compare the negative attributes of *a* and *b*.
- Pseudo-aversion: The pair of alternatives (*a,b*) belongs to the *pseudo-aversion* relation *IZ* with an intensity of $iz = T\left(i, z\right)$, if and only if the individual desires *a* as much as *b* and at the same time rejects *a* more than *b*.
- Strong indifference: The pair of alternatives (*a,b*) belongs to the *strong indifference* relation *IG* with an intensity of $ig = T\left(i, g\right)$, if and only if the individual desires *a* as much as *b* and at the same time rejects *a* as much as *b*.
- Positive indifference: The pair of alternatives (*a,b*) belongs to the *positive indifference* relation *IH* with an intensity of $ih = T\left(i, h\right)$, if and only if the individual desires *a* as much as *b* and at the same time he cannot compare the negative attributes of *a* and *b*.
- Semi-strong aversion: The pair of alternatives (*a,b*) belongs to the *semi-strong aversion* relation *JZ* with an intensity of $jz = T\left(j, z\right)$, if and only if the individual cannot compare the positive attributes of *a* and *b* and at the same time he rejects *a* more than *b*.
- Negative indifference: The pair of alternatives (*a,b*) belongs to the *negative indifference* relation *JG* with an intensity of $jg = T\left(j, g\right)$, if

and only if the individual cannot compare the positive attributes of *a* and *b* and at the same time he rejects *a* as much as *b*.

- Strong incomparability: The pair of alternatives (*a,b*) belongs to the *strong incomparability* relation *JH* with an intensity of $jh = T(j,h)$, if and only if the individual cannot compare neither the positive or the negative attributes of *a* and *b*.

In summary, under the P-A model's semantics (dropping the A-IFS semantic restriction between its opposite reference poles μ_R and ν_R), different types of *decision neutrality* appear. Hence, the different types of neutralities that can be represented, which allow describing the decision problem, depend on the *semantic relation* between such *meaningful opposites* [23], [28].
In particular, under the P-A representation of *R*, which is decomposed into the meaningful opposites of *desire* and *aversion*, we find *nine* distinct types for *decision neutralitiy*. In this way, in between strong preference and its inverse (which notice that can also be labeled as *strong aversion*), we may find *ambivalence*, *strong/positive/negative indifference*, *semi-strong preference/aversion*, *pseudo preference/aversion* or *strong incomparability*.
Besides, under the fuzzy environment in which it is formulated, the P-A model is also able to represent different types of *hesitation* on distinct levels. On a first level, it allows *hesitation* between the preference/desire components and also, between the preference/aversion ones; and on a second level, between the sixteen different components of the P-A structure.
Regarding *ignorance* [8], we have to make some previous considerations. In general, examining the solutions for (1), (3) and the respective ones for (14), (16), we know that $R = \langle \mu_R, \nu_R \rangle = \langle (p,i,j),(z,g,h) \rangle$ obtain lower and upper bounds given by,

$$T^L\left(\mu_R, n\left(\mu_{R^{-1}}\right)\right) \le p\left(\mu_R, \mu_{R^{-1}}\right) \le T^M\left(\mu_R, n\left(\mu_{R^{-1}}\right)\right),$$

$$T^L\left(\mu_R, \mu_{R^{-1}}\right) \le i\left(\mu_R, \mu_{R^{-1}}\right) \le T^M\left(\mu_R, \mu_{R^{-1}}\right),$$

$$T^L\left(n\left(\mu_R\right), n\left(\mu_{R^{-1}}\right)\right) \le j\left(\mu_R, \mu_{R^{-1}}\right) \le T^M\left(n\left(\mu_R\right), n\left(\mu_{R^{-1}}\right)\right),$$

$$T^L\left(\nu_R, n\left(\nu_{R^{-1}}\right)\right) \le z\left(\nu_R, \nu_{R^{-1}}\right) \le T^M\left(\nu_R, n\left(\nu_{R^{-1}}\right)\right),$$

$$T^L\left(\nu_R, \nu_{R^{-1}}\right) \le g\left(\nu_R, \nu_{R^{-1}}\right) \le T^M\left(\nu_R, \nu_{R^{-1}}\right),$$

$$T^L\left(n\left(\nu_R\right), n\left(\nu_{R^{-1}}\right)\right) \le h\left(\nu_R, \nu_{R^{-1}}\right) \le T^M\left(n\left(\nu_R\right), n\left(\nu_{R^{-1}}\right)\right).$$

Then, treating such lower and upper bounds as intervals, we may examine also interval valued fuzzy sets (IVFS) [20], where an interval in $[0,1]$ is assigned instead of one unique membership value. Hence, it is not required to give one unique degree, allowing some vagueness over its precise specification. In this way, we have that the intervals for $\langle p,i,j\rangle$, under the standard model (1)-(3) can be represented as,

$$p\left(\mu_R,\mu_{R^{-1}}\right)=\left[p^L,p^U\right]=\left[T^L\left(\mu_R,n\left(\mu_{R^{-1}}\right)\right),T^M\left(\mu_R,n\left(\mu_{R^{-1}}\right)\right)\right],$$

$$i\left(\mu_R,\mu_{R^{-1}}\right)=\left[i^L,i^U\right]=\left[T^L\left(\mu_R,\mu_{R^{-1}}\right),T^M\left(\mu_R,\mu_{R^{-1}}\right)\right],$$

$$j\left(\mu_R,\mu_{R^{-1}}\right)=\left[j^L,j^U\right]=\left[T^L\left(n\left(\mu_R\right),n\left(\mu_{R^{-1}}\right)\right),T^M\left(n\left(\mu_R\right),n\left(\mu_{R^{-1}}\right)\right)\right],$$

and in an analogous way, we have that the intervals for $\langle z,g,h\rangle$, under the system of equations (14)-(16), can be represented as,

$$z\left(\nu_R,\nu_{R^{-1}}\right)=\left[z^L,z^U\right]=\left[T^L\left(\nu_R,n\left(\nu_{R^{-1}}\right)\right),T^M\left(\nu_R,n\left(\nu_{R^{-1}}\right)\right)\right],$$

$$g\left(\nu_R,\nu_{R^{-1}}\right)=\left[g^L,g^U\right]=\left[T^L\left(\nu_R,\nu_{R^{-1}}\right),T^M\left(\nu_R,\nu_{R^{-1}}\right)\right],$$

$$h\left(\nu_R,\nu_{R^{-1}}\right)=\left[h^L,h^U\right]=\left[T^L\left(n\left(\nu_R\right),n\left(\nu_{R^{-1}}\right)\right),T^M\left(n\left(\nu_R\right),n\left(\nu_{R^{-1}}\right)\right)\right],$$

where the superscripts *L* and *U* denote respectively the lower and upper bounds for $\langle(p,i,j),(z,g,h)\rangle$.

In consequence, we can recover the respective estimations for μ_R and ν_R using a *t*-conorm S^{IV}, such that for every $x,y,z,w\in[0,1]$, $S^{IV}([x,y],[z,w])=[S(x,z),S(y,w)]$, where it respectively follows from (1) and (14) that

$$\mu_R=\left[\mu_R^L,\mu_R^U\right]=\left[S\left(p^L,i^L\right),S\left(p^U,i^U\right)\right],$$

and

$$\nu_R=\left[\nu_R^L,\nu_R^U\right]=\left[S\left(z^L,g^L\right),S\left(z^U,g^U\right)\right].$$

Notice that the respective values for $\left[\mu_R^L,\mu_R^U\right]$ and $\left[\nu_R^L,\nu_R^U\right]$ can be directly measured/elicited by/from the individual, assigning an explicit interval valuation for the verification of the preference predicate *R*. In this way, the length of the intervals $\left[\mu_R^L,\mu_R^U\right]$ and $\left[\nu_R^L,\nu_R^U\right]$ refers to the *ignorance* [6], [8], [17] associated to the pair of alternatives (under the P-A decomposition of *R*).
In conclusion, the P-A model's semantics allows representing different situations for decision neutrality, hesitation and ignorance. Such situations are the result of recognizing the existence of two separate and independent poles configuring the meaning of the preference predicate.

Remark 1. It has been shown in [5], [10], that A-IFS are formally/syntactically equivalent to IVFS, once the couple $(\mu_R,\nu_R)\in[0,1]$ is mapped into the interval $[\mu_R,1-\nu_R]$. This is, taking the *ignorance* in $\left[\mu_R^L,\mu_R^U\right]$, defined as (see e.g., [8])

$$\varphi_R=\mu_R^U-\mu_R^L,$$

and the A-IFS *uncertainty degree* (see e.g., [11]), given by,

$$\pi_R=1-\mu_R-\nu_R,$$

we obtain that if $\mu_R^L=\mu_R$ and $1-\nu_R=\mu_R^U$, then

$$\varphi_R=\mu_R^U-\mu_R^L=1-\nu_R-\mu_R=\pi_R.$$

Nonetheless, such syntactic equivalence is dependent of the condition that the upper bound μ_R^U (also *plausibility* of *R* [35]) of membership to *R* is at the same time the strong negation of the non membership degree, i.e., everything that is not ν_R is at most μ_R. In other words, it forces the particular interpretation of $\nu_R=1-\mu_R^U$, which is not purely syntactic, but also semantic. It is evident that the implicit A-IFS/IVFS's semantic interpretation of $\nu_R=1-\mu_R^U$ does not necessarily hold.

6 Final Comments

We have explored A-IFS and fuzzy preference structures, paying special attention to the semantic relation that exists between the membership and non membership components of the A-IFS structure. As it has been explored in [10], but also in [30], we stress that the formal/syntactic equivalence between A-IFS and IVFS comes from a particular semantic interpretation, which has to be explicitly addressed in order to avoid confusion. In this sense, the *dichotomy*

between syntactical and semantic aspects, i.e., of *form* and *content* in modelling, should be avoided, due to the close and interacting relationship between both.
It remains to explore in more detail the intuitionistic model for fuzzy preference structures, where the semantic relation between the meaningful opposites of preference can be represented by two separate measures μ_R and ν_R, but related under the inequality condition $\mu_R + \nu_R \leq 1$. In this way, different types of semantic relations between the opposite poles of the preference predicate can be examined, like for example, through the use of distinct types of intuitonistic fuzzy negations [4], or by their specific relation of being *antonyms* [36] or *antagonists* [31].

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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